Volume and surface area

G12, G16, G17 · Geometry and measures · revision notes

Volume and surface area of cuboids, prisms, cylinders, pyramids, cones and spheres, and the faces, edges and vertices of 3D shapes. Questions often add a context such as filling a tank or working out a cost. The cone, sphere and pyramid formulas are given in the question when you need them: they are not on the formulae sheet.

Key facts

- Prism: volume = area of cross-section × length
- Pyramid: V = ⅓ × area of base × perpendicular height
- Sphere: V = $\frac{4}{3}\pi r^3$; surface area = 4πr² (given in the question)
- 1 cm³ = 1 ml; 1 litre = 1000 cm³; 1 m³ = 1 000 000 cm³
- Cylinder: V = πr²h; curved surface area = 2πrh
- Cone: V = ⅓πr²h; curved surface area = πrl (given in the question)
- Cone: l² = r² + h²

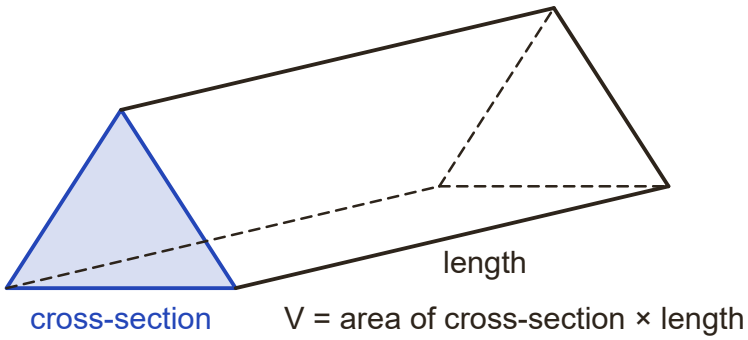
Notes

3D shapes

- **Faces** are the flat surfaces, **edges** are where two faces meet and **vertices** are the corners.
- Cuboid: 6 faces, 12 edges, 8 vertices. Triangular prism: 5 faces, 9 edges, 6 vertices. Square-based pyramid: 5 faces, 8 edges, 5 vertices.
- Curved solids: a cylinder has 2 flat circular faces and 1 curved surface; a cone has 1 flat circular face, 1 curved surface and 1 vertex (the apex); a sphere has 1 curved surface.
- A **prism** has the same cross-section all the way through. A cylinder is like a prism with a circular cross-section.

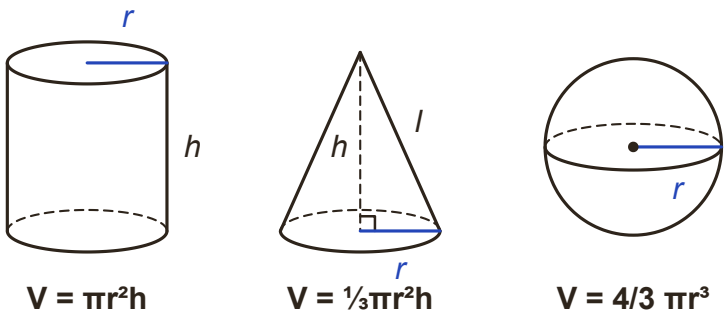
Volume

- Cuboid: V = length × width × height.
- Prism: V = area of cross-section × length. Cylinder: $V = \pi r^2 h$



The cross-section is the same all the way through the prism.

- Pyramid (any base): $V = \frac{1}{3} \times \text{area of base} \times h$, where h is the perpendicular height.
- Cone: $V = \frac{1}{3}\pi r^2 h$. Sphere: $V = \frac{4}{3}\pi r^3$. A hemisphere is half a sphere.



Cylinder, cone and sphere: h is the perpendicular height; l is the cone's slant height.

- Volume units are cubed. 1 cm³ = 1 ml, 1 litre = 1000 cm³ and 1 m³ = 1 000 000 cm³.

Surface area

- Surface area is the total area of all the faces. List the faces (or sketch a net) so none is missed.
- Cylinder: curved surface = $2\pi rh$ (it unrolls into a rectangle); total = $2\pi rh + 2\pi r^2$
- Cone: curved surface = πrl, where l is the slant height. Add πr^2 for the base.
- Sphere: surface area = $4\pi r^2$. A solid hemisphere has total surface area $2\pi r^2 + \pi r^2 = 3\pi r^2$

Composite solids

- Find the volume of each part and add. For surface area, leave out any faces that are joined together.
- The slant height l, radius r and perpendicular height h of a cone are linked by Pythagoras: $l^2 = r^2 + h^2$
- A **frustum** is a cone with its top cut off parallel to the base: volume = large cone − small cone. The two cones are similar, so use ratios to find a missing radius or height. Grade 7+ only

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Volume and surface area of a prism

2 to 3 marks each | **Grade 4**

1. Find the area of the cross-section (the end face).
2. Volume: multiply by the length.
3. Surface area: add the two end faces and every rectangular face (each rectangle is one edge of the cross-section × the length).

Example. A prism has a cross-section that is a right-angled triangle with base 6 cm and height 8 cm. The prism is 20 cm long.
(a) Work out the volume of the prism. (2 marks)
(b) Work out the total surface area of the prism. (3 marks)

Model answer.

(a) Cross-section = ½ × 6 × 8 = 24 cm²; volume = 24 × 20 (M1) = 480 cm³ (A1)

(b) Third side of the triangle = $\sqrt{(6^2 + 8^2)}$ = 10 cm (M1)

Area = 2 × 24 + 6 × 20 + 8 × 20 + 10 × 20 (M1) = 48 + 480 = 528 cm² (A1)

Cylinder in context (capacity and filling)

4 marks | **Grade 5**

1. Work out the volume in cm³ (all lengths in cm).
2. Convert to litres: divide by 1000.
3. Use the rate or price given to answer the actual question.

Example. A water tank is a cylinder with radius 40 cm and height 90 cm. The empty tank is filled with water at a rate of 12 litres per minute.
How long does it take to fill the tank? Give your answer to the nearest tenth of a minute.

Model answer.

Volume = π × 40² × 90 (M1) = 452 389 cm³ (A1)

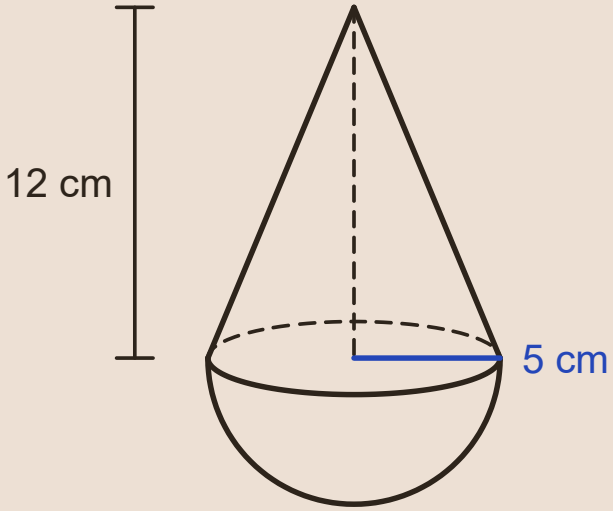
452 389 ÷ 1000 = 452.389 litres, and 452.389 ÷ 12 (M1)

= 37.7 minutes (A1)

Cones, spheres and composite solids (formula given) 3 marks each **Grade 6**

1. Copy the formula from the question and substitute.
2. Find each part separately (halve the sphere for a hemisphere).
3. Add the parts; leave π in if the answer is wanted in terms of π.

Example. A solid is made from a cone on top of a hemisphere. The cone and the hemisphere both have radius 5 cm. The perpendicular height of the cone is 12 cm.



[Volume of a cone = ⅓πr²h, curved surface area of a cone = πrl, volume of a sphere = $\frac{4}{3}\pi r^3$, surface area of a sphere = 4πr²]

(a) Work out the volume of the solid. Give your answer in terms of π. (3 marks)

(b) Work out the total surface area of the solid. Give your answer in terms of π. (3 marks)

Model answer.

(a) Cone = ⅓ × π × 5² × 12 = 100π (M1)

Hemisphere = ½ × $\frac{4}{3}$ × π × 5³ = $\frac{250\pi}{3}$ (M1)

Total = $\frac{550\pi}{3}$ cm³ (A1)

(b) Slant height l = √(5² + 12²) = 13 cm (M1)

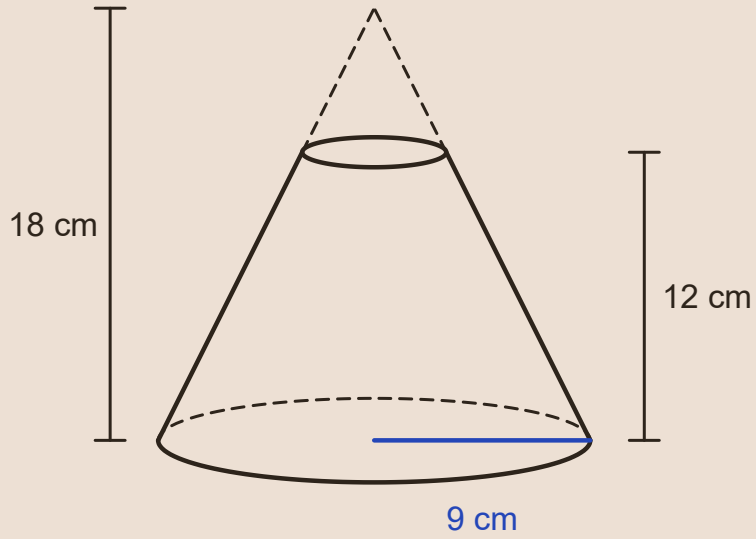
π × 5 × 13 + ½ × 4 × π × 5² (M1) = 65π + 50π = 115π cm² (A1)

Volume of a frustum

4 marks | Grade 7

1. Use similar triangles to find any missing radius or height of the small cone.
2. Work out the volume of the large cone and of the small cone.
3. Subtract.

Example. A solid cone has radius 9 cm and perpendicular height 18 cm. The top of the cone is cut off, parallel to the base, 12 cm above the base, leaving a frustum.



Work out the volume of the frustum. Give your answer in terms of π. [Volume of a cone = ⅓πr²h]

Model answer.

The small cone has height 18 − 12 = 6 cm, so its radius = 9 × 6 ÷ 18 = 3 cm	(M1)
Large cone = ⅓ × π × 9² × 18 = 486π	(M1)
Small cone = ⅓ × π × 3² × 6 = 18π	(M1)
Frustum = 486π − 18π = 468π cm³	(A1)

Don't lose marks

- Using the slant height instead of the perpendicular height in the volume of a cone or pyramid.
- Forgetting the ⅓ for cones and pyramids.
- Finding only the curved surface when the total surface area is asked for, or including faces that are joined together.
- Mixing units, e.g. a radius in cm with a height in m.
- Dividing by 100 instead of 1000 to change cm³ to litres.

More tips

Memory tricks

- The prism rule works for any prism: area of the end face × length.
- Check faces, edges and vertices with F + V − E = 2 (cuboid: 6 + 8 − 12 = 2).
- Look for Pythagorean triples in cones: 3-4-5 or 5-12-13 often link r, h and l.
- A 10 cm cube holds 1000 cm³, which is exactly 1 litre; use this to sense check capacities.

Exam technique

- Copy any formula given in the question and substitute your values into it clearly.
- 'In terms of π' means leave π in and keep fractions exact, e.g. 550π/3.
- In filling or cost problems, make sure your final answer is what was asked (time, number of tins, cost), not just the volume.
- Give units: cm³ for volume and cm² for surface area.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 2 — **Count faces, edges and vertices**
e.g. a cuboid has 6 faces, 12 edges and 8 vertices.

Grade 3 — **Find the volume of a cuboid**
Volume = length × width × height (or count the centimetre cubes).

Grade 4 — **Find volumes of prisms and cylinders**
Volume = area of cross-section × length; for a cylinder, $V = \pi r^2 h$.

Grade 4 — **Find the surface area of a prism**
Add the areas of all the faces; sketch a net so you do not miss any.

Grade 5 — **Find the surface area of a cylinder**
Two circles plus the curved surface: $2\pi r^2 + 2\pi rh$.

Grade 5 — **Use formulas for pyramids, cones and spheres**
Substitute into the formula, e.g. volume of a cone = $\frac{1}{3}\pi r^2 h$.

Grade 6 — **Solve problems with composite solids**
e.g. a cone on a hemisphere, or finding a length when the volume is given.

Grade 7 — **Find the volume of a frustum**
Volume of the large cone minus the small cone, using similar triangles for any missing lengths.

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