Venn diagrams and set notation

P6, P9 · Probability · revision notes

Venn diagrams sort items into sets and show how the sets overlap, and set notation (ξ, $\cap$, $\cup$, A') describes the regions. On both tiers you complete Venn diagrams from lists or totals and read probabilities from them. Harder questions use three sets or algebra, and on Higher you also find conditional probabilities ('given that') from a Venn diagram.

Key facts

- ξ = the universal set (everything, the rectangle)
- $A \cup B$ = in A or B or both
- $(A \cup B)'$ = in neither A nor B
- P(region) = number in the region ÷ total in ξ
- $A \cap B$ = in A and in B (the overlap)
- A' = not in A
- A only = total in A − number in $A \cap B$
- Grade 7+ only P(B given A) = number in $A \cap B$ ÷ number in A

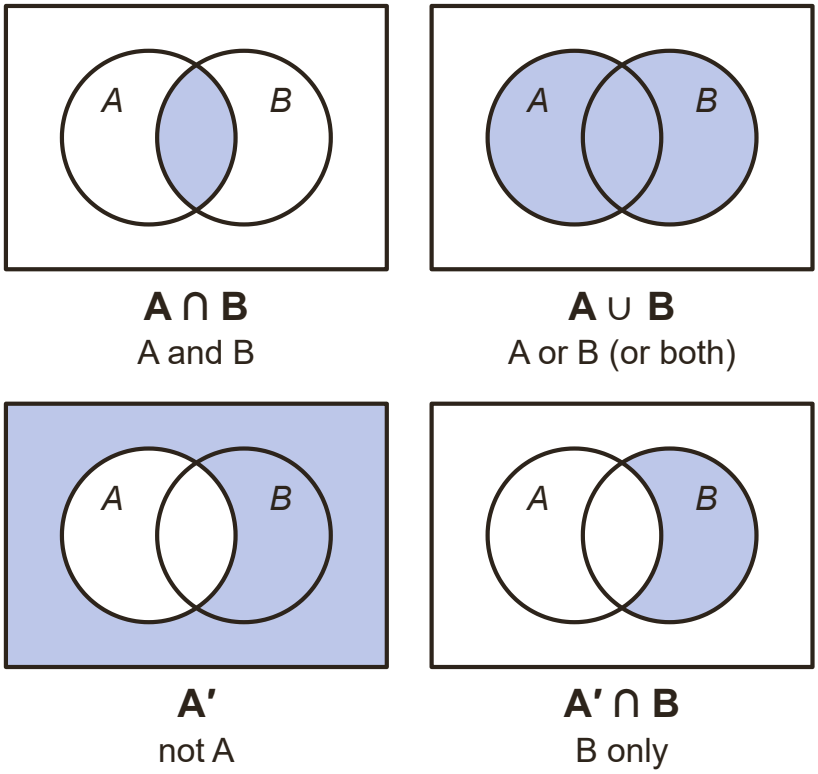
Notes

Sets and Venn diagrams

- A **set** is a collection of items (elements), listed in curly brackets, e.g. $A = \{2, 4, 6, 8\}$
- The **universal set** ξ is everything being considered. It is the rectangle round the diagram.
- Each circle is one set. Items in both sets go in the overlap; items in neither set go inside the rectangle but outside the circles.
- Every item goes in exactly one region. A Venn diagram can show the items themselves or just how many items are in each region.

Set notation

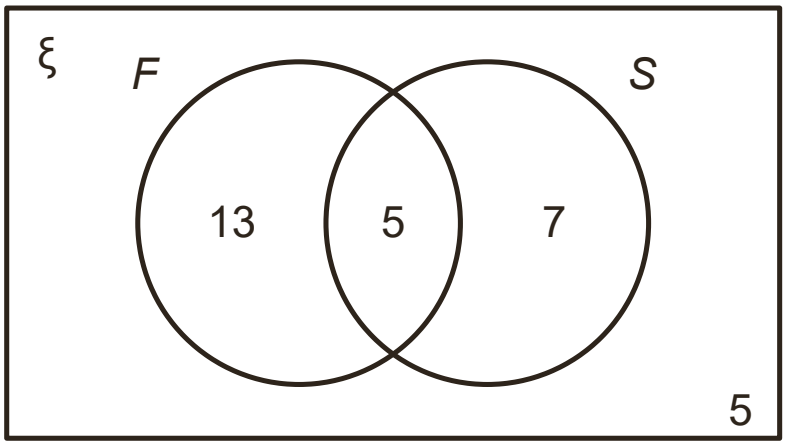
- $A \cap B$ (intersection): in A **and** in B. This is the overlap.
- $A \cup B$ (union): in A **or** B or both. This is everything inside the circles.
- A' (complement): **not** in A. This includes the region outside both circles.
- Combinations: $A' \cap B$ is in B but not in A ('B only'), and $(A \cup B)'$ is outside both circles.



The shaded region is the set named under each diagram.

Filling in and finding probabilities

- Start with the overlap. Then 'A only' = total in A − overlap, and the same for B. Then outside = grand total − everything in the circles.
- Example: of 30 students, 18 study French, 12 study Spanish and 5 study both. French only = 13, Spanish only = 7, neither = 30 − 13 − 5 − 7 = 5.

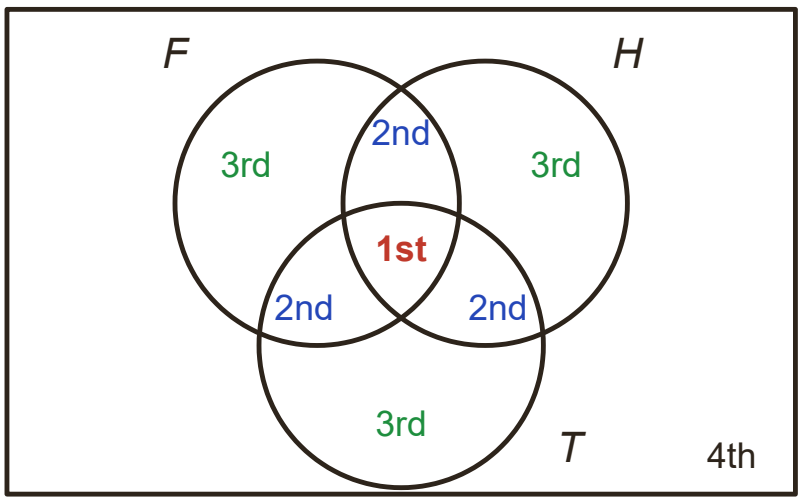


F = French, S = Spanish. The 5 outside the circles study neither.

- P(region) = number in the region ÷ total number in ξ. Here P(studies French or Spanish) = $\frac{25}{30} = \frac{5}{6}$
- Check: all the numbers, including the one outside the circles, add up to the total.

Three sets and algebra Grade 6+ only

- Three overlapping circles make 8 regions, including the outside.
- Fill from the centre outwards: in all three first, then each 'exactly two' region, then each 'only' region, then the outside.



Fill in this order: 1st the centre, 2nd the 'exactly two' regions, 3rd the 'only' regions, 4th outside.

- A statement like '9 play football and tennis' includes anyone who plays all three, so subtract the centre to get 'football and tennis only'.
- If a region is unknown, call it x, write the other regions in terms of x, add them all to make the total and solve. Grade 7+ only

Conditional probability from a Venn diagram Grade 7+ only

- 'Given that the student studies French' means you only look inside the French circle: the denominator becomes the number in that circle.
- In the French and Spanish example, P(studies Spanish given that they study French) = $\frac{5}{18}$: the overlap ÷ the whole French circle.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

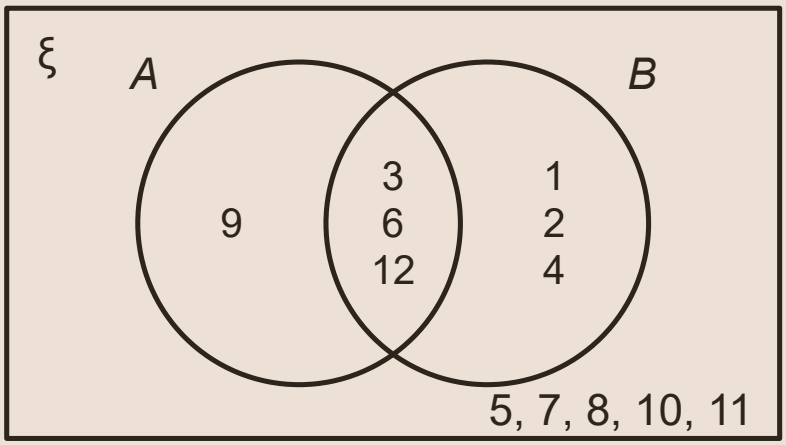
Complete a Venn diagram from lists and use set notation

4 to 5 marks | **Grade 4**

1. List the members of each set first.
2. Put members of both sets in the overlap, then the rest of each set, then everything else in ξ outside the circles.
3. Tick off each member of ξ as you place it, so nothing is missed or written twice.
4. For a probability, count the members in the region and divide by the number of members of ξ

Example. ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
A = {multiples of 3}
B = {factors of 12}
(a) Draw a Venn diagram to show this information. (2 marks)
(b) List the members of $A \cap B$. (1 mark)
(c) A number is chosen at random from ξ. Work out the probability that the number is in $A \cup B$. (2 marks)

Model answer.



(a) A = {3, 6, 9, 12} and B = {1, 2, 3, 4, 6, 12}.
Overlap: 3, 6, 12. A only: 9. B only: 1, 2, 4. Outside: 5, 7, 8, 10, 11 (B2 for a fully correct diagram, B1 if one or two numbers are misplaced)
(b) 3, 6, 12 (B1)
(c) $A \cup B$ = {1, 2, 3, 4, 6, 9, 12}, which is 7 numbers (M1), so the probability is $\frac{7}{12}$ (A1)

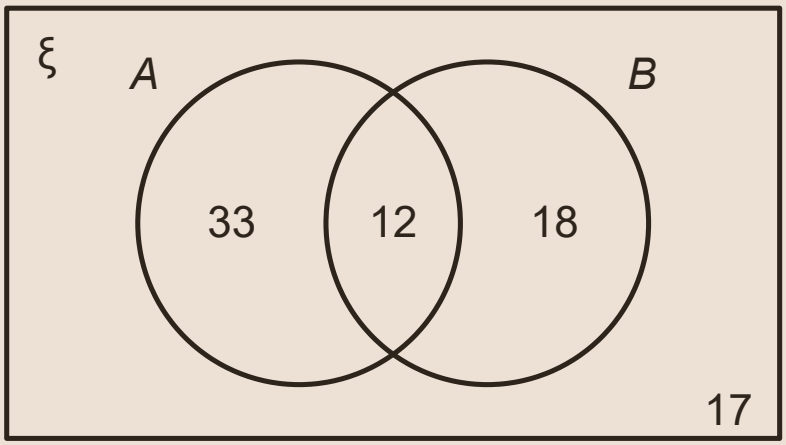
Complete a Venn diagram from totals and find a probability

4 to 5 marks | Grade 5

1. Write the 'both' number in the overlap first.
2. Subtract it from each set's total to get the 'only' regions.
3. Subtract everything in the circles from the grand total to get the number outside.
4. Work out which region the notation describes (shade it if that helps), then divide by the grand total.

Example. 80 people at a gym were asked whether they use the pool and whether they use the sauna. 45 use the pool, 30 use the sauna and 12 use both. A is the set of people who use the pool and B is the set of people who use the sauna.
(a) Draw a Venn diagram to show this information. (3 marks)
(b) One of the 80 people is chosen at random. Work out the probability that this person is in $A' \cap B$. (2 marks)

Model answer.



(a) Overlap: 12 (B1)

Pool only: 45 − 12 = 33 and sauna only: 30 − 12 = 18 (B1)

Outside: 80 − 33 − 12 − 18 = 17 (B1)

(b) $A' \cap B$ is 'sauna but not pool', which is 18 people (M1), so the probability is $\frac{18}{80} = \frac{9}{40}$ (A1)

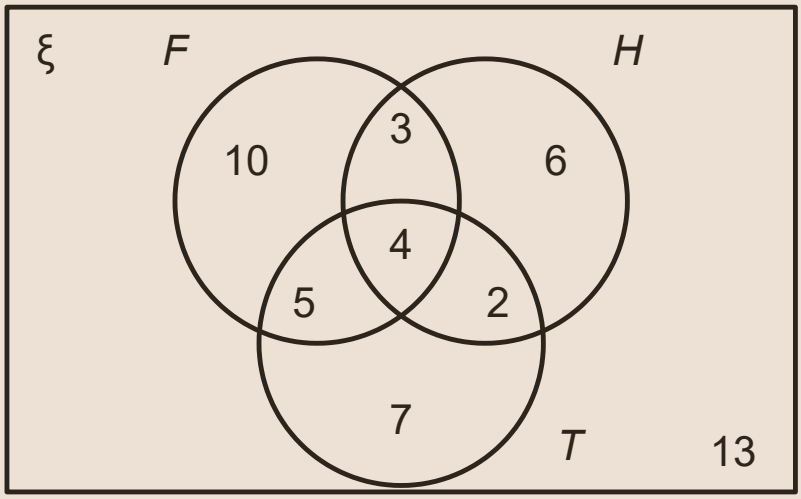
Three-set Venn diagram

4 to 5 marks | **Grade 6**

1. Put the 'all three' number in the centre.
2. For each pair, subtract the centre: '7 play football and hockey' includes the people who play all three.
3. For each 'only' region, subtract every number already in that circle from the circle's total.
4. Subtract the total inside the circles from the grand total for the outside region.

Example. 50 students were asked which of three sports they play: football (F), hockey (H) and tennis (T).
4 play all three sports. 7 play football and hockey. 6 play hockey and tennis. 9 play football and tennis. 22 play football, 15 play hockey and 18 play tennis.
(a) Draw a Venn diagram to show this information. (3 marks)
(b) A student is chosen at random. Work out the probability that this student plays exactly one of the sports. (2 marks)

Model answer.



(a) Centre: 4. Football and hockey only: 7 − 4 = 3. Hockey and tennis only: 6 − 4 = 2. Football and tennis only: 9 − 4 = 5 (B1)

Football only: 22 − 4 − 3 − 5 = 10. Hockey only: 15 − 4 − 3 − 2 = 6. Tennis only: 18 − 4 − 2 − 5 = 7 (B1)

Outside: 50 − (4 + 3 + 2 + 5 + 10 + 6 + 7) = 50 − 37 = 13 (B1)

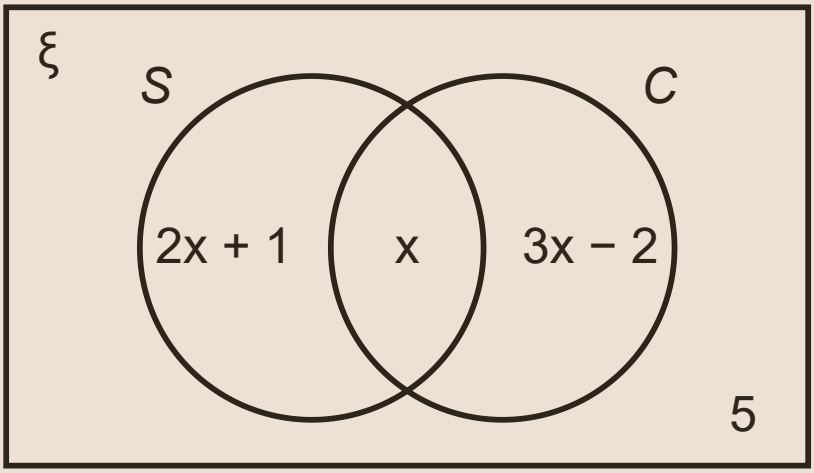
(b) Exactly one sport: 10 + 6 + 7 = 23 (M1), so the probability is $\frac{23}{50}$ (A1)

Venn diagram with algebra

4 to 5 marks | **Grade 7**

1. Add every region, including the outside, and set the sum equal to the total.
2. Collect like terms and solve for x
3. Substitute x into each region and check that they add up to the total.
4. Use the numbers, not the expressions, to find the probability.

Example. There are 40 people in a club. The Venn diagram shows how many of them can swim (S) and how many can cycle (C).



(a) Work out the value of x. (3 marks)
(b) A person is chosen at random. Work out the probability that this person can swim. (2 marks)

Model answer.

(a) $(2x + 1) + x + (3x - 2) + 5 = 40$ (M1)

$6x + 4 = 40$, so $6x = 36$ (M1)

$x = 6$ (A1)

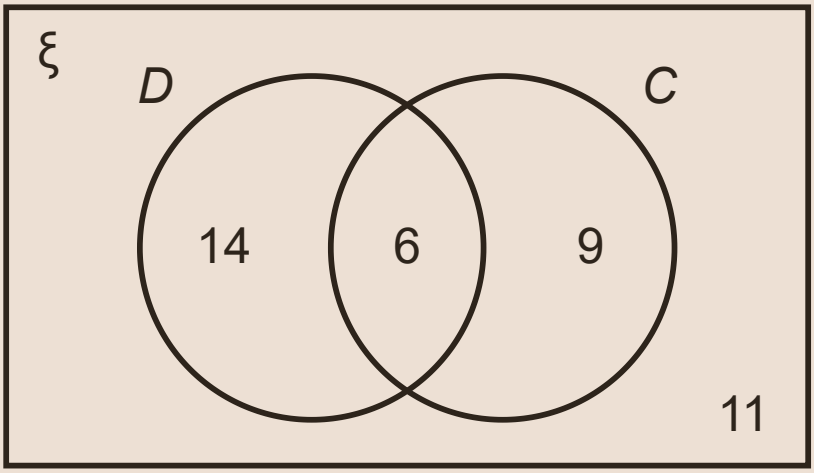
(b) S only = 13 and both = 6, so 13 + 6 = 19 people can swim (M1). The probability is $\frac{19}{40}$ (A1)

'Given that …' from a Venn diagram

2 marks **Grade 7**

1. Find the given set and add up every number inside it: this is the denominator.
2. Find the part of that set that is also in the event you want: this is the numerator.
3. Write the fraction. Do not divide by the grand total.

Example. The Venn diagram shows information about the pets of 40 students. D is the set of students who have a dog and C is the set of students who have a cat.



One of the students is chosen at random. Given that this student has a cat, work out the probability that the student also has a dog.

Model answer.

Students with a cat: 6 + 9 = 15 (M1)

P(dog given cat) = $\frac{6}{15} = \frac{2}{5}$ (A1)

Don't lose marks

- Putting the whole total for A in the 'A only' region, so the overlap is counted twice.
- Forgetting the number outside the circles when you find the total, or leaving that region blank.
- Mixing up $\cap$ (and: the overlap) with $\cup$ (or: everything in the circles).
- Thinking A' is just 'B only': it also includes everything outside both circles.
- Writing how many members a set has when the question says 'list the members'.
- Higher: dividing by the grand total in a 'given that' question instead of by the number in the given set.

More tips

Memory tricks

- ∪ looks like a U, for Union: everything in either set. ∩ is the other one: only the overlap, in both.
- Work from the inside out: overlap first, then the 'only' regions, then the outside.
- Shade the region the notation describes before you count. It stops you mixing up ∩ and ∪
- Check that every number in the diagram, including the outside, adds up to the total.

Exam technique

- When you complete a Venn diagram of numbers, fill every region, including the outside. Write 0 if a region is empty.
- 'List the members' means write the actual elements, each one once.
- For a probability, the denominator is the total in ξ, unless the question says 'given that' (Higher), when it is the number in the given set.
- In maths, 'or' includes both: 'plays football or tennis' means $F \cup T$, which includes the people who play both.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 3

Sort items into a two-set Venn diagram

Put items in both sets in the overlap, the rest of each set in its own circle, and items in neither set outside the circles.

Grade 4

Understand ξ, intersection and union

ξ is everything, $A \cap B$ is the overlap, and $A \cup B$ is everything inside either circle.

Grade 4

Find a probability from a Venn diagram

Divide the number in the region you want by the total in ξ, including anything outside the circles.

Grade 5

Complete a Venn diagram from given totals

Start with the overlap, subtract it from each set's total to get the 'only' regions, then work out the number outside.

Grade 5

Use complements such as A′ in set notation

A' is everything not in A, so $A' \cap B$ is the part of B outside A

Grade 6

Complete a three-set Venn diagram

Fill the centre (in all three sets) first, then the regions in exactly two sets, then each 'only' region and the outside.

Grade 7

Use algebra to find missing Venn numbers

Write each region in terms of x, add them all to make the total and solve the equation.

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