Vectors

G25 · Geometry and measures · revision notes

A vector has size and direction. You add, subtract and multiply column vectors, draw and use them on a grid, and on Higher write routes through a diagram in terms of vectors such as a and b to prove that lines are parallel or that points lie on a straight line.

Key facts

- Column vector: top = across (right +), bottom = up (+) or down (−)
- Add or subtract: top with top, bottom with bottom
- $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \mathbf{b} - \mathbf{a}$
- Grade 8+ only Parallel: one vector is a multiple of the other
- $\overrightarrow{BA} = -\overrightarrow{AB}$
- ka: multiply both numbers by k; ka is parallel to a
- Grade 7+ only P divides AB in the ratio m : n → $\overrightarrow{AP} = \frac{m}{m+n}\overrightarrow{AB}$
- Grade 8+ only Collinear: parallel vectors that share a point

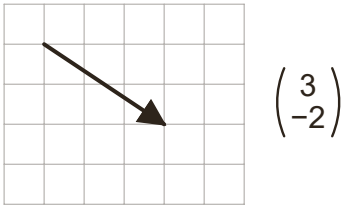
Notes

Vector basics

- A **vector** has size (magnitude) and direction. A scalar is just a number.
- The column vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ means 3 right and 2 down.
- Vectors are written $\overrightarrow{AB}$ (from A to B) or with a bold letter such as $\mathbf{a}$. When you write by hand, underline the letter.
- $\overrightarrow{BA} = -\overrightarrow{AB}$: the same size in the opposite direction.

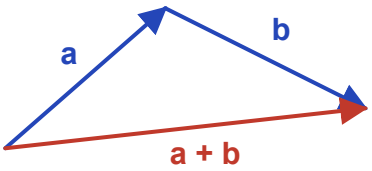
Calculating with vectors

- Add or subtract the top numbers and the bottom numbers separately: $\begin{pmatrix}3\\-2\end{pmatrix}+\begin{pmatrix}1\\5\end{pmatrix}=\begin{pmatrix}4\\3\end{pmatrix}$
- Multiplying by a scalar multiplies both numbers: $3\begin{pmatrix}2\\-1\end{pmatrix}=\begin{pmatrix}6\\-3\end{pmatrix}$. The result is parallel to the original and 3 times as long.
- On a diagram, a + b means follow a, then follow b from where a ends. The result (the resultant) goes straight from the start to the end.



$\begin{pmatrix}3\\-2\end{pmatrix}$

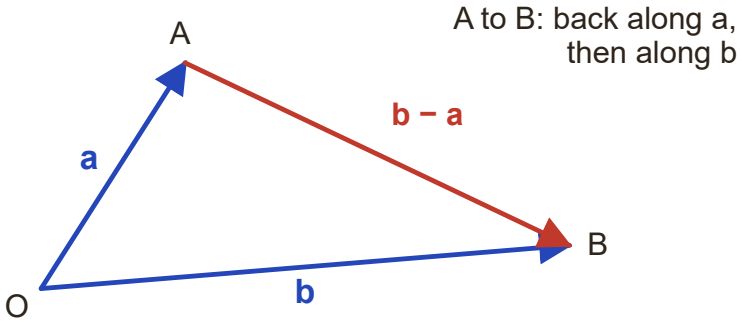
3 right, 2 down



Follow a, then b: the resultant goes straight from the start to the end.

Vector paths

- To find a vector such as $\overrightarrow{AB}$, go from A to B along a route of known vectors. Going against an arrow makes that vector negative.
- With $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$: $\overrightarrow{AB}=\overrightarrow{AO}+\overrightarrow{OB}=-\mathbf{a}+\mathbf{b}=\mathbf{b}-\mathbf{a}$



To go from A to B, go back along a to O, then along b.

- Opposite sides of a parallelogram are equal vectors (same length and direction).
- If M is the midpoint of AB, $\overrightarrow{AM}=\frac{1}{2}\overrightarrow{AB}$
- If P divides AB in the ratio 2 : 3, then $\overrightarrow{AP}=\frac{2}{5}\overrightarrow{AB}$ Grade 7+ only

Vector proofs Grade 8+ only

- Two vectors are **parallel** if one is a multiple of the other, e.g. $\mathbf{6a - 4b = 2(3a - 2b)}$, so it is parallel to $\mathbf{3a - 2b}$ and twice as long.
- Points A, B and C are **collinear** (on one straight line) if $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel and share the point B.
- Factorise to show the multiple clearly, and finish with a sentence stating what you have shown.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, C = a communication mark.

Calculate with column vectors 2 marks **Grade 5**

1. Multiply each vector by its scalar (both numbers).
2. Add or subtract the top numbers, then the bottom numbers.
3. Write the answer as a column vector.

Example. $\mathbf{a} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

Work out $\mathbf{2a - 3b}$ as a column vector.

Model answer.

$\mathbf{2a} = \begin{pmatrix} 8 \\ -2 \end{pmatrix}$ and $\mathbf{3b} = \begin{pmatrix} -6 \\ 9 \end{pmatrix}$ (M1)

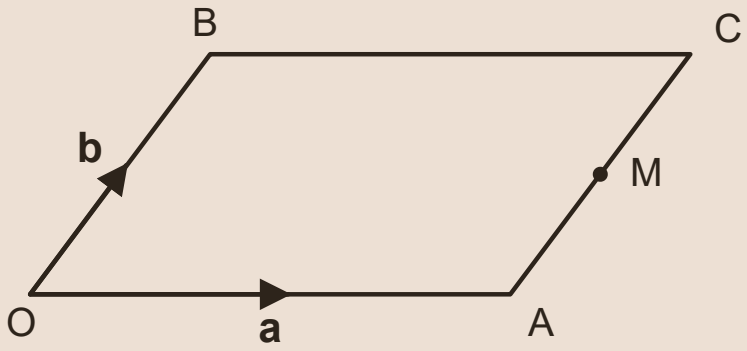
$\mathbf{2a - 3b} = \begin{pmatrix} 14 \\ -11 \end{pmatrix}$ (A1)

Write vectors in terms of a and b

1 mark each | **Grade 5**

1. Choose a route from the start point to the end point along vectors you know.
2. Use a minus sign for any part of the route that goes against an arrow.
3. Simplify: collect the a terms and the b terms.

Example. OACB is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. M is the midpoint of AC.



Write in terms of a and b: (a) $\overrightarrow{AB}$ (b) $\overrightarrow{OC}$ (c) $\overrightarrow{OM}$

Model answer.

(a) $\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$ (B1)

(b) $\overrightarrow{OC} = \mathbf{a} + \mathbf{b}$ (B1)

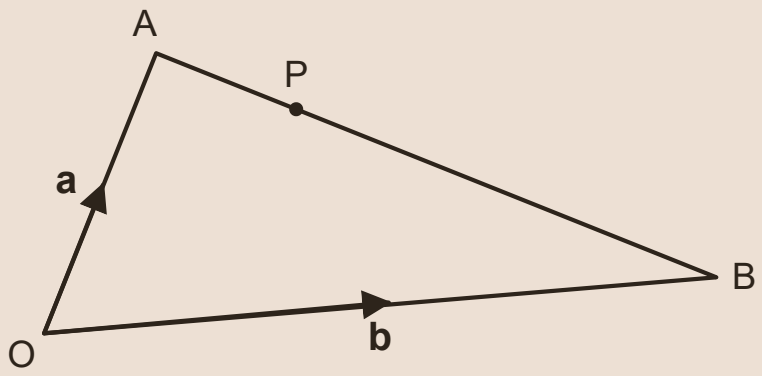
(c) $\overrightarrow{OM} = \mathbf{a} + \frac{1}{2}\mathbf{b}$ (B1)

Use a ratio to find a vector

3 marks | Grade 7

1. Find the vector along the whole line, e.g. $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$
2. Take the right fraction of it from the ratio.
3. Add it to a route to the start of the line and simplify.

Example. OAB is a triangle. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. P is the point on AB such that AP : PB = 1 : 3.



Find $\overrightarrow{OP}$ in terms of a and b. Give your answer in its simplest form.

Model answer.

$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ (M1)

$\overrightarrow{OP} = \mathbf{a} + \frac{1}{4}(\mathbf{b} - \mathbf{a})$ (M1)

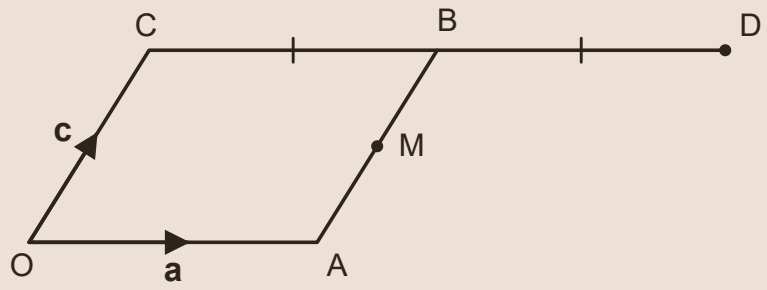
$= \frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$ (A1)

Prove that points are collinear

4 marks | Grade 8

1. Find vectors between the points, e.g. $\overrightarrow{OM}$ and $\overrightarrow{OD}$, in terms of the given vectors.
2. Factorise to show that one is a multiple of the other.
3. Write a conclusion: the vectors are parallel and share a point, so the points are collinear.

Example. OABC is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. M is the midpoint of AB. The line CB is extended to the point D so that CB = BD.



Prove that O, M and D lie on a straight line.

Model answer.

$$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = \mathbf{a} + \frac{1}{2}\mathbf{c} \quad \text{(M1)}$$

$$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD} = \mathbf{c} + 2\mathbf{a} \quad \text{(M1)}$$

$$\overrightarrow{OD} = 2\left(\mathbf{a} + \frac{1}{2}\mathbf{c}\right) = 2\overrightarrow{OM} \quad \text{(M1)}$$

So OD is parallel to OM and they share the point O, so O, M and D lie on a straight line (C1)

Don’t lose marks

- Writing $\overrightarrow{AB} = \mathbf{a} - \mathbf{b}$ instead of $\mathbf{b} - \mathbf{a}$
- Drawing a fraction line inside a column vector; it is not a fraction.
- Not simplifying the final answer (collect the a terms and the b terms).
- Higher: using a ratio of 1 : 3 as ⅓ of the line instead of ¼.
- Higher: saying lines are parallel without showing one vector is a multiple of the other.

More tips

Memory tricks

- Going the wrong way along an arrow? Put a minus sign in front.
- End minus start: if $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, then $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$
- Higher: a ratio of 1 : 3 splits the line into 1 + 3 = 4 parts, so AP is ¼ of AB.
- Higher: collinear checklist: (1) a multiple, (2) a common point, (3) a concluding sentence.

Exam technique

- Write the route you use, e.g. $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$, before substituting; it earns method marks.
- Give vector answers in their simplest form, collecting like terms.
- Higher: end a proof with a sentence that uses the words 'multiple', 'parallel' and, for collinear points, 'common point'.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 4 — **Read and draw column vectors**

The top number is the move across and the bottom number the move up or down.

Grade 4 — **Add and subtract column vectors**

Add or subtract the top numbers and the bottom numbers separately.

Grade 5 — **Multiply by a scalar and combine vectors**

Multiply both numbers by the scalar, e.g. work out 2a − 3b as a column vector.

Grade 5 — **Write a vector path using a and b**

Go along known vectors, e.g. $\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$

Grade 7 — **Use ratios to find vectors along lines**

If P divides AB in the ratio 1 : 3, then $\overrightarrow{AP} = \frac{1}{4}\overrightarrow{AB}$

Grade 8 — **Prove lines parallel or points collinear**

Show one vector is a multiple of another; for collinear points they must also share a point.

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