Right-angled trigonometry

G20, G21 · Geometry and measures · revision notes

Sine, cosine and tangent link the angles and sides of a right-angled triangle. You use SOH CAH TOA to find a missing side or angle, learn exact values for 0°, 30°, 45°, 60° and 90° for the non-calculator paper and, on Higher, solve problems in 3D.

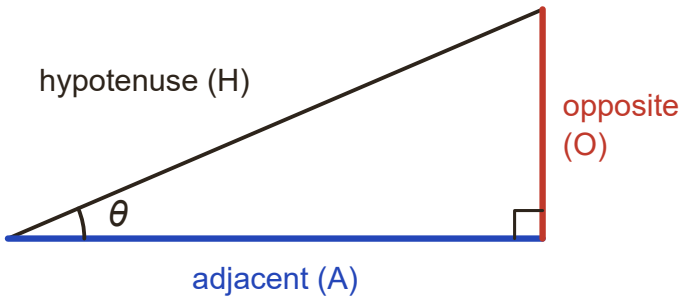
Key facts

- SOH CAH TOA: $\sin\theta = O \div H$, $\cos\theta = A \div H$, $\tan\theta = O \div A$
- Missing angle: $\theta = \sin^{-1}(O \div H)$, $\cos^{-1}(A \div H)$ or $\tan^{-1}(O \div A)$
- $\cos$ 0°, 30°, 45°, 60°, 90° = 1, $\frac{\sqrt{3}}{2}$, $\frac{\sqrt{2}}{2}$, $\frac{1}{2}$, 0
- Elevation: up from the horizontal; depression: down from the horizontal
- Unknown side on the top of the fraction: multiply; on the bottom: divide
- $\sin$ 0°, 30°, 45°, 60°, 90° = 0, $\frac{1}{2}$, $\frac{\sqrt{2}}{2}$, $\frac{\sqrt{3}}{2}$, 1
- $\tan$ 0°, 30°, 45°, 60° = 0, $\frac{\sqrt{3}}{3}$, 1, $\sqrt{3}$
- Grade 7+ only 3D: the angle between a line and a plane uses the line's projection onto the plane

Notes

Labelling the sides

- The **hypotenuse** (H) is the longest side, opposite the right angle.
- The **opposite** (O) is the side opposite the angle you are using.
- The **adjacent** (A) is the side next to that angle that is not the hypotenuse.
- The opposite and adjacent swap if you use the other angle, so always label from the angle in the question.



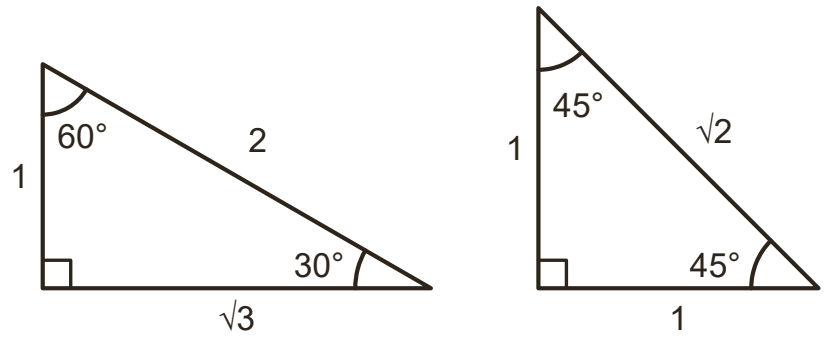
Label O and A from the angle you are using.

SOH CAH TOA

- $\sin\theta = \frac{O}{H}$, $\cos\theta = \frac{A}{H}$, $\tan\theta = \frac{O}{A}$
- Pick the ratio that uses the side you know and the side you want.
- Unknown on the top: multiply. $\sin 40° = \frac{x}{12}$ gives $x = 12\sin 40° = 7.71$
- Unknown on the bottom: divide. $\cos 25° = \frac{9}{x}$ gives $x = \frac{9}{\cos 25°} = 9.93$
- Missing angle: use the inverse. $\tan\theta = \frac{5}{8}$ gives $\theta = \tan^{-1}(0.625) = 32.0°$
- Check your calculator is in degrees: sin 30 should give 0.5.

Exact values (non-calculator)

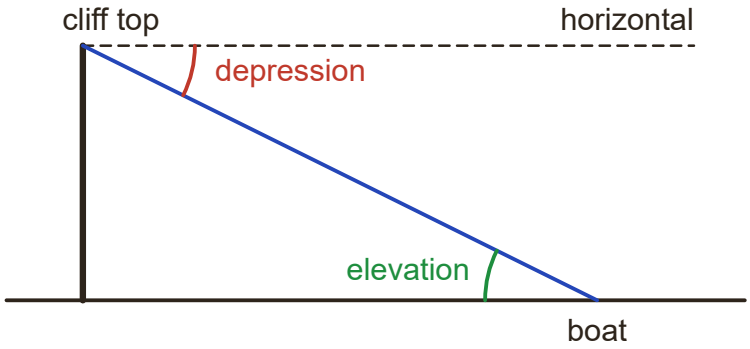
- sin 30° = cos 60° = $\frac{1}{2}$; sin 60° = cos 30° = $\frac{\sqrt{3}}{2}$; sin 45° = cos 45° = $\frac{\sqrt{2}}{2}$
- tan 30° = $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$; tan 45° = 1; tan 60° = $\sqrt{3}$
- sin 0° = 0, sin 90° = 1, cos 0° = 1, cos 90° = 0, tan 0° = 0.



Half of an equilateral triangle of side 2, and a right-angled isosceles triangle with shorter sides 1: read off sin, cos and tan.

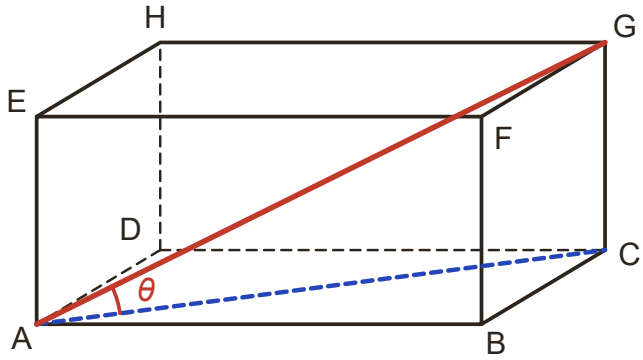
Elevation, depression and 3D

- An angle of **elevation** is measured up from the horizontal. An angle of **depression** is measured down from the horizontal.



Both angles are measured from the horizontal, and they are equal (alternate angles).

- In 3D, find the right-angled triangle that contains the angle or length you want, and draw it flat. Grade 7+ only
- The angle between a line and a plane is the angle between the line and its projection on the plane (the line's 'shadow' straight down onto it). Grade 7+ only



AC (dashed) is the 'shadow' of AG on the base, so θ is the angle between AG and the base.

Grade 7+ only

- You usually need Pythagoras first, e.g. to find a diagonal of the base. Grade 7+ only

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Calculate the length of a side

3 marks **Grade 5**

1. Label O, A and H from the given angle.
2. Choose the ratio that uses the known side and the unknown side, and write the equation.
3. Rearrange (multiply or divide) and calculate, then round.

Example. Triangle PQR has a right angle at Q. PR = 14 cm and angle QPR = 38°.
Work out the length of QR. Give your answer correct to 3 significant figures.

Model answer.

QR is opposite the 38° angle and PR is the hypotenuse, so sin 38° = QR ÷ 14 (M1)
QR = 14 × sin 38° (M1)
= 8.62 cm (A1)

Calculate the size of an angle

3 marks **Grade 5**

1. Label the two sides you know as O, A or H from the angle you want.
2. Write the ratio as a fraction, e.g. tan θ = O ÷ A.
3. Use the inverse function and round, usually to 1 decimal place.

Example. A ramp rises 0.8 m vertically over a horizontal distance of 5.5 m.
Work out the angle the ramp makes with the horizontal. Give your answer correct to 1 decimal place.

Model answer.

$\tan \theta = 0.8 \div 5.5$ (M1)
$\theta = \tan^{-1}(0.8 \div 5.5)$ (M1)
$= 8.3°$ (A1)

Non-calculator: use exact values

2 to 4 marks | Grade 5

1. Write the trig equation as normal.
2. Replace the trig value with its exact value (e.g. $\sin 30° = \frac{1}{2}$).
3. Simplify, or match a ratio to an exact value to find the angle.

Example. Do not use a calculator.
(a) ABC is a triangle with a right angle at B. Angle BAC = 30° and AC = 18 cm. Work out the length of BC. (2 marks)
(b) DEF is a triangle with a right angle at E. DE = 7 cm and EF = 7 cm. Show that angle EDF = 45°. (2 marks)

Model answer.

(a) $\sin 30° = BC \div 18$, so $BC = 18 \times \frac{1}{2}$ (M1) = 9 cm (A1)

(b) EF is opposite angle D and DE is adjacent to it, so $\tan D = 7 \div 7 = 1$ (M1)

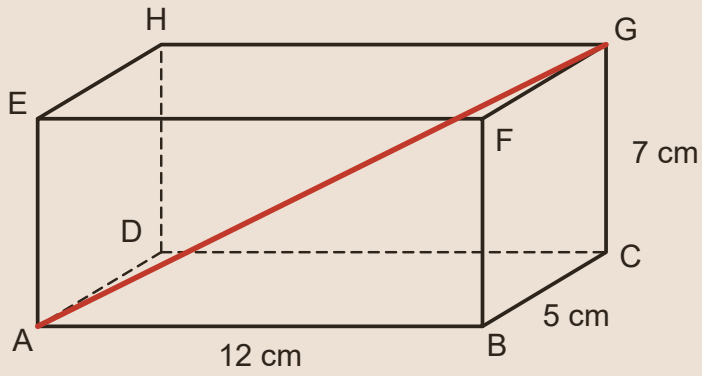
$\tan 45° = 1$, so angle EDF = 45° (A1)

3D: the angle between a line and a plane

4 marks | Grade 7

1. Find the right-angled triangle made by the line, its projection on the plane and a vertical edge.
2. Use Pythagoras to find the side of that triangle that lies in the plane.
3. Use SOH CAH TOA to find the angle.

Example. ABCDEFGH is a cuboid. The base ABCD is horizontal with AB = 12 cm and BC = 5 cm. G is vertically above C and CG = 7 cm.



Work out the size of the angle between AG and the base ABCD. Give your answer correct to 1 decimal place.

Model answer.

$AC^2 = 12^2 + 5^2$ (M1), so AC = 13 cm (A1)

$\tan \theta = 7 \div 13$ (M1)

$\theta = \tan^{-1}(7 \div 13) = 28.3°$ (A1)

Don’t lose marks

- Calculator in radians or gradians mode.
- Labelling the opposite and adjacent from the wrong angle.
- Writing $\theta = \tan(0.625)$ instead of $\theta = \tan^{-1}(0.625)$.
- Multiplying when the unknown is on the bottom: if $\cos 25° = 9 \div x$, then $x = 9 \div \cos 25°$.
- Rounding the ratio before using the inverse: $\tan^{-1}(0.15) = 8.5°$, but $\tan^{-1}(0.8 \div 5.5) = 8.3°$.

More tips

Memory tricks

- Formula triangles: O on top with S and H below (SOH), A on top with C and H below (CAH), O on top with T and A below (TOA). Cover the one you want to find.
- Exact sin values follow a pattern: $\frac{\sqrt{0}}{2}, \frac{\sqrt{1}}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, \frac{\sqrt{4}}{2}$ for 0°, 30°, 45°, 60°, 90°. Cos is the same list backwards.
- Sin and cos are never bigger than 1. If $\sin^{-1}$ or $\cos^{-1}$ gives a maths error, you have divided the wrong way round.
- Test your calculator mode at the start: sin 30 = 0.5 means degrees.

Exam technique

- Write the trig equation first (e.g. $\sin 38° = QR \div 14$); it is usually the first method mark.
- Unless told otherwise, give angles to 1 decimal place and lengths to 3 significant figures.
- On the non-calculator paper, any trig with 30°, 45° or 60° needs the exact values, so learn them as a table.
- Use a previous answer at full accuracy (the ANS key or calculator memory), not a rounded one.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 4 — **Label the hypotenuse, opposite and adjacent sides**
The opposite is across from the angle, the hypotenuse is opposite the right angle, and the adjacent is the other side next to the angle.

Grade 5 — **Find a missing side using SOH CAH TOA**
Choose the ratio from the side you know and the side you want, then rearrange, e.g. $x = 12 \times \sin 40°$.

Grade 5 — **Find a missing angle using inverse trig**
Use $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ on your calculator, e.g. $\theta = \tan^{-1}(5 \div 8)$.

Grade 5 — **Recall exact trig values**
Know sin and cos of 0°, 30°, 45°, 60° and 90°, and tan of 0°, 30°, 45° and 60°, without a calculator.

Grade 5 — **Solve angle of elevation and depression problems**
Draw the right-angled triangle with the angle measured from the horizontal.

Grade 7 — **Find lengths and angles in 3D shapes**
Find the angle between a line and a plane using a right-angled triangle inside the shape.

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