Tree diagrams and combined events

P6–P8 · Probability · revision notes

Combined events are two or more events together, such as rolling two dice or taking two counters from a bag. You list outcomes systematically, use sample space diagrams and draw tree diagrams, multiplying along branches for 'and' and adding for 'or'. Independent events and simple 'without replacement' questions are on both tiers; longer three-event and algebraic problems appear on Higher papers at grades 6 to 9.

Key facts

- And: multiply along the branches
- The branches from one point add up to 1
- Independent events: P(A and B) = P(A) × P(B)
- Two fair six-sided dice: 36 equally likely outcomes
- Or: add the probabilities of the separate paths
- All the end-of-path probabilities add up to 1
- Without replacement: the total goes down by 1 for each pick
- At least one = 1 − P(none)

Notes

Listing outcomes and sample spaces

- List outcomes **systematically** so you do not miss any: keep the first item fixed while you go through every option for the second.
- A **sample space diagram** is a grid showing every outcome of two events. Two fair six-sided dice give 6 × 6 = 36 equally likely outcomes.
- Count the cells you want: a total of 9 comes from 3 + 6, 4 + 5, 5 + 4 and 6 + 3, so P(total of 9) = $\frac{4}{36} = \frac{1}{9}$

Dice 2

Dice 1 +	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	**9**
4	5	6	7	8	**9**	10
5	6	7	8	**9**	10	11
6	7	8	**9**	10	11	12

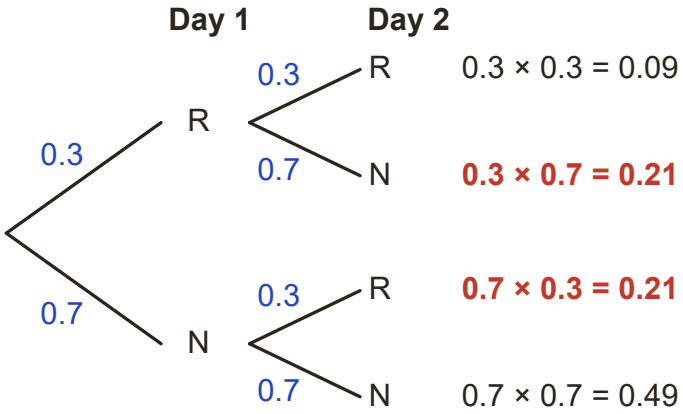
36 equally likely outcomes
P(total of 9) = 4/36

The four cells with a total of 9 are shaded.

- This only works when every outcome is equally likely (fair dice, fair spinners).

Tree diagrams

- Each set of branches shows one event, with a probability on each branch.
- The branches coming from one point always add up to 1.
- **And**: multiply along the branches to get the probability of that path.
- **Or**: add the probabilities of the different paths. All the end-of-path probabilities add up to 1.
- Example: P(rain) = 0.3 each day, independently. P(rain on exactly one of two days) = 0.3 × 0.7 + 0.7 × 0.3 = 0.21 + 0.21 = 0.42.



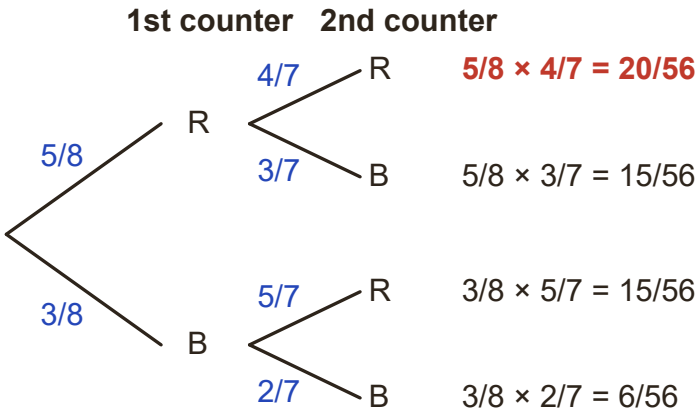
Exactly one day of rain: add the two red paths, 0.21 + 0.21 = 0.42.

- **At least one** is quickest as 1 − P(none): P(rain on at least one day) = 1 − 0.7 × 0.7 = 1 − 0.49 = 0.51.

Independent and dependent events

- **Independent**: one event does not affect the other, e.g. the counter is put back. Then P(A and B) = P(A) × P(B), and every second set of branches is the same.
- **Dependent**: the first outcome changes the probabilities for the second, e.g. taking counters without replacement, or eating a sweet.
- Without replacement, the total goes down by 1 for the second pick. The count for a colour goes down only if that colour was taken first.
- Example: 5 red and 3 blue counters, two taken without replacement. P(both red) =

$$\frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \frac{5}{14}$$



Without replacement the second branches are out of 7, and they depend on the first counter.

- Using the same probabilities on every set of branches assumes the events are independent: you may be asked to state this.

Longer problems Grade 6+ only

- Three events: multiply three probabilities along each path. If each shot hits with probability 0.4, P(at least one hit in three shots) = 1 − 0.6 × 0.6 × 0.6 = 1 − 0.216 = 0.784.
- 'Exactly one hit' in three shots has three paths (HMM, MHM, MMH, where H = hit and M = miss). Work out each path and add.
- Algebra: with n counters in a bag, taking two without replacement gives denominators n and $n-1$. Setting the probability equal to the value given usually leads to a quadratic equation. Grade 8+ only

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Use a sample space diagram

2 marks | **Grade 3**

1. Draw a grid: one event across the top, the other down the side.
2. Fill in every cell (the total, the product or the pair of outcomes).
3. Count the cells you want and write that number over the total number of cells.

Example. Spinner A is numbered 1, 2 and 3. Spinner B is numbered 1, 2, 3 and 4. Both spinners are fair. Kai spins both spinners and multiplies the two numbers.
Work out the probability that the product is greater than 5.

Model answer.

Sample space of products, with spinner A down the side and spinner B across the top (M1 for all 12 outcomes shown):

×	**1**	**2**	**3**	**4**
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12

Greater than 5: 6, 8, 6, 9 and 12, which is 5 out of 12.

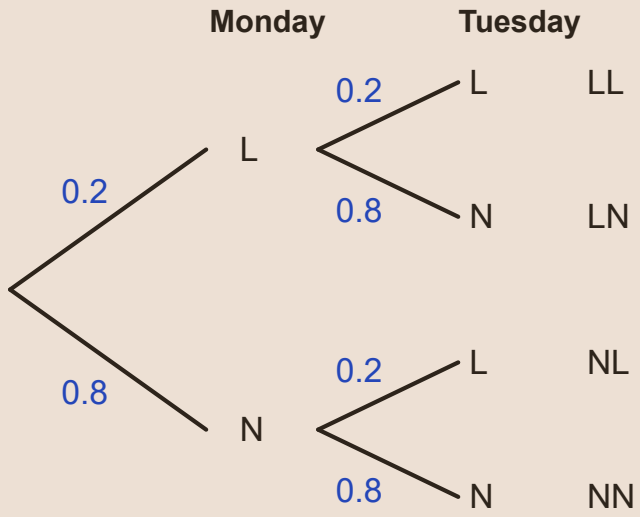
P(product greater than 5) = $\frac{5}{12}$ (A1)

Complete a tree diagram and find probabilities (independent events) 4 to 6 marks **Grade 4**

1. Write the missing branch probabilities: each pair adds up to 1.
2. For independent events, copy the first set of probabilities onto every second set.
3. Multiply along a path for 'and'.
4. Add paths for 'or': 'exactly one' is late then on time, plus on time then late.

Example. The probability that Leah's bus is late on any day is 0.2. Whether it is late is independent from one day to the next.
(a) Draw a tree diagram for Monday and Tuesday. (2 marks)
(b) Work out the probability that the bus is late on both days. (2 marks)
(c) Work out the probability that the bus is late on exactly one of the two days. (2 marks)

Model answer.



(a) L = late, N = not late. Monday: late 0.2, not late 0.8 (B1). From each Monday branch, Tuesday: late 0.2, not late 0.8 (B1).
(b) 0.2 × 0.2 (M1) = 0.04 (A1)
(c) 0.2 × 0.8 + 0.8 × 0.2 (M1) = 0.16 + 0.16 = 0.32 (A1)

Two picks without replacement

3 marks | Grade 5

1. Draw the tree: the second pick is out of one fewer (here 10, then 9).
2. Reduce the numerator only for the colour already taken.
3. Find each path you need (here RR and YY), then add them.
4. Keep the same denominator (here 90) so the fractions are easy to add.

Example. A bag contains 4 red counters and 6 yellow counters. Tom takes a counter at random and does not put it back. He then takes a second counter at random.
Work out the probability that the two counters are the same colour.

Model answer.

P(RR) = $\frac{4}{10} \times \frac{3}{9} = \frac{12}{90}$ (M1 for either product)

P(YY) = $\frac{6}{10} \times \frac{5}{9} = \frac{30}{90}$

$\frac{12}{90} + \frac{30}{90}$ (M1 for adding both products) $= \frac{42}{90} = \frac{7}{15}$ (A1)

Three events: 'at least one' and 'exactly two'

3 marks per part | Grade 6

1. For 'at least one', work out P(none) by multiplying, then subtract it from 1.
2. For 'exactly two', work out one path (e.g. HHT) and count how many orders there are.
3. Multiply the path probability by the number of orders (or add the paths).

Example. A biased coin has P(heads) = 0.3. Ria flips the coin three times.
(a) Work out the probability that she gets at least one head. (3 marks)
(b) Work out the probability that she gets exactly two heads. (3 marks)

Model answer.

(a) P(no heads) = 0.7 × 0.7 × 0.7 = 0.343 (M1)

1 − 0.343 (M1) = 0.657 (A1)

(b) One path: 0.3 × 0.3 × 0.7 = 0.063 (M1)

There are three orders (HHT, HTH, THH), so 3 × 0.063 (M1) = 0.189 (A1)

Show that … (algebra on a tree diagram) 5 marks **Grade 8**

1. Write each probability in terms of the letter; without replacement the second denominator is one less.
2. Multiply the fractions along the path and set the product equal to the probability given.
3. Clear the fractions, expand and rearrange until you reach the given equation exactly.
4. Solve the quadratic and reject any impossible value (negative, or not a whole number of items).

Example. There are 12 counters in a bag. x of the counters are green. Sam takes a counter at random and does not replace it. He then takes a second counter at random. The probability that both counters are green is $\frac{1}{11}$.

(a) Show that $x^2 - x - 12 = 0$. (3 marks)

(b) Work out the number of green counters in the bag. (2 marks)

Model answer.

(a) P(both green) = $\frac{x}{12} \times \frac{x-1}{11}$ (M1)

$\frac{x(x-1)}{132} = \frac{1}{11}$ (M1)

$x(x-1) = 12$, so $x^2 - x - 12 = 0$ (A1)

(b) $(x-4)(x+3) = 0$ (M1)

x cannot be negative, so $x = 4$: there are 4 green counters (A1)

Don’t lose marks

- Adding along the branches instead of multiplying.
- Forgetting that 'one of each' has two paths: red then yellow, and yellow then red.
- Without replacement: not reducing the total for the second pick, or reducing the count of the wrong colour.
- Adding the denominators when adding fractions: $\frac{12}{90} + \frac{30}{90} = \frac{42}{90}$, not $\frac{42}{180}$
- Writing branch probabilities that do not add up to 1, such as 0.3 and 0.6.
- In a 'show that' question, jumping straight to the given equation without showing the lines in between.

More tips

Memory tricks

- 'And' means multiply, 'or' means add.
- Quick check: every pair of branches adds up to 1, and all the end results add up to 1.
- Without replacement, the denominators count down: 10, then 9, then 8.
- Leave fractions unsimplified until the end: with a common denominator (like 90) the paths are easy to add.
- For 'at least one', 1 − P(none) is much quicker than adding every other path.

Exam technique

- Write a probability on every branch, even ones you think you will not need: completing the tree often earns marks on its own.
- Write down each path you are adding, e.g. P(RR) + P(YY), so your method marks are clear even if you make a slip.
- Look for the words 'replaces', 'does not replace' and 'eats': they tell you whether the events are independent.
- Give exact answers (fractions, or exact decimals) where you can. If you have to round, keep at least 3 significant figures.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 3 — **List all the outcomes systematically**
Fix the first item and list every option for the second, then change the first, e.g. HH, HT, TH, TT for two coins.

Grade 3 — **Use a sample space diagram**
Draw a grid for two dice or spinners, then count the outcomes you want out of the total number of cells.

Grade 4 — **Complete a tree diagram for independent events**
Each pair of branches adds up to 1, and the second set of branches repeats the first when the events are independent.

Grade 5 — **Multiply for 'and', add for 'or'**
Multiply along each path, then add the probabilities of all the paths that give the result you want.

Grade 5 — **Use tree diagrams without replacement**
For the second pick the total goes down by 1, and so does the count of the colour already taken.

Grade 6 — **Solve problems with three events**
Multiply three probabilities along each path, count the paths you need, and use 1 − P(none) for 'at least one'.

Grade 8 — **Form and solve an equation from probabilities**
Write the branch probabilities in terms of an unknown, set up an equation (often a quadratic) and solve it.

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