Transformations

G7, G8, G24 · Geometry and measures · revision notes

Translations, reflections, rotations and enlargements move or resize a shape. You carry them out on a coordinate grid, describe them fully, and deal with combinations of transformations; Higher also includes enlargements with negative scale factors.

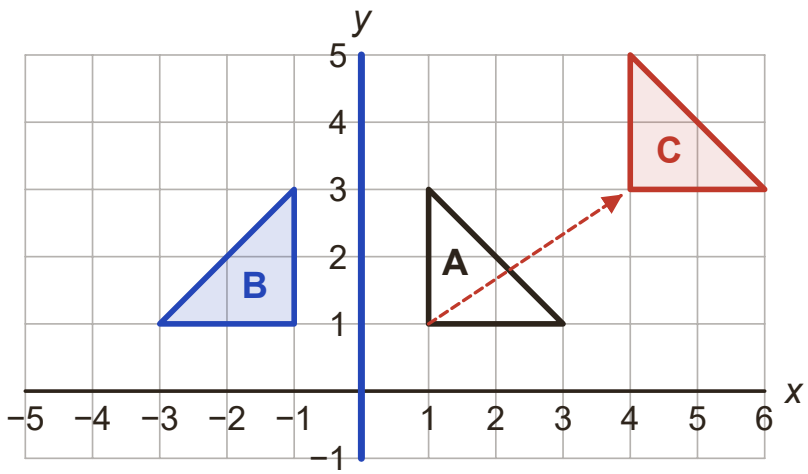
Key facts

- Translation: column vector (top: right +/left −; bottom: up +/down −)
- Reflection: equation of the mirror line
- Rotation: angle, direction and centre
- Enlargement: scale factor and centre
- x = a is vertical; y = b is horizontal
- Image distance from the centre = scale factor × original distance
- Invariant point: a point that does not move
- Grade 7+ only Negative scale factor: image on the opposite side of the centre, upside down

Notes

Translation and reflection

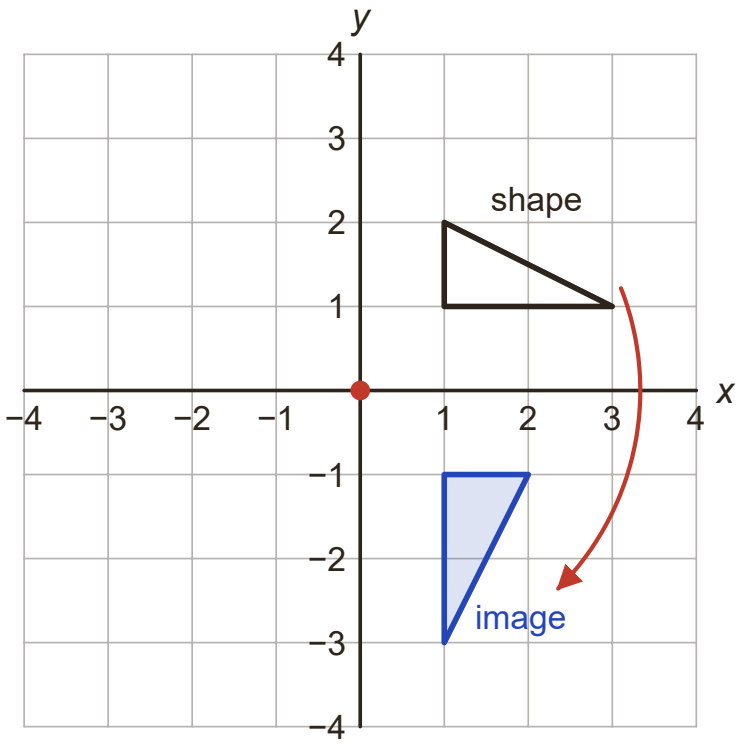
- A **translation** slides a shape. It is described by a column vector $\begin{pmatrix} x \\ y \end{pmatrix}$: x across (right is positive) and y up (down is negative).
- A **reflection** flips a shape in a mirror line. Each image point is the same perpendicular distance from the line as the original point, on the other side.
- x = a is a vertical line and y = b is a horizontal line. Also know the diagonal lines y = x and y = −x.



Each point of B is the same distance from the mirror line as the matching point of A.

Rotation

- A **rotation** turns a shape through an angle, clockwise or anticlockwise, about a **centre of rotation**.
- Use tracing paper: trace the shape, hold the centre still with your pencil point and turn the paper.
- To find the centre, try points with tracing paper until the shape lands on its image.
- A rotation of 180° needs no direction, as clockwise and anticlockwise give the same image.

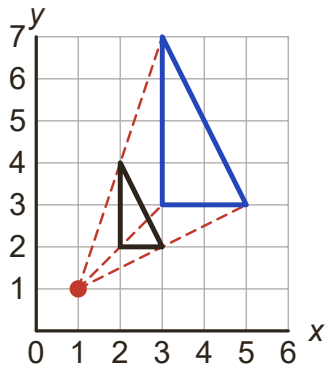


Rotation 90° clockwise about (0, 0)

Give the angle, the direction and the centre.

Enlargement

- An **enlargement** changes the size of a shape by a scale factor from a **centre of enlargement**. Angles stay the same, so the image is similar to the original.
- Multiply the distance (or the vector) from the centre to each vertex by the scale factor.
- A scale factor between 0 and 1 makes the shape smaller; ½ halves every length.
- To find the centre, draw straight lines through corresponding vertices of the shape and image; they meet at the centre.



Enlargement, scale factor 2, centre (1, 1)

Lines through matching corners meet at the centre.

Every distance from the centre is doubled.

- Negative scale factor: measure from the centre in the opposite direction, so the image is on the other side of the centre and upside down. Scale factor −1 has the same effect as a rotation of 180° about the centre.

Grade 7+ only

Describing and combining

- Describe fully: translation + vector; reflection + equation of the mirror line; rotation + angle, direction and centre; enlargement + scale factor and centre.
- For a combination, do the first transformation, then the second on the image. The result can often be described as one single transformation.
- **Invariant points** stay in the same place, e.g. points on the mirror line in a reflection, or the centre of a rotation.
- Translations, reflections and rotations give congruent images; enlargements give similar images.

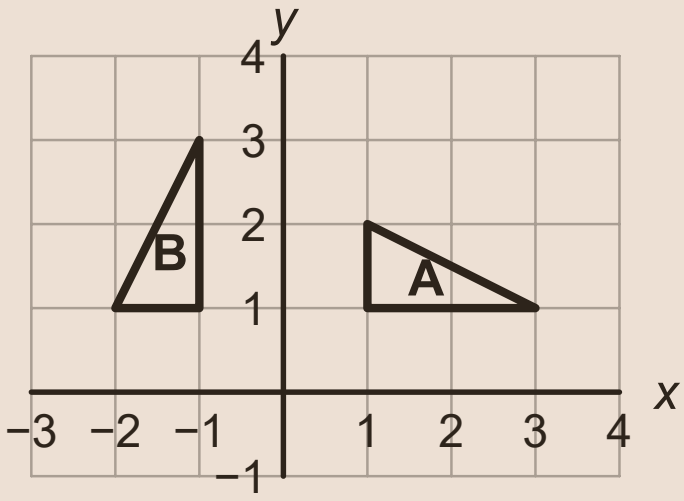
How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Describe fully the single transformation 2 to 3 marks **Grade 5**

1. Decide the type: same size and same way round (translation), flipped (reflection), turned (rotation) or resized (enlargement).
2. Find every detail: vector, mirror line, angle, direction, centre or scale factor.
3. Write one transformation only, with all its details.

Example. Triangle A has vertices (1, 1), (3, 1) and (1, 2). Triangle B has vertices (−1, 1), (−1, 3) and (−2, 1).



Describe fully the single transformation that maps triangle A onto triangle B.

Model answer.

Rotation	(B1)
90° anticlockwise	(B1)
about (0, 0)	(B1)

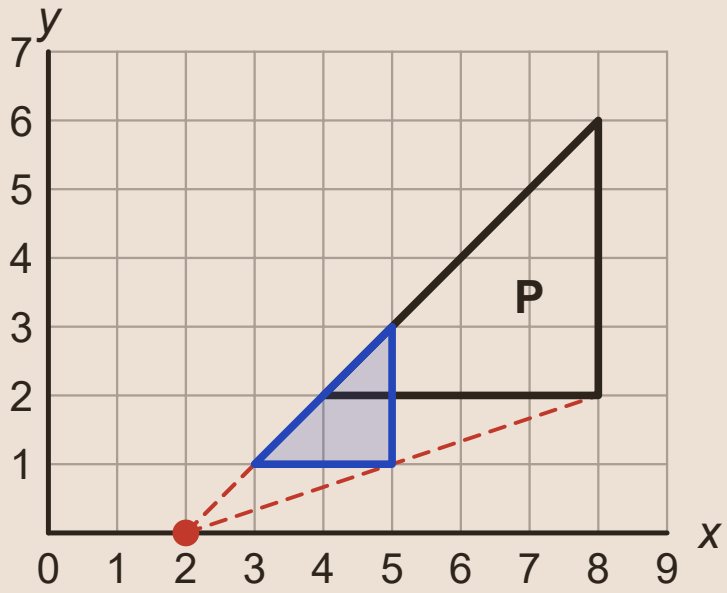
Enlarge a shape by a fractional scale factor

2 marks | Grade 5

1. Write the vector from the centre to each vertex.
2. Multiply each vector by the scale factor.
3. Add each new vector to the centre to plot the image.

Example. Triangle P has vertices (4, 2), (8, 2) and (8, 6).
Enlarge triangle P by scale factor ½ with centre (2, 0).

Model answer.



Vectors from (2, 0) to the vertices: (2, 2), (6, 2), (6, 6); halved: (1, 1), (3, 1), (3, 3)
Image vertices (3, 1), (5, 1) and (5, 3) (B2)
(B1 for a triangle of the correct size and orientation in the wrong position)

Combined transformations

2 to 3 marks | Grade 5

1. Carry out the first transformation, then apply the second to its image.
2. Compare the final image with the original shape.
3. Describe the single transformation that maps one onto the other.

Example. Shape S is reflected in the y-axis to give shape T. Shape T is then reflected in the x-axis to give shape U.
Describe fully the single transformation that maps shape S onto shape U.

Model answer.

e.g. the point (2, 1) goes to (−2, 1) and then to (−2, −1), so every (x, y) goes to (−x, −y)
Rotation (B1) 180° about (0, 0) (B1)

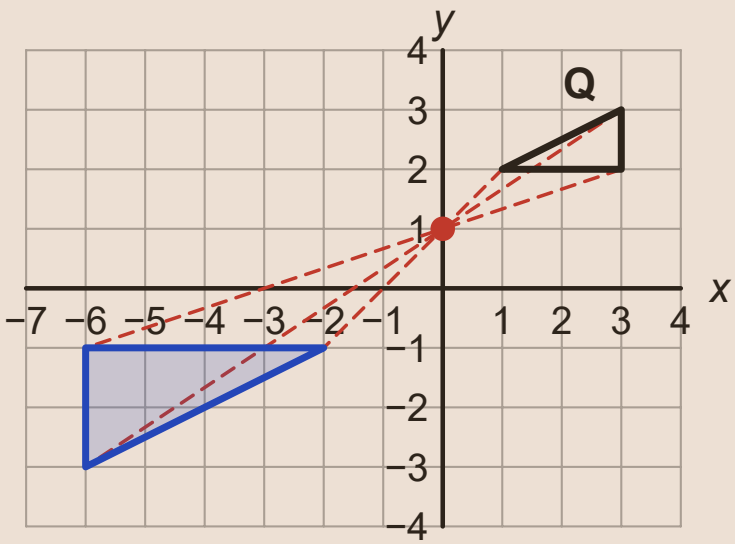
Enlarge by a negative scale factor

2 marks | Grade 7

1. Write the vector from the centre to each vertex.
2. Multiply by the negative scale factor, which reverses both directions.
3. Plot the image from the centre; it should be upside down on the other side.

Example. Triangle Q has vertices (1, 2), (3, 2) and (3, 3).
Enlarge triangle Q by scale factor −2 with centre (0, 1).

Model answer.



Vectors from (0, 1): (1, 1), (3, 1), (3, 2); multiplied by −2: (−2, −2), (−6, −2), (−6, −4)

Image vertices (−2, −1), (−6, −1) and (−6, −3) (B2)

(B1 for two correct vertices, or the correct size and orientation in the wrong position)

Don’t lose marks

- Describing two transformations when the question asks for a single transformation; this scores no marks.
- Mixing up x = 3 (a vertical line) and y = 3 (a horizontal line).
- Leaving out a detail, such as the direction or centre of a rotation, or the centre of an enlargement.
- Enlarging from the origin when the centre is somewhere else.
- Writing 'turn', 'flip' or 'move' instead of rotation, reflection or translation.
- Writing a translation as a coordinate such as (3, −2) instead of a column vector.

More tips

Memory tricks

- Rotations about (0, 0): 90° clockwise takes (x, y) to (y, −x); 90° anticlockwise takes (x, y) to (−y, x); 180° takes (x, y) to (−x, −y).
- Reflection in y = x swaps the coordinates: (x, y) goes to (y, x). In y = −x: (x, y) goes to (−y, −x).
- Ask for tracing paper for rotations and reflections.
- Check an enlargement with straight lines from the centre through each vertex: the image vertices must lie on them.

Exam technique

- 'Describe fully' needs the name of the transformation and every detail; use the number of marks as a check.
- Only describe one transformation when the question says 'single transformation'.
- Draw images accurately and check two or three vertices using their coordinates.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 3 — **Reflect a shape in a mirror line**
Reflect in lines such as x = 2, y = −1, y = x and y = −x; each image point is the same distance from the line.

Grade 3 — **Translate a shape using a column vector**
The top number moves right (+) or left (−); the bottom number moves up (+) or down (−).

Grade 4 — **Rotate a shape about a given centre**
Use tracing paper, the angle and the direction (clockwise or anticlockwise).

Grade 4 — **Enlarge a shape by a positive scale factor**
Multiply the distance from the centre of enlargement to each vertex by the scale factor.

Grade 5 — **Describe a single transformation fully**
Give the type and every detail, e.g. rotation 90° clockwise about (1, 2).

Grade 5 — **Enlarge by a fractional scale factor**
A scale factor between 0 and 1 makes the shape smaller, still measured from the centre.

Grade 5 — **Carry out and describe combined transformations**
Do one transformation then the next, then describe the single transformation that has the same effect.

Grade 7 — **Enlarge by a negative scale factor**
The image is on the opposite side of the centre and upside down.

Free notes, Full packs, practice papers and mock exams at chhetriacademy.com. Written for this site; not taken from past papers. Stuck? 1-to-1 tutoring with a free 30-minute trial lesson.