Transforming graphs

A13 · Algebra · revision notes

Higher only. Sketching translations and reflections of a graph $y = f(x)$, such as $y = f(x) + a$, $y = f(x + a)$, $y = -f(x)$ and $y = f(-x)$, and working out where points such as turning points move to.

Key facts

- $y = f(x) + a$: up a (down if a is negative)
- $y = -f(x)$: reflect in the x-axis, $(p, q) \to (p, -q)$
- $y = f(x + a) + b$: translation by $\begin{pmatrix} -a \\ b \end{pmatrix}$
- $y = f(x + a)$: left a; $y = f(x - a)$: right a
- $y = f(-x)$: reflect in the y-axis, $(p, q) \to (-p, q)$
- Outside the bracket: y changes the way you expect. Inside: x changes the opposite way

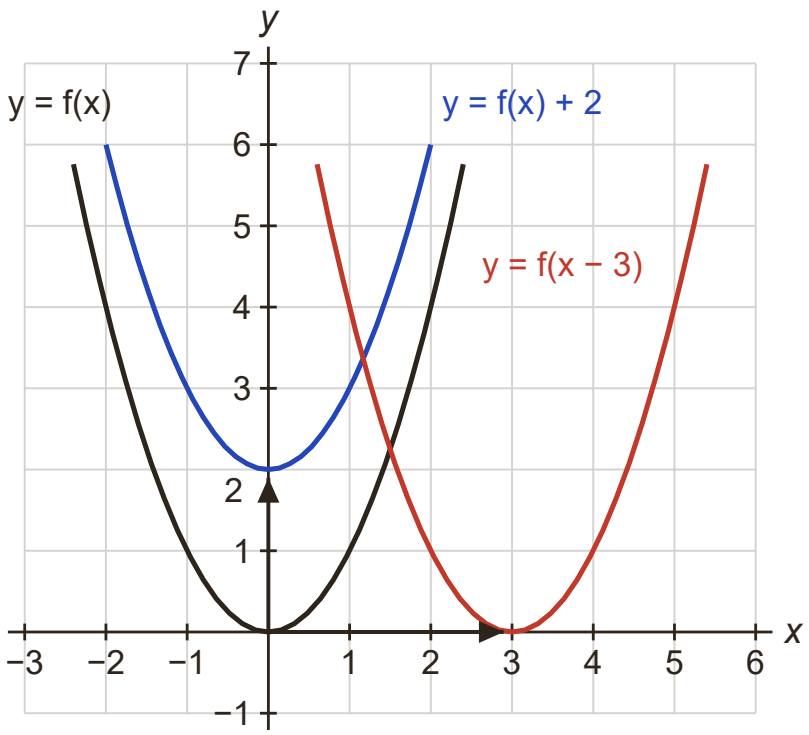
Notes

Function notation

- $y = f(x)$ stands for any graph. Changes **outside** the bracket change the y-values (vertical moves); changes **inside** the bracket change the x-values (horizontal moves).
- Changes inside the bracket work the opposite way to what you might expect.
- You need translations and reflections only; stretches are not on the GCSE.

Translations

- $y = f(x) + a$: translation by $\begin{pmatrix} 0 \\ a \end{pmatrix}$. The point (p, q) moves to $(p, q + a)$
- $y = f(x + a)$: translation by $\begin{pmatrix} -a \\ 0 \end{pmatrix}$. The point (p, q) moves to $(p - a, q)$
- So $y = f(x - 3)$ moves the graph 3 units to the **right**.

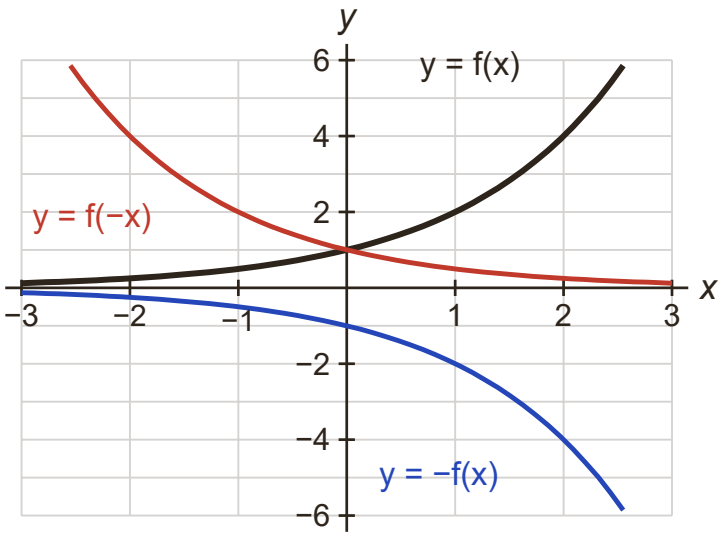


$f(x) + 2$: up 2. $f(x - 3)$: right 3 (the opposite way to the sign).

- Combined: $y = f(x - 2) + 5$ is a translation by $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$

Reflections

- $y = -f(x)$: reflection in the x-axis. (p, q) moves to $(p, -q)$
- $y = f(-x)$: reflection in the y-axis. (p, q) moves to $(-p, q)$



$-f(x)$ flips in the x-axis; $f(-x)$ flips in the y-axis.

- Points on the mirror line do not move: under $y = -f(x)$, the points where the graph crosses the x-axis stay put.

With a known function

- If $f(x) = x^2$, then $f(x + 3) = (x + 3)^2$, and its turning point moves from (0, 0) to (−3, 0).
- $y = (x - 4)^2 + 1$ is $y = x^2$ translated by $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$, so its turning point is (4, 1).
- $y = \sin x + 2$ is the sine curve moved up 2, so it goes between 1 and 3.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Write down the new coordinates of a point

1 mark each **Grade 7**

1. Decide whether the change is inside or outside the bracket.
2. Change only the x-coordinate (inside) or only the y-coordinate (outside).

Example. The graph of $y = f(x)$ has a turning point at (2, −3). Write down the coordinates of the turning point of

(a) $y = f(x) + 4$ (b) $y = f(x + 5)$ (c) $y = -f(x)$ (d) $y = f(-x)$

Model answer.

(a) (2, 1) B1

(b) (−3, −3) B1

(c) (2, 3) B1

(d) (−2, −3) B1

Sketch a transformed graph

2 marks **Grade 7**

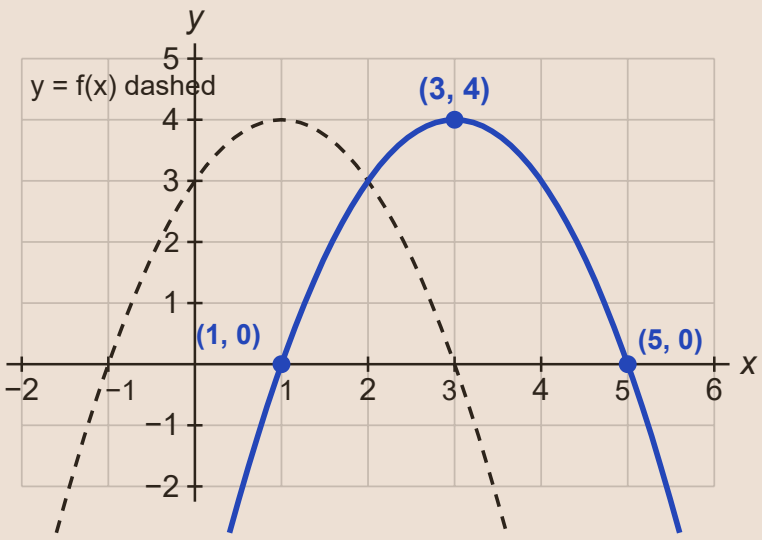
1. Move the key points (turning point, intercepts).
2. Draw the same shape through the new points and label them.

Example. The graph of $y = f(x)$ is a ∩-shaped curve that crosses the x-axis at (−1, 0) and (3, 0) and has a maximum point at (1, 4). Sketch the graph of $y = f(x - 2)$

Model answer.

The same ∩ shape moved 2 units to the right B1

Crosses the x-axis at (1, 0) and (5, 0), with maximum point (3, 4) B1



Describe the transformation fully

2 marks | Grade 7

1. Name it: translation or reflection.
2. Give the vector (for a translation) or the mirror line (for a reflection).

Example. Describe fully the single transformation that maps the graph of $y = f(x)$ onto the graph of $y = f(x + 1) - 6$

Model answer.

Translation B1

by the vector $\begin{pmatrix} -1 \\ -6 \end{pmatrix}$ B1

Transform a trigonometric graph

2 marks | Grade 8

1. Start from the known key points of the sin, cos or tan graph.
2. Apply the translation to each key point.

Example. The graph of $y = \cos x$ has a minimum point at (180°, −1).
(a) Write down the coordinates of the corresponding minimum point on $y = \cos(x - 60^\circ)$.
(b) Write down the maximum value of $y = 3 + \cos x$

Model answer.

(a) (240°, −1) B1

(b) 4 B1

Don’t lose marks

- Moving $y = f(x + a)$ to the right instead of the left.
- Mixing up the reflections: $-f(x)$ reflects in the x-axis; $f(-x)$ reflects in the y-axis.
- Changing the shape or size of the curve: translations and reflections keep the shape the same.
- Not labelling key points on a sketch.

More tips

Memory tricks

- Inside the bracket: the opposite way. Outside the bracket: the way you expect.
- Track key points (turning points and intercepts), then draw the same shape through the new points.
- Check with one point: for $y = f(x - 3)$, the value of f at 0 now appears at $x = 3$

Exam technique

- 'Describe fully' needs the name of the transformation and its vector or mirror line.
- On a sketch, label the new coordinates of the turning point and where the graph crosses the axes.
- If you know f(x), you can check by substituting, e.g. $f(x) + 4$ should add 4 to every y-value.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 6 — **Translate a graph vertically**

$y = f(x) + a$ moves the graph up a units (down if a is negative).

Grade 7 — **Translate a graph horizontally**

$y = f(x + a)$ moves the graph a units to the left; $y = f(x - a)$ moves it a units to the right.

Grade 7 — **Reflect a graph in either axis**

$y = -f(x)$ reflects it in the x-axis; $y = f(-x)$ reflects it in the y-axis.

Grade 7 — **Find where a point moves to**

Apply the transformation to the coordinates, e.g. under $y = f(x - 3)$, (2, 5) moves to (5, 5).

Grade 8 — **Describe a combined translation as a vector**

$y = f(x + 2) - 1$ is a translation by $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$

Grade 8 — **Transform trigonometric graphs**

e.g. $y = \cos(x - 90^\circ)$ is the cos graph moved 90° to the right, which gives the same graph as $y = \sin x$

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