Changing the subject

A5 · Algebra · revision notes

Rearranging a formula so that a different letter is on its own, e.g. making r the subject of $A = \pi r^2$. Simple rearrangements are on both tiers; Higher questions include harder powers and roots and the subject appearing twice.

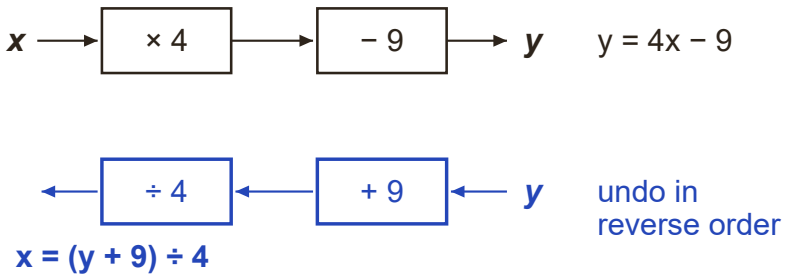
Key facts

- Do the same to both sides
- Inverse pairs: + and −, × and ÷, squaring and square rooting
- Clear fractions first: multiply both sides by the denominator
- Undo operations in the reverse order
- The final answer starts with the new subject on its own, e.g. $r = \ldots$
- Grade 7+ only Subject twice: collect the subject terms, factorise, divide

Notes

The method

- The subject is the letter on its own on one side: in $v = u + at$, v is the subject.
- Rearranging is like solving an equation: do the same to both sides, using inverse operations.
- Undo the operations in the reverse order to the order they were applied to the new subject.
- Make x the subject of $y = 4x - 9$: add 9 to get $y + 9 = 4x$, then divide by 4 to get $x = \frac{y+9}{4}$



The operations done to x, then the inverses in reverse order (blue).

Fractions, brackets, powers and roots

- Clear fractions first by multiplying both sides by the denominator: $T = \frac{k}{m}$ gives $Tm = k$, so $m = \frac{k}{T}$
- With a bracket, divide by the number outside or expand: $P = 2(l + w)$ gives $l = \frac{P}{2} - w$
- Undo a square with a square root: $A = \pi r^2$ gives $r^2 = \frac{A}{\pi}$, so $r = \sqrt{\frac{A}{\pi}}$
- Undo a square root by squaring: $t = \sqrt{\frac{h}{5}}$ gives $t^2 = \frac{h}{5}$, so $h = 5t^2$

When the subject appears twice Grade 7+ only

- Clear any fractions, then expand any brackets.
- Move every term containing the subject to one side and every other term to the other side.
- Factorise out the subject, then divide by the bracket.
- Make a the subject of $3a + b = ac$: $3a - ac = -b$, so $a(3 - c) = -b$ and $a = \frac{-b}{3 - c}$, which is the same as $\frac{b}{c - 3}$

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Make a letter the subject (linear) 2 marks **Grade 4**

1. Move the terms without the new subject to the other side.
2. Divide by the number multiplying the new subject.

Example. Make t the subject of the formula $w = 5t + 3k$

Model answer.

$w - 3k = 5t$ M1

$t = \frac{w - 3k}{5}$ A1

Rearrange a formula with a fraction

2 marks | Grade 5

1. Multiply both sides by the denominator.
2. Divide by any letter multiplying the bracket.
3. Isolate the new subject.

Example. Make u the subject of $s = \frac{(u+v)t}{2}$

Model answer.

$2s = (u+v)t$ M1

$u = \frac{2s}{t} - v$ A1

(or $u = \frac{2s - vt}{t}$)

Rearrange a formula with a square or root

3 marks | Grade 6

1. Clear fractions.
2. Get the squared term on its own.
3. Square root both sides as the last step.

Example. Make r the subject of $V = \frac{1}{3}\pi r^2 h$

Model answer.

$3V = \pi r^2 h$ M1

$r^2 = \frac{3V}{\pi h}$ M1

$r = \sqrt{\frac{3V}{\pi h}}$ A1

Make x the subject when it appears twice

4 marks | Grade 7

1. Multiply both sides by the denominator.
2. Expand, then collect all the x terms on one side.
3. Factorise out x.
4. Divide by the bracket.

Example. Make x the subject of $y = \dfrac{2x+1}{x-3}$

Model answer.

$y(x-3) = 2x+1$	M1
$xy - 3y = 2x+1$, so $xy - 2x = 3y+1$	M1
$x(y-2) = 3y+1$	M1
$x = \dfrac{3y+1}{y-2}$	A1

Don't lose marks

- Only dividing one term: from $y = 3x+2$, writing $\dfrac{y}{3} = x+2$
- Square rooting term by term: $\sqrt{a^2+b^2}$ is not $a+b$
- Leaving the subject on both sides, e.g. $x = \dfrac{y-x}{2}$, which is not a finished rearrangement.
- Dropping a minus sign when moving a term to the other side.

More tips

Memory tricks

- Check with numbers: choose a value for x, work out y from the original formula, then put y into your rearranged formula. You should get your x back.
- Draw a function machine for the new subject, then run it backwards using inverse operations.
- Higher: if the subject appears twice, say 'collect, factorise, divide'.

Exam technique

- Write each step on a new line; each correct step can earn a method mark.
- Your final answer must have the new subject on its own on one side and nowhere on the other side.
- If a later part uses your rearranged formula, substitute into it carefully and keep brackets.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 3 — **Rearrange a one-step formula**
Use the inverse operation, e.g. $P = x+7$ gives $x = P-7$

Grade 4 **Rearrange a two-step formula**

Undo the operations in reverse order, e.g. $y = 3x - 5$ gives $x = \dfrac{y+5}{3}$

Grade 5 **Rearrange formulae with brackets or fractions**

Clear the fraction or bracket first, e.g. $P = \dfrac{a+b}{2}$ gives $a = 2P - b$

Grade 5 **Rearrange formulae with squares or square roots**

Undo a square with a square root and a square root by squaring, e.g. $A = \pi r^2$ gives $r = \sqrt{\dfrac{A}{\pi}}$

Grade 6 **Rearrange multi-step formulae with powers**

e.g. $v^2 = u^2 + 2as$ gives $u = \sqrt{v^2 - 2as}$

Grade 7 **Rearrange when the subject appears twice**

Collect the subject terms on one side and factorise, e.g. $ax + b = cx + d$ gives $x = \dfrac{d-b}{a-c}$

Grade 8 **Rearrange fractions with the subject twice**

e.g. $y = \dfrac{x+2}{x-3}$ gives $x = \dfrac{3y+2}{y-1}$

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