Sine rule, cosine rule and ½ab sin C

G22, G23 · Geometry and measures · revision notes

Higher tier only. The sine rule, the cosine rule and area = ½ab sin C let you find sides, angles and areas in triangles that are not right-angled. All three are on the formulae sheet in every paper, but the sheet does not tell you which rule a question needs or how to rearrange it.

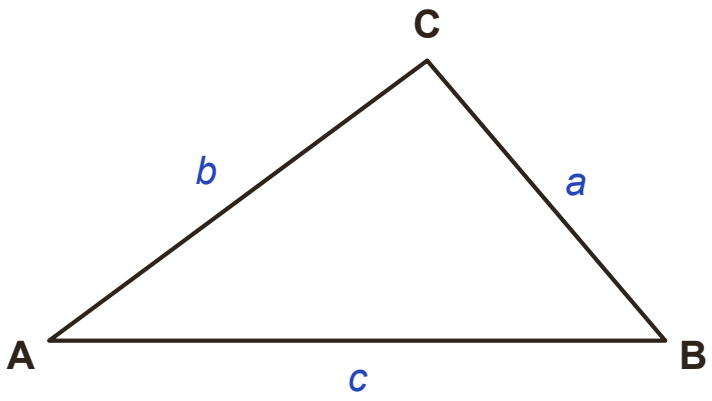
Key facts

- Sine rule (finding a side): $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
- Cosine rule (finding a side): $a^2 = b^2 + c^2 - 2bc\cos A$
- Area of a triangle = $\frac{1}{2}ab\sin C$ (C is between a and b)
- Matching pair → sine rule; two sides and included angle, or three sides → cosine rule
- Sine rule (finding an angle): $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- Cosine rule (finding an angle): $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
- Side a is always opposite angle A
- sin x = sin(180° − x), so an acute and an obtuse angle can have the same sine

Notes

Labelling any triangle

- Label the angles with capital letters A, B and C, and the sides with lower-case letters: side a is opposite angle A, b is opposite B, c is opposite C.



Each side has the same letter as the angle opposite it.

- These rules work in any triangle, including right-angled ones.

The sine rule

- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, or upside down:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

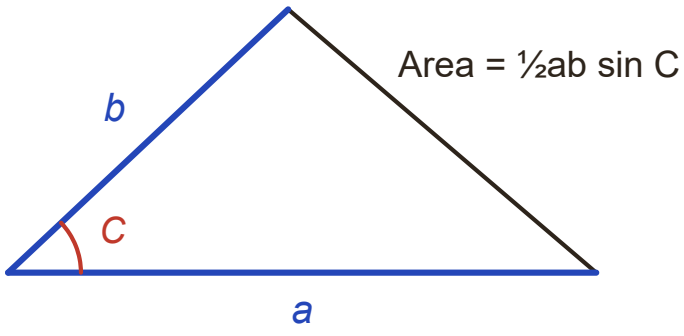
- Use it when you know a side and the angle **opposite** it (a matching pair), plus one more side or angle.
- Finding a side: use the version with the sides on top. Finding an angle: use the version with the sines on top.
- In a triangle, $\sin^{-1}$ gives an acute angle. If the angle you want is obtuse, the answer is 180° minus the calculator value, because sin x = sin(180° − x). Grade 8+ only

The cosine rule

- $a^2 = b^2 + c^2 - 2bc\cos A$: use it to find a side when you know the other two sides and the angle **between** them.
- Rearranged: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$: use it to find an angle when you know all three sides.
- Side a is opposite angle A; b and c are the two sides that make angle A.
- If A is obtuse, cos A is negative, so a is the longest side. If A = 90°, cos A = 0 and the rule becomes Pythagoras.

Area and choosing a rule

- Area of any triangle = $\frac{1}{2}ab\sin C$, where C is the angle between sides a and b.



C must be the angle between the two sides a and b.

- A matching side-and-opposite-angle pair: sine rule. Two sides and the included angle, or all three sides: cosine rule.
- In a multi-step problem, work one triangle at a time and keep full calculator accuracy until the end.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Find a side using the sine rule

3 marks **Grade 7**

1. Label the triangle and find the matching pair (a side and its opposite angle).
2. Write the sine rule with the sides on top and substitute.
3. Rearrange and calculate, then round.

Example. In triangle ABC, angle BAC = 42°, angle ABC = 73° and BC = 9.6 cm.
Work out the length of AC. Give your answer correct to 3 significant figures.

Model answer.

BC is opposite A and AC is opposite B, so AC ÷ sin 73° = 9.6 ÷ sin 42°	(M1)
AC = 9.6 × sin 73° ÷ sin 42°	(M1)
= 13.7 cm	(A1)

Find an angle using the cosine rule

3 marks **Grade 7**

1. Identify the side opposite the angle you want: that is a.
2. Substitute into $\cos A = (b^2 + c^2 - a^2) \div 2bc$.
3. Use $\cos^{-1}$ and round to 1 decimal place.

Example. In triangle PQR, PQ = 8 cm, QR = 11 cm and PR = 6 cm.
Work out the size of angle PQR. Give your answer correct to 1 decimal place.

Model answer.

PR is opposite angle Q, so $\cos Q = (8^2 + 11^2 - 6^2) \div (2 \times 8 \times 11)$	(M1)
$\cos Q = 149 \div 176$	(M1)
$Q = \cos^{-1}(149 \div 176) = 32.2°$	(A1)

Use area = ½ab sin C (often backwards)

3 marks | Grade 7

1. Check the angle is between the two sides you are using.
2. Substitute into area = ½ab sin C.
3. If the area is given, solve for sin C (or a side), then use $\sin^{-1}$ if you need the angle.

Example. Triangle XYZ has XY = 7 cm, XZ = 10 cm and area 28 cm². Angle YXZ is acute.
Work out the size of angle YXZ. Give your answer correct to 1 decimal place.

Model answer.

½ × 7 × 10 × sin X = 28	(M1)
35 sin X = 28, so sin X = 0.8	(M1)
X = $\sin^{-1}(0.8)$ = 53.1°	(A1)

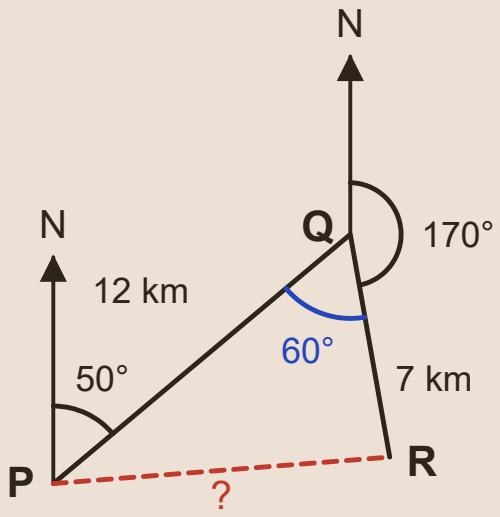
Multi-step problem (e.g. bearings)

4 to 5 marks | Grade 8

1. Draw a clear diagram with north lines and mark all known lengths and angles.
2. Use angle facts (e.g. co-interior angles on parallel north lines) to find the angle inside the triangle.
3. Choose the sine or cosine rule and calculate, keeping full accuracy.

Example. A ship sails 12 km from port P to point Q on a bearing of 050°. It then sails 7 km from Q to point R on a bearing of 170°.
Work out the distance PR. Give your answer correct to 3 significant figures.

Model answer.



The bearing of P from Q is 050° + 180° = 230°, so angle PQR = 230° − 170° = 60°	(B1)
$PR^2 = 12^2 + 7^2 - 2 \times 12 \times 7 \times \cos 60°$	(M1)
$PR^2 = 144 + 49 - 84 = 109$	(M1)
PR = √109 = 10.4 km	(A1)

Don’t lose marks

- Using the cosine rule with an angle that is not between the two known sides.
- Working out $b^2 + c^2 - 2bc$ first and then multiplying by cos A; 2bc cos A must be worked out before subtracting.
- Forgetting to square root at the end when finding a side with the cosine rule.
- Calculator in the wrong mode, or rounding sine and cosine values part-way through.
- Giving the acute answer from $\sin^{-1}$ when the diagram shows the angle is obtuse.

More tips

Memory tricks

- The cosine rule is Pythagoras with a correction term: if the angle is 90°, cos 90° = 0 and you get $a^2 = b^2 + c^2$.
- No matching pair of side and opposite angle? Use the cosine rule.
- For the area formula think 'side, angle, side': the angle is sandwiched between the two sides.
- Type the angle version of the cosine rule in one go with brackets: $(8^2 + 11^2 - 6^2) \div (2 \times 8 \times 11)$.
- Check: the longest side is opposite the largest angle.

Exam technique

- Write the rule with the numbers substituted before you calculate; this is usually the first method mark.
- Sketch the triangle and label a, b and c opposite A, B and C before choosing a rule.
- Round only at the end: lengths to 3 significant figures and angles to 1 decimal place unless told otherwise.
- In bearings questions, draw a north line at every point and use parallel-line angle facts to get the angle inside the triangle.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 6 — **Find a triangle's area using ½ab sin C**
Use two sides and the angle between them: area = ½ × a × b × sin C.

Grade 7 — **Find a missing side with the sine rule**
Use a ÷ sin A = b ÷ sin B when you know an angle and the side opposite it.

Grade 7 — **Find a missing angle with the sine rule**
Use sin A ÷ a = sin B ÷ b, then $\sin^{-1}$.

Grade 7 — **Find a missing side with the cosine rule**
Use $a^2 = b^2 + c^2 - 2bc \cos A$ when you know two sides and the angle between them.

Grade 7 — **Find a missing angle with the cosine rule**
Use $\cos A = (b^2 + c^2 - a^2) \div 2bc$ when you know all three sides.

Grade 8 — **Choose and combine rules in multi-step problems**
Pick the right rule at each stage, e.g. cosine rule for a side, then ½ab sin C for an area, including bearings problems.

Grade 9 — **Work backwards and handle obtuse angles**
Find an angle or side from a given area, and use 180° minus the $\sin^{-1}$ value when the angle is obtuse.

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