Similar shapes (length, area, volume)

G19, R12 · Geometry and measures · revision notes

Similar shapes are the same shape but a different size: their angles are equal and all their lengths are in the same ratio. You use the scale factor to find missing lengths and, on Higher, areas and volumes of similar shapes and solids.

Key facts

- Similar: equal angles and sides in the same ratio
- Two pairs of equal angles → similar triangles
- Grade 7+ only Area scale factor = k^2
- Grade 7+ only Lengths in ratio a : b → areas a^2 : b^2 → volumes a^3 : b^3
- Length scale factor k = new length ÷ corresponding original length
- A line parallel to a side of a triangle makes a similar triangle
- Grade 7+ only Volume scale factor = k^3

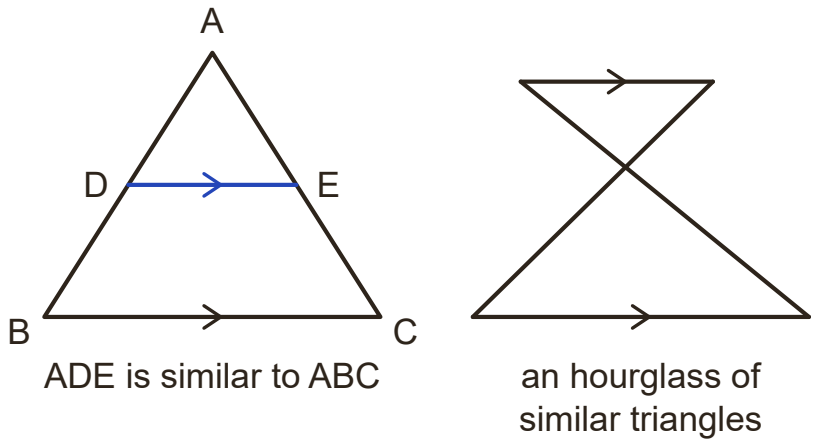
Notes

Similar shapes

- Two shapes are **similar** if one is an enlargement of the other: corresponding angles are equal and corresponding sides are in the same ratio.
- Length scale factor $k = \dfrac{\text{length on the new shape}}{\text{corresponding length on the original}}$
- Match corresponding sides carefully. If one shape is turned or flipped, redraw them the same way round.
- Two triangles are similar if their angles are equal. Two pairs of equal angles is enough, as the third angles must then be equal too.

Missing lengths

- Find k from a pair of corresponding sides you know. Multiply by k to go from the small shape to the large one; divide by k to go back.
- A line parallel to one side of a triangle cuts off a smaller, similar triangle (the corresponding angles are equal).
- Two parallel lines crossed by two straight lines that meet between them make an 'hourglass' of two similar triangles (alternate angles and vertically opposite angles are equal).

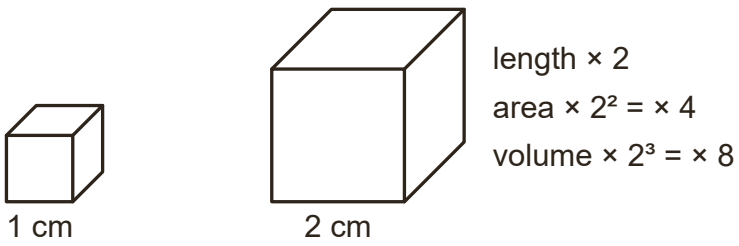


Parallel lines make similar triangles.

- Use whole sides: if AD = 4 cm and DB = 6 cm on the same line, the full side AB is 10 cm.

Areas and volumes Grade 7+ only

- If the length scale factor is k, the **area** scale factor is k^2 and the **volume** scale factor is k^3
- For example, k = 3: areas are 9 times larger and volumes 27 times larger.



Double the lengths: areas × 4, volumes × 8.

- Surface areas and areas of faces or labels use k^2. Capacity, and the mass of solids made of the same material, use k^3
- Going backwards: area factor 16 gives $k = \sqrt{16} = 4$; volume factor 8 gives $k = \sqrt[3]{8} = 2$
- Area to volume: square root the area factor to get k, then cube k.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Find missing lengths in similar shapes

2 marks each | **Grade 5**

1. Find a pair of corresponding sides you know both of.
2. Work out the scale factor (keep it as a fraction if it is not exact).
3. Multiply to go small to large; divide to go large to small.

Example. Triangle ABC is similar to triangle PQR. AB corresponds to PQ, BC to QR and AC to PR. AB = 6 cm, BC = 9 cm and PQ = 10 cm.
(a) Work out the length of QR. (2 marks)
(b) PR = 12.5 cm. Work out the length of AC. (2 marks)

Model answer.

(a) Scale factor = $10 \div 6 = \frac{5}{3}$, so QR = $9 \times \frac{5}{3}$ (M1) = 15 cm (A1)

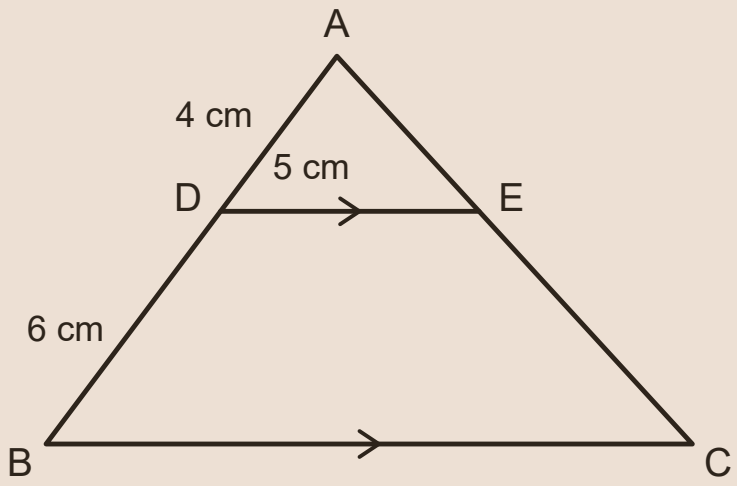
(b) AC = $12.5 \div \frac{5}{3}$ (M1) = 7.5 cm (A1)

Similar triangles from parallel lines

3 marks | **Grade 5**

1. Redraw the small triangle and the large triangle separately, the same way round.
2. Use whole side lengths for the large triangle.
3. Find the scale factor and use it.

Example. In triangle ABC, D is a point on AB and E is a point on AC so that DE is parallel to BC. AD = 4 cm, DB = 6 cm and DE = 5 cm.



Work out the length of BC.

Model answer.

AB = 4 + 6 = 10 cm (M1)
Scale factor = 10 ÷ 4 = 2.5 (M1)
BC = 5 × 2.5 = 12.5 cm (A1)

Areas and volumes of similar solids

4 to 5 marks | Grade 7

1. Find the length scale factor k from corresponding lengths.
2. Use k^2 for any area and k^3 for any volume or mass.
3. Multiply or divide, depending on which way you are going.

Example. Two bottles are mathematically similar. The small bottle has height 12 cm and volume 250 cm³. The large bottle has height 18 cm.
(a) Work out the volume of the large bottle. (3 marks)
(b) The label on the large bottle has area 81 cm². Work out the area of the label on the small bottle. (2 marks)

Model answer.

(a) k = 18 ÷ 12 = 1.5 (M1)
Volume = 250 × 1.5³ (M1) = 843.75 cm³ (A1)
(b) Area = 81 ÷ 1.5² (M1) = 36 cm² (A1)

Find the length scale factor from areas, then find a volume

4 marks | Grade 8

1. Divide the areas to get the area scale factor.
2. Square root it to get k.
3. Cube k to get the volume scale factor and use it.

Example. Solids A and B are mathematically similar. A has surface area 45 cm² and B has surface area 125 cm². The volume of A is 54 cm³.
Work out the volume of B.

Model answer.

Area scale factor = 125 ÷ 45 = $\frac{25}{9}$ (M1)

k = $\sqrt{\frac{25}{9}} = \frac{5}{3}$ (M1)

Volume scale factor = $\left(\frac{5}{3}\right)^3 = \frac{125}{27}$ (M1)

Volume of B = 54 × $\frac{125}{27}$ = 250 cm³ (A1)

Don't lose marks

- Using the short piece DB instead of the whole side AB when a small triangle sits inside a large one.
- Adding the difference in lengths instead of multiplying by the scale factor.
- Matching the wrong sides when one triangle is turned or flipped.

- Higher: using the length scale factor for an area or a volume.
- Higher: square rooting a volume factor instead of cube rooting it.

More tips

Memory tricks

- Redraw overlapping triangles as two separate triangles, the same way round, and label matching vertices.
- Sense check: if the new shape is bigger, your answer must be bigger than the original length.
- Keep scale factors as fractions (e.g. 5/3) to avoid rounding errors.
- Higher: 'length k, area k^2, volume k^3'. The power matches the units: cm, cm^2, cm^3.

Exam technique

- Write the scale factor clearly (e.g. k = 18 ÷ 12 = 1.5); it is usually the first method mark.
- To show triangles are similar, show that the angles are equal, giving a reason for each (e.g. corresponding angles are equal).
- Check which way you are going: small to large, multiply; large to small, divide.
- Higher: the masses of solids made of the same material scale like volumes, so use k^3.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 4 — **Recognise similar shapes and find scale factors**
Check that corresponding sides are all in the same ratio; scale factor = new length ÷ original length.

Grade 5 — **Find missing lengths in similar shapes**
Multiply or divide a length by the scale factor.

Grade 5 — **Use similar triangles made by parallel lines**
Parallel lines give equal corresponding or alternate angles, so the triangles are similar.

Grade 7 — **Use the area scale factor**
If lengths are multiplied by k, areas are multiplied by k^2.

Grade 7 — **Use the volume scale factor**
If lengths are multiplied by k, volumes (and masses of the same material) are multiplied by k^3.

Grade 8 — **Find length scale factors from areas or volumes**
Square root the area factor or cube root the volume factor to get k, then use it.

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