Simultaneous equations

A19 · Algebra · revision notes

Solving two equations with two unknowns at the same time. Both tiers solve two linear equations (by elimination, substitution or from a graph) and form them from worded problems; Higher also solves a linear equation with a quadratic one.

Key facts

- Same signs: subtract. Different signs: add
- Substitute back to find the second letter
- Graph method: the solution is the intersection point
- Match coefficients by multiplying every term of an equation
- Check both values in the other equation
- Grade 7+ only Linear and quadratic: substitute the linear into the quadratic; expect two pairs of answers

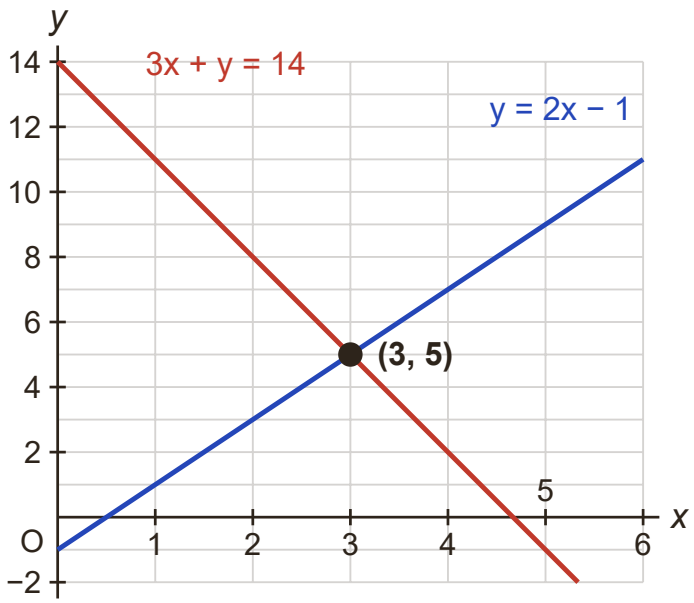
Notes

Elimination

- Label the equations (1) and (2).
- Make the coefficients of one letter the same size, by multiplying one or both whole equations.
- If the signs are the same, subtract the equations; if they are different, add them.
- Solve for the letter that is left, then substitute into either original equation to find the other.
- $3x + 2y = 16$ and $5x - 2y = 16$: adding gives $8x = 32$, so $x = 4$. Then $12 + 2y = 16$, so $y = 2$
- If nothing matches, multiply both: for $2x + 3y = 12$ and $3x + 4y = 17$, do (1) × 3 and (2) × 2 to get $6x$ in both, then subtract: $y = 2$ and $x = 3$

Substitution and graphs

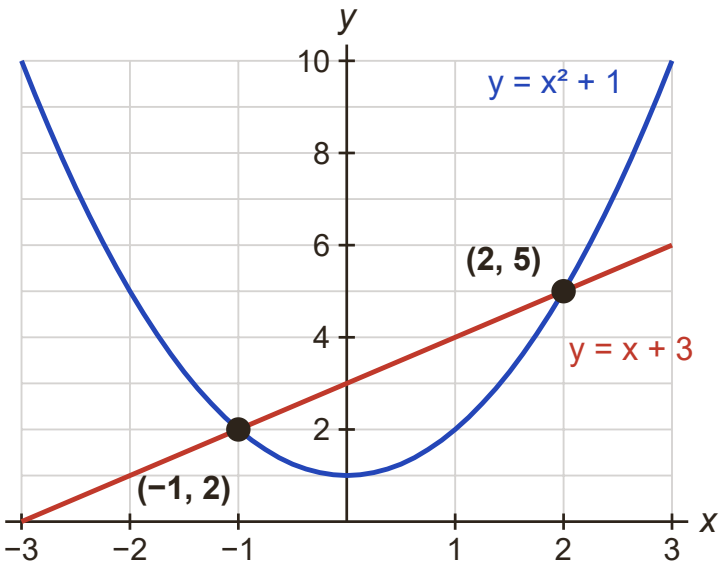
- If one equation is already $y = \ldots$ (or $x = \ldots$), substitute it into the other.
- $y = 2x - 1$ and $3x + y = 14$: $3x + 2x - 1 = 14$, so $x = 3$ and $y = 5$
- On a graph, the solution is the point where the two lines intersect. Solutions read from a graph are estimates.



The lines cross at $(3, 5)$, so $x = 3$ and $y = 5$

One linear and one quadratic Grade 7+ only

- Rearrange the linear equation to $y = \ldots$ (or $x = \ldots$) and substitute it into the quadratic.
- Rearrange to a quadratic equal to 0 and solve it.
- Substitute each x into the **linear** equation to find its y, and give the answers as pairs.
- $y = x + 3$ and $y = x^2 + 1$: $x^2 + 1 = x + 3$, so $x^2 - x - 2 = 0$ and $(x - 2)(x + 1) = 0$. Solutions: $x = 2, y = 5$ and $x = -1, y = 2$
- On a graph, these are the points where the line meets the curve.



Two meeting points, so two pairs of solutions.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, P = a process mark.

Solve two linear equations 3 marks Grade 5

1. Multiply one or both equations so that one letter has the same coefficient.
2. Add or subtract to eliminate it, then solve.
3. Substitute back to find the other letter, and check.

Example. Solve the simultaneous equations

$4x + 3y = 5$
$3x - 2y = 8$

Model answer.

(1) × 2: $8x + 6y = 10$; (2) × 3: $9x - 6y = 24$. Add: $17x = 34$ M1

$x = 2$, then $8 + 3y = 5$ M1

$x = 2$, $y = -1$ A1

Form and solve (worded problem)

4 marks | Grade 5

1. Choose letters and write two equations, using the same units throughout.
2. Solve by elimination.
3. Answer the question asked, with units.

Example. At a café, 3 teas and 2 coffees cost £8.10. 2 teas and 3 coffees cost £8.65. Work out the cost of one coffee.

Model answer.

$3t + 2c = 810$ and $2t + 3c = 865$ (pence)	P1
$6t + 4c = 1620$ and $6t + 9c = 2595$	P1
Subtract: $5c = 975$, $c = 195$	P1
One coffee costs £1.95	A1

Solve a linear and a quadratic equation

5 marks | Grade 7

1. Substitute the linear equation into the quadratic.
2. Rearrange to equal 0 and solve.
3. Find the matching y for each x and pair them.

Example. Solve the simultaneous equations

$y = x^2 - 3x + 4$

$y = 2x - 2$

Model answer.

$x^2 - 3x + 4 = 2x - 2$	M1
$x^2 - 5x + 6 = 0$	M1
$(x - 2)(x - 3) = 0$	M1
$x = 2$ or $x = 3$	A1
$x = 2, y = 2$ and $x = 3, y = 4$	A1

Solve with a circle equation

5 marks | Grade 8

1. Substitute the line into $x^2 + y^2 = r^2$, squaring the bracket carefully.
2. Simplify to a quadratic equal to 0 and solve.
3. Find y from the line for each x.

Example. Solve the simultaneous equations

$x^2 + y^2 = 25$

$y = x + 1$

Model answer.

$x^2 + (x+1)^2 = 25$	M1
$2x^2 + 2x - 24 = 0$, so $x^2 + x - 12 = 0$	M1
$(x+4)(x-3) = 0$	M1
$x = 3, y = 4$	A1
$x = -4, y = -3$	A1

Don't lose marks

- Multiplying only some terms of an equation when matching coefficients.
- Subtracting when the signs are different (or adding when they are the same).
- Slipping with negatives when subtracting: $5 - (-3) = 8$
- Giving x and y when the question asks for the cost of one item.
- Higher: mixing up the pairs, e.g. writing $x = 2, y = 4$ when $x = 2$ goes with $y = 2$

More tips

Memory tricks

- Check your pair in the equation you did not use to find the second value.
- Eliminate the letter that is easiest: one that already has opposite signs, or where one coefficient is a multiple of the other.
- In money problems, work in pence to avoid decimals.
- Higher: a line and a curve usually give two pairs of answers; keep each x with its own y.

Exam technique

- When the question says 'You must show all your working', answers found by trial and improvement (or a calculator's solver alone) may score no marks.
- Write the answer clearly as $x = \ldots$, $y = \ldots$, or answer the question in context with units.
- For the graph method, read both coordinates as accurately as the grid allows.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 4 **Read a solution from two line graphs**
The solution is the coordinates of the point where the two lines cross.

Grade 5 **Solve linear simultaneous equations by elimination**
Make the coefficients of one letter the same, then add or subtract the equations.

Grade 5 **Form and solve simultaneous equations in context**
e.g. 3 teas and 2 coffees cost £8.10 gives $3t + 2c = 810$ (in pence).

Grade 7 **Solve linear and quadratic simultaneous equations**
Substitute the linear equation into the quadratic, solve, then find the matching values.

Grade 8 **Solve a line and a circle simultaneously**
Substitute into $x^2 + y^2 = r^2$, e.g. $y = x + 1$ and $x^2 + y^2 = 13$ meet at $(2, 3)$ and $(-3, -2)$

Free notes, Full packs, practice papers and mock exams at chhetriacademy.com. Written for this site; not taken from past papers. Stuck? 1-to-1 tutoring with a free 30-minute trial lesson.