Sequences and the nth term

A23–A25 · Algebra · revision notes

Generating terms from term-to-term and position-to-term rules, recognising special sequences, and finding the nth term of linear sequences (both tiers) and quadratic sequences (Higher). Includes arithmetic, geometric and Fibonacci-type sequences.

Key facts

- Linear nth term: $dn + c$, where d is the common difference and c = first term − d
- Geometric: multiply by the same number each time, e.g. 2, 4, 8, 16 has nth term 2^n
- Is x a term? Solve nth term = x; n must be a positive whole number
- Square numbers n^2, cube numbers n^3, triangular numbers $\frac{n(n+1)}{2}$
- Fibonacci-type: add the previous two terms
- Grade 7+ only Quadratic nth term $an^2 + bn + c$: a is half the second difference

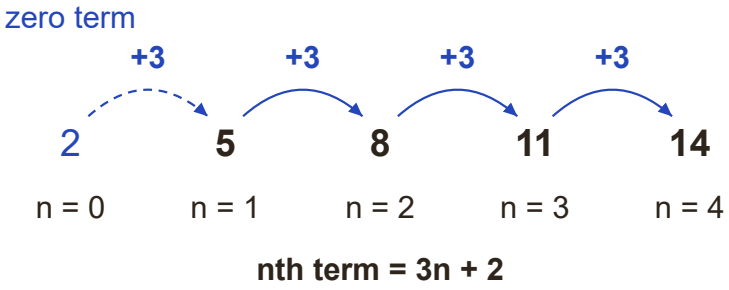
Notes

Types of sequence

- **Arithmetic (linear)**: add or subtract the same amount each time, e.g. 7, 10, 13, 16 (common difference 3).
- **Geometric**: multiply by the same number r each time, e.g. 3, 6, 12, 24 (r = 2) or 80, 40, 20, 10 ($r = \frac{1}{2}$).
- The ratio r can be a surd: 2, $2\sqrt{3}$, 6, $6\sqrt{3}$, 18 multiplies by $\sqrt{3}$ each time. Grade 8+ only
- **Fibonacci-type**: each term is the sum of the two before it, e.g. 1, 1, 2, 3, 5, 8.
- **Quadratic**: the differences change but the second differences are constant, e.g. 2, 5, 10, 17 (differences 3, 5, 7).
- Special sequences: square numbers 1, 4, 9, 16, 25; cube numbers 1, 8, 27, 64; triangular numbers 1, 3, 6, 10, 15.

Linear sequences and the nth term

- A term-to-term rule links each term to the one before. The nth term (position-to-term rule) gives any term from its position n.
- Find the common difference d. The nth term is $dn + c$, where c = first term − d.
- 5, 8, 11, 14: $d = 3$ and $c = 5 - 3 = 2$, so the nth term is $3n + 2$

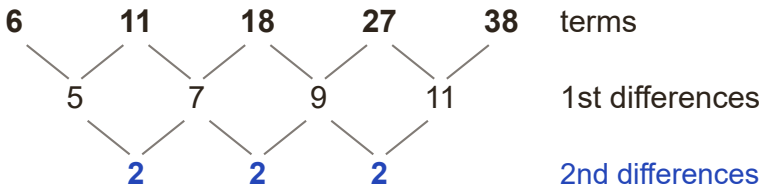


Difference 3 gives $3n$; step back one term to get the number to add (2).

- Decreasing sequences have a negative difference: 20, 16, 12, 8 has nth term $24 - 4n$
- Is 100 in the sequence $3n + 2$? $3n + 2 = 100$ gives $n = \frac{98}{3}$, which is not a whole number, so no.

Quadratic sequences Grade 7+ only

- Find the first and second differences. The coefficient of n^2 is half the second difference.
- Subtract an^2 from each term. What is left is linear: find its nth term and add it on.
- 6, 11, 18, 27, 38: differences 5, 7, 9, 11, second difference 2, so n^2. Subtracting n^2 (1, 4, 9, 16, 25) leaves 5, 7, 9, 11, 13, which is $2n + 3$. The nth term is $n^2 + 2n + 3$



2nd difference 2, so the n² term is ½ × 2 = 1n²

Constant second differences mean a quadratic sequence.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, C = a communication mark.

Find the nth term (linear)

2 marks | Grade 4

1. Find the common difference: this is the number in front of n.
2. Compare with the times table to find what to add or subtract.
3. Check with the first two terms.

Example. Here are the first five terms of a sequence: 2, 9, 16, 23, 30. Find an expression, in terms of n, for the nth term.

Model answer.

Common difference 7, so $7n + k$ M1

$7n - 5$ A1

Is this number a term? Give a reason

2 marks | Grade 5

1. Set the nth term equal to the number.
2. Solve for n.
3. Conclude: it is a term only if n is a positive whole number.

Example. The nth term of a sequence is $4n + 3$. Is 125 a term in the sequence? Give a reason for your answer.

Model answer.

$4n + 3 = 125$ gives $n = 30.5$ M1

No, because n is not a whole number (the 30th term is 123 and the 31st is 127) C1

Fibonacci-type sequence with an unknown

3 marks | Grade 5

1. Write the terms in terms of the unknown, adding the two before each time.
2. Set the given term equal to its value.
3. Solve.

Example. The rule for a sequence is: add the two previous terms. The first term is 3 and the second term is b. The fifth term is 21. Find b.

Model answer.

Terms: 3, b, $3 + b$, $3 + 2b$, $6 + 3b$ M1

$6 + 3b = 21$ M1

$b = 5$ A1

Find the nth term (quadratic)

3 marks | Grade 7

1. Find the second difference and halve it for the n^2 coefficient.
2. Subtract an^2 from each term.
3. Find the nth term of what is left and add it on.

Example. Find an expression for the nth term of the quadratic sequence 5, 12, 23, 38, 57.

Model answer.

Differences 7, 11, 15, 19; second difference 4, so $2n^2$	M1
Subtract $2n^2$: 3, 4, 5, 6, 7, which is $n + 2$	M1
$2n^2 + n + 2$	A1

Don't lose marks

- Writing $n + 3$ for a sequence that goes up in 3s: the difference multiplies n, so it is $3n + \ldots$
- Giving the term-to-term rule ('add 3') when the question asks for the nth term.
- Forgetting the negative for decreasing sequences: 20, 16, 12 has nth term $24 - 4n$, not $4n + 24$
- Higher: using the whole second difference as the n^2 coefficient instead of half of it.

More tips

Memory tricks

- Check an nth term by putting in $n = 1$ and $n = 2$: you should get the first two terms.
- Zero term trick: work back one step from the first term to find c. For 5, 8, 11, the zero term is 2, so the nth term is $3n + 2$
- $3n + 2$ is the 3 times table with 2 added to each term.
- Higher: 'half the second difference' gives the n^2 coefficient.

Exam technique

- 'Find an expression in terms of n' needs an answer containing n, e.g. $6n - 1$, not 'add 6'.
- When asked 'is x a term? Give a reason', show your calculation and finish with a clear yes or no.
- Read whether you are given terms or the nth term before you start.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 2 — **Continue a sequence using a term-to-term rule**
e.g. 'add 4' starting at 3 gives 3, 7, 11, 15, …; 'multiply by 2' starting at 5 gives 5, 10, 20, 40, …

Grade 3 — **Recognise square, cube and triangular numbers**
Squares 1, 4, 9, 16, …; cubes 1, 8, 27, 64, …; triangular numbers 1, 3, 6, 10, …

Grade 4

Generate terms from an nth term

Substitute $n = 1, 2, 3, \ldots$, e.g. $3n + 2$ gives 5, 8, 11, …

Grade 4

Find the nth term of a linear sequence

The common difference multiplies n, then adjust: 5, 9, 13, 17, … has nth term $4n + 1$

Grade 5

Decide if a number is in a sequence

Set the nth term equal to the number and check whether n is a positive whole number.

Grade 5

Continue Fibonacci-type and geometric sequences

Fibonacci-type: add the previous two terms. Geometric: multiply by the same number each time.

Grade 7

Find the nth term of a quadratic sequence

Halve the second difference to get the coefficient of n^2, subtract that n^2 term, then find the linear part.

Grade 8

Continue geometric sequences involving surds

e.g. 2, $2\sqrt{3}$, 6, $6\sqrt{3}$, …: multiply by $\sqrt{3}$ each time.

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