Rounding, estimation and error intervals

N14, N15 · Number · revision notes

Rounding to decimal places and significant figures, estimating answers by rounding to 1 significant figure, and writing error intervals for rounded or truncated values using inequalities. These come up on both papers, often as quick 1 to 3 mark questions.

Key facts

- Next digit 5 or more → round up; 4 or less → round down
- Estimate: round every number to 1 s.f.
- Truncated to 1 d.p.: 7.3 gives $7.3 \leq x < 7.4$
- ÷ 0.1 is the same as × 10; ÷ 0.2 is × 5; ÷ 0.5 is × 2
- Significant figures start at the first non-zero digit
- Rounded to 1 d.p.: 7.3 gives $7.25 \leq x < 7.35$
- ≈ means 'is approximately equal to'

Notes

Rounding

- Find the last digit you are keeping and look at the next digit. 5 or more: round up. 4 or less: leave the digit as it is.
- Decimal places (d.p.) count the digits after the decimal point: 12.3649 = 12.36 (2 d.p.).
- Significant figures (s.f.) start at the first non-zero digit: 0.004718 = 0.0047 (2 s.f.) and 28,461 = 28,000 (2 s.f.).
- Keep the place value: 28,461 to 2 s.f. is 28,000, not 28. The zeros hold the place.
- Rounding can carry: 4.97 to 1 d.p. is 5.0, and you must write the 0.
- If a question doesn't say how to round, write down the full answer, then give it to 3 s.f. (money to 2 d.p., e.g. £4.50).

Estimating

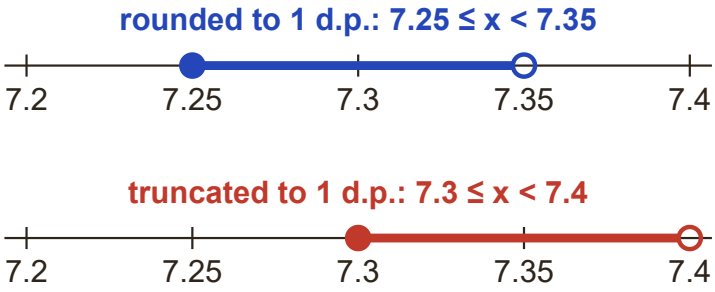
- Round every number to 1 significant figure, then calculate:

$$\frac{61.3 \times 4.82}{0.198} \approx \frac{60 \times 5}{0.2}$$
$$= \frac{300}{0.2}$$
$$= 1500$$

- Dividing by 0.2 is the same as multiplying by 5; dividing by 0.5 is the same as multiplying by 2.
- For a square root, round to a nearby square number: $\sqrt{24.6} \approx \sqrt{25} = 5$
- Use ≈ ('is approximately equal to') for an estimate.
- Overestimate or underestimate? Making the numbers you add or multiply bigger, or the number you divide by smaller, makes the answer bigger (an overestimate).

Error intervals

- A rounded value could have come from a range of values: this is its **error interval**.
- Rounding: go half a unit down and half a unit up. 'x = 7.3 to 1 d.p.' gives $7.25 \leq x < 7.35$. The upper value is not included, because 7.35 would round to 7.4.
- To the nearest 10: 'n = 80' gives $75 \leq n < 85$
- Truncating means chopping off digits without rounding. 'x = 7.3, truncated to 1 d.p.' gives $7.3 \leq x < 7.4$



Filled circle: the value is included ($\leq$). Open circle: it is not included ($<$).

- Write error intervals with $\leq$ on the left and $<$ on the right.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, C = a communication mark.

Round to decimal places or significant figures

1 mark each | Grade 3

1. Count to the last digit you keep (for s.f., start at the first non-zero digit).
2. Look at the next digit to decide whether to round up.
3. Put back zeros to keep the place value of large numbers.

Example. (a) Round 0.06058 to 2 significant figures.
(b) Round 1396.7 to 3 significant figures.

Model answer.

(a) 0.061 (B1)
(b) 1400 (B1)

Work out an estimate, and say if it is too big or too small

3 to 4 marks | Grade 4

1. Round every number to 1 significant figure and write the rounded numbers down.
2. Work out the calculation with the rounded numbers.
3. For over or under: look at which way each number was rounded and what that does to the answer, and give that as your reason.

Example. (a) Work out an estimate for $\dfrac{3.87 \times 59.6}{0.203}$
(b) Is your answer to (a) an overestimate or an underestimate? Give a reason for your answer.

Model answer.

(a) $\dfrac{4 \times 60}{0.2}$ (M1 for at least two of 4, 60, 0.2)

$= \dfrac{240}{0.2} = 1200$ (A1)

(b) Overestimate: 3.87 and 59.6 were rounded up, and 0.203 was rounded down, so you divide a bigger number by a smaller number (C1)

Write down the error interval

1 to 2 marks each | Grade 5

1. Find the unit it was rounded to (e.g. 0.1 for 1 d.p.).
2. Rounding: subtract and add half of that unit. Truncation: the value itself up to one whole unit more.
3. Write it as an inequality with the letter in the middle: $\leq$ on the left, $<$ on the right.

Example. (a) The length of a pencil, L cm, is 8.4 cm correct to 1 decimal place. Write down the error interval for L.
(b) Nia's time, t seconds, is 12 seconds when truncated to a whole number of seconds. Write down the error interval for t

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Model answer.

(a) $8.35 \leq L < 8.45$ (B2, or B1 for 8.35 and 8.45 with the wrong signs, or one end correct)
(b) $12 \leq t < 13$ (B1)

Don't lose marks

- Losing the place value: 28,461 to 2 s.f. is 28,000, not 28.
- Counting zeros at the front as significant: 0.00471 to 1 s.f. is 0.005.
- Not writing down the rounded numbers in an estimate: the method mark is for seeing them.
- Rounding to 1 d.p. instead of 1 s.f. in an estimate, or using the exact values.
- Writing $\leq$ at both ends of a rounding error interval, e.g. $8.35 \leq L \leq 8.45$
- Writing 3.96 to 1 d.p. as 3.10 or 4 instead of 4.0.

More tips

Memory tricks

- For s.f., the first significant figure is the first digit that isn't 0; after that, zeros count.
- Error interval for rounding: half a unit down, half a unit up; $\leq$ on the left, $<$ on the right.
- Truncation just chops: the value can't be below the truncated number, and goes up to (but not including) the next one.
- Dividing by a decimal in an estimate? Multiply top and bottom by 10: 240 ÷ 0.2 = 2400 ÷ 2 = 1200.

Exam technique

- In 'work out an estimate' questions, show the rounded numbers. An exact answer from a calculator gets no marks.
- When asked for 3 significant figures, write the full calculator value first, then round.
- For error intervals, use the letter from the question and put it in the middle of the inequality.
- If asked 'overestimate or underestimate?', give a reason linked to how each number was rounded.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 1

Round to the nearest 10, 100 or 1000

Look at the next digit: 5 or more rounds up, so 3472 is 3500 to the nearest 100.

Grade 2

Round to a number of decimal places

Count digits after the decimal point: 6.4851 is 6.49 to 2 decimal places.

Grade 3

Round to significant figures

Start counting at the first non-zero digit: 0.03064 is 0.0306 to 3 significant figures.

Grade 3

Estimate by rounding to 1 significant figure

Round every number to 1 s.f., then calculate: 38.2 × 5.9 ≈ 40 × 6 = 240.

Grade 4

Decide if an estimate is over or under

Use how each number was rounded, e.g. rounding up the numbers you multiply gives an overestimate.

Grade 5

Write error intervals for rounded values

7.3 to 1 decimal place means $7.25 \leq x < 7.35$

Grade 5

Write error intervals for truncated values

7.3 truncated to 1 decimal place means $7.3 \leq x < 7.4$

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