Solving quadratic equations

A18 · Algebra · revision notes

Solving equations such as $x^2 - 5x + 6 = 0$. Foundation: factorising $x^2 + bx + c$ and reading solutions from a graph. Higher: factorising $ax^2 + bx + c$, the quadratic formula, and equations that need rearranging first (completing the square has its own notes).

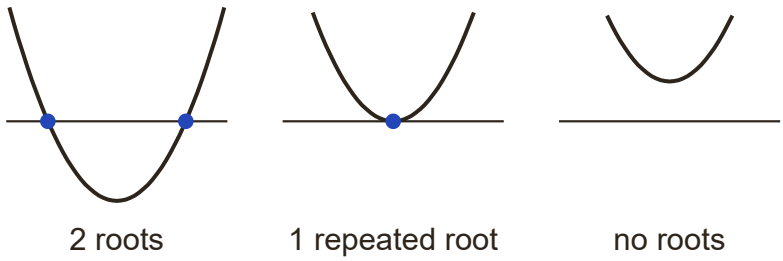
Key facts

- Rearrange to $ax^2 + bx + c = 0$ first
- $x^2 = k$ gives $x = \pm\sqrt{k}$
- Grade 6+ only Answers to d.p. or s.f. asked for: use the formula
- If $pq = 0$, then $p = 0$ or $q = 0$
- Grade 6+ only $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (on the formulae sheet)
- Roots are the x-coordinates where the curve crosses the x-axis

Notes

Rearrange to equal 0

- A quadratic equation has an x^2 term and no higher power of x. Get everything on one side so that it equals 0.
- $x^2 + 3x = 10$ becomes $x^2 + 3x - 10 = 0$
- A quadratic usually has two solutions (roots). Some have one repeated root, and some have no solutions.



The roots are where the graph meets the x-axis.

Solving by factorising

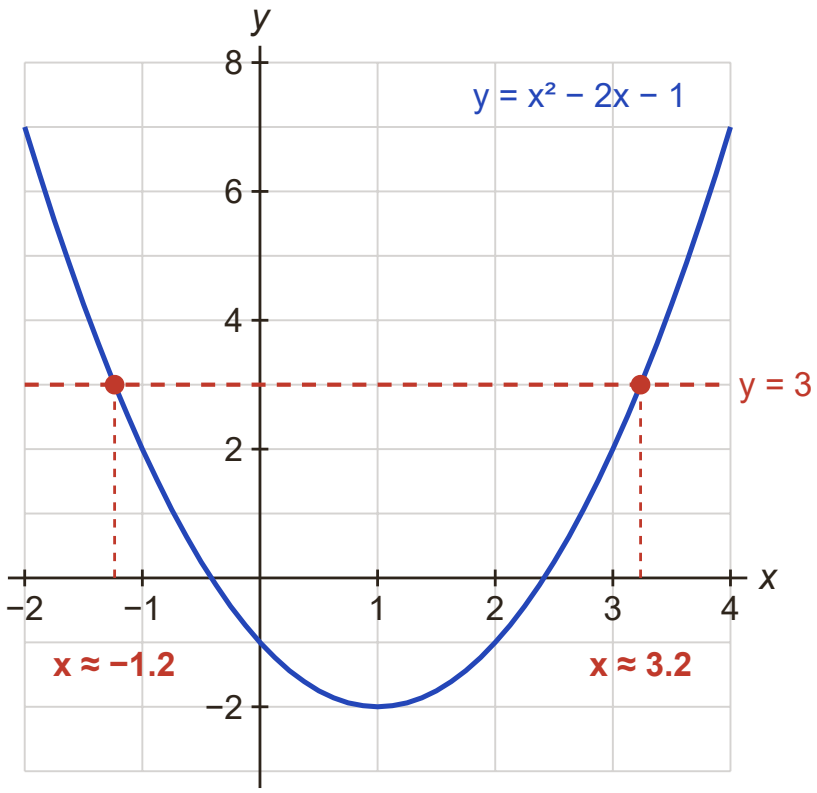
- Factorise, then use the fact that if two things multiply to give 0, one of them must be 0.
- $x^2 + 3x - 10 = 0$ gives $(x + 5)(x - 2) = 0$, so $x = -5$ or $x = 2$
- Common factor: $x^2 - 7x = 0$ gives $x(x - 7) = 0$, so $x = 0$ or $x = 7$. Don't divide by x, or you lose $x = 0$
- $x^2 = 36$ has two solutions: $x = 6$ or $x = -6$
- Harder: $3x^2 - 10x - 8 = 0$ gives $(3x + 2)(x - 4) = 0$, so $x = -\dfrac{2}{3}$ or $x = 4$ Grade 6+ only

The quadratic formula Grade 6+ only

- For $ax^2 + bx + c = 0$: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. It is on the formulae sheet: you must pick out a, b and c correctly and substitute them.
- Use it when the answers must be given to decimal places or significant figures, as the quadratic will not factorise.
- Write down a, b and c with their signs first. If $b = -3$, then $-b = 3$ and $b^2 = 9$
- $2x^2 - 3x - 4 = 0$: $x = \dfrac{3 \pm \sqrt{9 + 32}}{4} = \dfrac{3 \pm \sqrt{41}}{4}$, so $x = 2.35$ or $x = -0.85$ (2 d.p.).
- If $b^2 - 4ac$ is negative, there are no solutions: the graph does not cross the x-axis.

Using a graph

- The solutions of $x^2 + bx + c = 0$ are the x-coordinates where the graph of $y = x^2 + bx + c$ crosses the x-axis.
- To solve $x^2 - 2x - 1 = 3$ using the graph of $y = x^2 - 2x - 1$, read the x-coordinates where the curve meets the line $y = 3$



Read down from where the curve meets $y = 3$: $x \approx -1.2$ or $x \approx 3.2$

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, P = a process mark.

Solve by factorising

3 marks | Grade 5

1. Check the equation equals 0.
2. Factorise.
3. Set each bracket equal to 0 and solve.

Example. Solve $x^2 + 2x - 24 = 0$

Model answer.

$(x + 6)(x - 4) = 0$ M1

$x = -6$ A1

$x = 4$ A1

Solve using the formula, to 2 decimal places

3 marks | Grade 6

1. Write down a, b and c.
2. Substitute into the formula, using brackets for negatives.
3. Work out both answers and round as asked.

Example. Solve $3x^2 + 5x - 1 = 0$. Give your solutions correct to 2 decimal places.

Model answer.

$x = \dfrac{-5 \pm \sqrt{5^2 - 4 \times 3 \times (-1)}}{2 \times 3}$ M1

$= \dfrac{-5 \pm \sqrt{37}}{6}$ M1

$x = 0.18$ or $x = -1.85$ A1

Rearrange, then solve

4 marks | Grade 6

1. Expand brackets (or clear fractions).
2. Collect everything on one side to make it equal 0.
3. Factorise (or use the formula) and solve.

Example. Solve $(x+2)^2 = 3x + 10$

Model answer.

$x^2 + 4x + 4 = 3x + 10$ M1

$x^2 + x - 6 = 0$ M1

$(x+3)(x-2) = 0$ M1

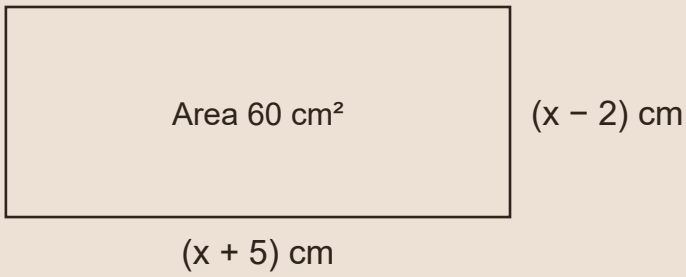
$x = -3$ or $x = 2$ A1

Form and solve a quadratic (area problem)

4 marks | Grade 7

1. Write an expression for the area and set it equal to the given value.
2. Rearrange to equal 0 and solve.
3. Reject any value that gives a negative length, then answer the question.

Example. A rectangle has length $(x+5)$ cm and width $(x-2)$ cm. Its area is 60 cm^2. Work out the perimeter of the rectangle.



Model answer.

$(x+5)(x-2) = 60$ P1

$x^2 + 3x - 70 = 0$ P1

$(x+10)(x-7) = 0$, so $x = 7$ ($x = -10$ gives a negative width) P1

Length 12 cm and width 5 cm, so the perimeter is 34 cm A1

Don't lose marks

- Not rearranging to equal 0 first, e.g. solving $x(x+1) = 12$ as $x = 12$ or $x + 1 = 12$
- Wrong signs: $(x+6)(x-4) = 0$ gives $x = -6$ and $x = 4$, not 6 and -4

- Dividing both sides by x and losing the solution $x = 0$
- Forgetting the negative square root: $x^2 = 25$ gives $x = 5$ or $x = -5$
- Higher: in the formula, the whole of $-b \pm \sqrt{b^2 - 4ac}$ is divided by $2a$

More tips

Memory tricks

- Check each root in the original equation: $4^2 + 2 \times 4 - 24 = 0$
- For $x^2 + bx + c = 0$, the two roots add up to $-b$ and multiply to give c: a quick check.
- If your calculator has an equation solver, use it to check, but always write out the method to get the marks.
- Higher: if the question asks for answers to 2 d.p. or 3 s.f., it won't factorise, so go straight to the formula.

Exam technique

- 'Solve' needs values of x: if you only factorise, you only get the method mark.
- Higher: write the formula with the numbers substituted before using your calculator: that line is the method mark.
- In context questions, say which solution you reject and why.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 4 — **Read solutions from a quadratic graph**
The solutions of $x^2 + bx + c = 0$ are the x-coordinates where $y = x^2 + bx + c$ crosses the x-axis.

Grade 5 — **Solve simple quadratics by factorising**
Factorise, then set each bracket equal to 0, e.g. $(x - 3)(x + 7) = 0$ gives $x = 3$ or $x = -7$

Grade 5 — **Solve quadratics with a missing term**
$x^2 = 49$ gives $x = \pm 7$; $x^2 - 5x = 0$ gives $x(x - 5) = 0$, so $x = 0$ or $x = 5$

Grade 6 — **Factorise harder quadratics to solve them**
e.g. $2x^2 + x - 6 = 0$ gives $(2x - 3)(x + 2) = 0$, so $x = 1.5$ or $x = -2$

Grade 6 — **Use the quadratic formula**
Substitute a, b and c into $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and round as instructed.

Grade 6 — **Rearrange an equation into a quadratic**
Expand brackets or clear fractions to get $ax^2 + bx + c = 0$ before solving.

Grade 7 — **Form and solve a quadratic in context**
e.g. from an area; reject any solution that is impossible, such as a negative length.

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