Pythagoras' theorem (incl. 3D)

G20 · Geometry and measures · revision notes

Pythagoras' theorem links the three sides of a right-angled triangle. You use it to find a missing side, the distance between two points and, on Higher, lengths inside 3D shapes such as cuboids and pyramids. It appears on both tiers, often inside a longer problem.

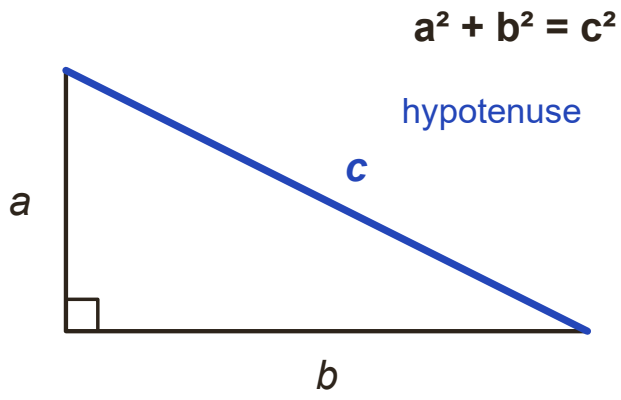
Key facts

- $a^2 + b^2 = c^2$, where c is the hypotenuse (opposite the right angle)
- Shorter side: $a = \sqrt{c^2 - b^2}$
- Right-angled test: (longest side)² = sum of the squares of the other two sides
- Grade 7+ only Cuboid space diagonal: $d = \sqrt{a^2 + b^2 + c^2}$
- Hypotenuse: $c = \sqrt{a^2 + b^2}$
- Distance between two points = $\sqrt{(\text{difference in } x)^2 + (\text{difference in } y)^2}$
- Common whole-number sides: 3, 4, 5; 5, 12, 13; 8, 15, 17 (and multiples, e.g. 6, 8, 10)

Notes

The theorem

- In a right-angled triangle, the **hypotenuse** is the longest side. It is opposite the right angle.
- Pythagoras' theorem: $a^2 + b^2 = c^2$, where c is the hypotenuse and a and b are the two shorter sides.



The hypotenuse c is always opposite the right angle.

- It works only in right-angled triangles.

Finding a missing side

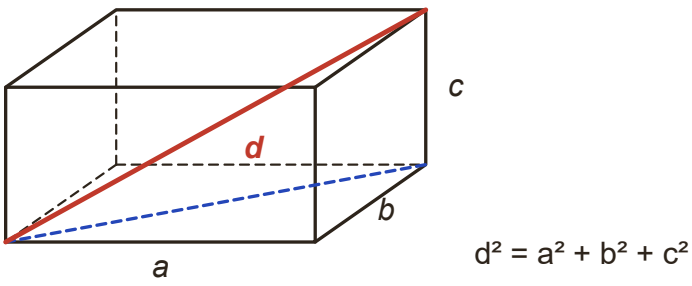
- Finding the hypotenuse: **add** the squares, then square root. Sides 6 cm and 8 cm give $c = \sqrt{36 + 64} = \sqrt{100} = 10$ cm.
- Finding a shorter side: **subtract** the squares (hypotenuse squared minus the other side squared), then square root. Hypotenuse 13 cm and side 5 cm give $\sqrt{169 - 25} = \sqrt{144} = 12$ cm.
- Sense check: the hypotenuse must be longer than each of the other two sides.
- For an exact answer, simplify the surd: $\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$ Grade 6+ only

Using it in problems

- Sketch the right-angled triangle on its own and label the sides you know.
- The diagonal of a rectangle makes two right-angled triangles. An isosceles triangle splits into two right-angled triangles along its line of symmetry.
- Distance between two points: find the horizontal and vertical differences and use them as a and b. From (1, 2) to (7, 10) the differences are 6 and 8, so the distance is 10 units.
- Converse: if $a^2 + b^2 = c^2$ where c is the longest side, the triangle is right-angled. If not, it is not right-angled.

Pythagoras in 3D Grade 7+ only

- Find a right-angled triangle that contains the length you want. You often need to find one of its sides first, using another right-angled triangle.
- Cuboid with edges a, b and c: space diagonal $d = \sqrt{a^2 + b^2 + c^2}$



First find the base diagonal (dashed), then use it with the height c to find d.

- For a pyramid with its apex directly above the centre of a rectangular base, the vertical height, half the base diagonal and a sloping edge form a right-angled triangle.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, C = a communication mark.

Work out the length of a side

3 marks | Grade 4

1. Find the hypotenuse: the side opposite the right angle.
2. Square the two known sides. Add them to find the hypotenuse; subtract them to find a shorter side.
3. Square root, then round as the question asks.

Example. ABC is a triangle with a right angle at B. AB = 7.2 cm and AC = 11.5 cm.
Work out the length of BC. Give your answer correct to 1 decimal place.

Model answer.

AC is the hypotenuse, so $BC^2 = 11.5^2 - 7.2^2 = 132.25 - 51.84 = 80.41$ (M1)
$BC = \sqrt{80.41}$ (M1)
$= 8.967\ldots = 9.0$ cm (A1)

Problem in context: decide and justify

3 to 4 marks | Grade 5

1. Draw the right-angled triangle and label the lengths from the question.
2. Use Pythagoras to find the missing length, keeping extra decimal places.
3. Compare with the value in the question and write a clear conclusion.

Example. A ladder 4.5 m long leans against a vertical wall. The foot of the ladder is on horizontal ground, 1.2 m from the wall. The ladder is safe to use if its top reaches at least 4.3 m up the wall.
Is the ladder safe to use? You must show your working.

Model answer.

$\text{Height}^2 = 4.5^2 - 1.2^2 = 20.25 - 1.44 = 18.81$ (M1)
$\text{Height} = \sqrt{18.81}$ (M1) $= 4.337\ldots$ m (A1)
4.337 m is more than 4.3 m, so yes, the ladder is safe (C1)

Pythagoras in 3D

3 to 4 marks | Grade 7

1. Find the right-angled triangle that contains the length you want.
2. Use Pythagoras on the base first to find a missing side of that triangle (for example the base diagonal).
3. Use Pythagoras again, keeping the first answer unrounded (or use $\sqrt{a^2 + b^2 + c^2}$ for a cuboid).

Example. A box is a cuboid measuring 8 cm by 6 cm by 5 cm.
Work out the length of the longest straight rod that will fit inside the box. Give your answer correct to 3 significant figures.

Model answer.

Diagonal of the 8 cm by 6 cm base $= \sqrt{(8^2 + 6^2)} = \sqrt{100} = 10$ cm (M1)
Rod $= \sqrt{(10^2 + 5^2)} = \sqrt{125}$ (M1)
$= 11.2$ cm (A1)

Don’t lose marks

- Adding the squares when finding a shorter side (the answer comes out longer than the hypotenuse).
- Forgetting to square root at the end.
- Doubling instead of squaring: 7^2 is 49, not 14.
- Using Pythagoras in a triangle that has no right angle.
- Rounding the first answer in a two-step problem and then using the rounded value.

More tips

Memory tricks

- Hypotenuse? Add the squares. Shorter side? Subtract the squares.
- Learn 3-4-5, 5-12-13 and 8-15-17 and their multiples; non-calculator questions often use them.
- Sense check: the hypotenuse is always the longest side.
- Type the whole calculation into your calculator in one go, e.g. $\sqrt{(11.5^2 - 7.2^2)}$, so you do not round too early.

Exam technique

- Write down the squares and the addition or subtraction before the answer; this earns method marks even if you slip later.
- In 'show that' or 'is it safe' questions, give more decimal places than the value you compare with, then write a conclusion.
- Round to the accuracy asked for and keep any final zero: 9.0 cm to 1 decimal place, not 9 cm.
- Higher: in 3D problems, sketch the right-angled triangle you are using as a flat 2D triangle.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 4 — **Find the hypotenuse of a right-angled triangle**
Square the two shorter sides, add, then square root: $c = \sqrt{a^2 + b^2}$

Grade 4 — **Find a shorter side of a right-angled triangle**
Square the hypotenuse, subtract the square of the other side, then square root.

Grade 5 — **Use Pythagoras in problems and on coordinates**
Spot right-angled triangles in rectangles, isosceles triangles and on grids, e.g. the distance between two points.

Grade 5 — **Test whether a triangle is right-angled**
Check whether the square of the longest side equals the sum of the squares of the other two sides.

Grade 6 — **Give exact answers as simplified surds**
On a non-calculator Higher paper, an exact length such as $\sqrt{45}$ is written as $3\sqrt{5}$

Grade 7 — **Use Pythagoras in 3D shapes**
Find the space diagonal of a cuboid or the height of a pyramid using two right-angled triangles.

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