Direct and inverse proportion

R10, R13 · Ratio, proportion and rates of change · revision notes

Two quantities are in direct proportion when they increase at the same rate (double one and the other doubles), and in inverse proportion when one halves as the other doubles. Both tiers use recipes, best buys and 'workers and days' problems; at Higher tier you also form and use equations such as $y = kx^2$ or $y = \frac{k}{\sqrt{x}}$

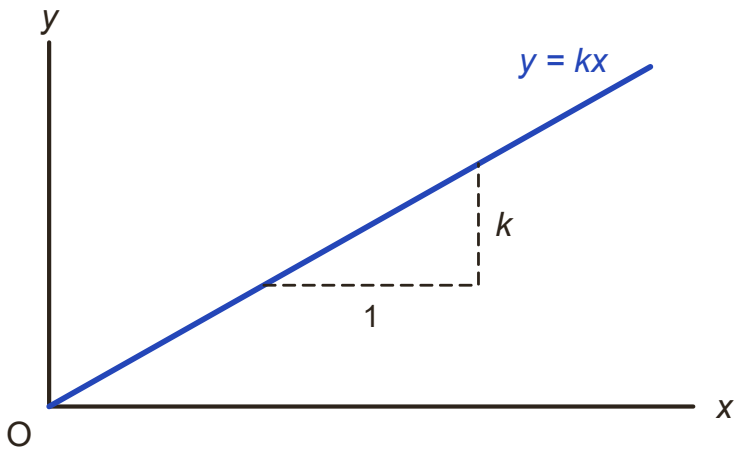
Key facts

- Direct proportion: $y = kx$; the graph is a straight line through the origin
- $\propto$ means 'is proportional to'
- Inverse problems: workers × time stays the same
- Grade 7+ only $y \propto x^2$ gives $y = kx^2$; $y \propto \sqrt{x}$ gives $y = k\sqrt{x}$; y inversely proportional to x^2 gives $y = \frac{k}{x^2}$
- Inverse proportion: $y = \frac{k}{x}$, so xy = k
- Unitary method: find the value for one, then multiply
- Grade 6+ only Find k by substituting one pair of values, then rewrite the formula with k as a number
- Grade 8+ only If $y = kx^n$ and x is multiplied by s, then y is multiplied by s^n

Notes

Direct proportion

- y is directly proportional to x, written $y \propto x$, when $y = kx$ for a constant k.
- If x doubles, y doubles; if x is halved, y is halved. The ratio y : x stays the same.
- The graph of $y = kx$ is a straight line through the origin, with gradient k.

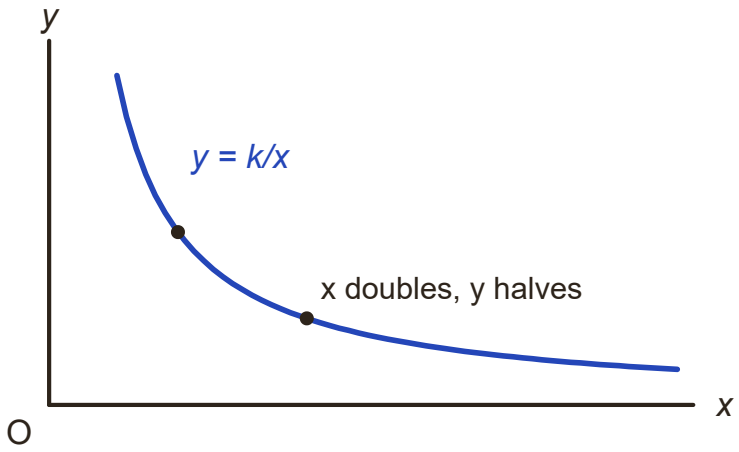


Direct proportion: a straight line through the origin with gradient k.

- Unitary method: find the value for one, then multiply. 6 pens cost £4.50, so 1 pen costs 75p and 10 pens cost £7.50.
- Best buys: compare the cost of the same amount (e.g. per 100 g), or the amount you get for £1.

Inverse proportion

- y is inversely proportional to x when $y = \frac{k}{x}$, so the product xy = k is always the same.
- If x doubles, y halves. Being inversely proportional to x is the same as being proportional to $\frac{1}{x}$
- The graph is a curve that gets closer and closer to both axes but never meets them.



Inverse proportion: the curve never touches either axis.

- Workers and days: 4 builders take 9 days, so the job is 36 'builder-days'. 6 builders take 36 ÷ 6 = 6 days, if they all work at the same rate.

Forming proportion equations Grade 6+ only

- Write the statement using k, e.g. 'p is directly proportional to q' means $p = kq$
- Substitute the pair of values you are given to find k, then write the formula with the number in place of k.
- Example: y is directly proportional to x and y = 12 when x = 4. 12 = 4k, so k = 3 and y = 3x.
- Powers and roots: 'proportional to the square of x' means $y = kx^2$; 'inversely proportional to the square root of x' means $y = \frac{k}{\sqrt{x}}$ Grade 7+ only
- Example: $y \propto x^2$ and y = 45 when x = 3. 45 = 9k, so k = 5 and $y = 5x^2$. When x = 4, y = 5 × 16 = 80. Grade 7+ only
- Effect of a change: if $y = kx^2$ and x is multiplied by 3, y is multiplied by $3^2 = 9$. If $y = \frac{k}{x^2}$ and x doubles, y is divided by 4. Grade 8+ only

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, C = a communication mark.

Scale a recipe (direct proportion)

2 marks | Grade 3

1. Find the amount for one (divide), or the scale factor (new number ÷ old number).
2. Multiply the quantity you need by the new number, or by the scale factor.
3. Check the answer is sensible: more flapjacks need more of each ingredient.

Example. Here are the ingredients needed to make 12 flapjacks.
150 g oats, 90 g butter, 60 g sugar, 45 g golden syrup
Kate wants to make 20 flapjacks.
How much butter does she need?

Model answer.

90 ÷ 12 = 7.5 g of butter for one flapjack (M1)
7.5 × 20 = 150 g (A1)

Decide which pack is the best value

3 marks | Grade 4

1. Work out the cost of one item (or of 100 g, or of 1 litre) for each pack, in the same units.
2. Or scale the packs up to the same size and compare the prices.
3. Write a conclusion naming the better value pack, with your figures.

Example. Tea bags are sold in two sizes.
A small box has 80 tea bags and costs £2.40.
A large box has 240 tea bags and costs £6.60.
Which box is the better value for money?
You must show how you get your answer.

Model answer.

Small box: 240p ÷ 80 = 3p per tea bag	(M1)
Large box: 660p ÷ 240 = 2.75p per tea bag	(M1)
The large box is better value, because 2.75p is less than 3p	(C1)

Solve an inverse proportion problem in context

2 marks | Grade 4

1. Check it is inverse: more machines (or workers) means less time.
2. Multiply to find the total amount of work, e.g. machines × hours.
3. Divide by the new number of machines or workers.

Example. It takes 5 machines 12 hours to complete an order.
How long would it take 3 machines to complete the same order?
Assume that all the machines work at the same rate.

Model answer.

5 × 12 = 60 machine-hours	(M1)
60 ÷ 3 = 20 hours	(A1)

Find a formula for a proportion and use it

4 to 5 marks | Grade 7

1. Write the relationship with a constant k, e.g. $d = kt^2$
2. Substitute the pair of values you are given and solve for k.
3. Write the formula with the value of k in it.
4. Substitute into your formula to answer the next part.

Example. d is directly proportional to the square of t.
When $t = 4$, $d = 40$.
(a) Find a formula for d in terms of t.
(b) Work out the value of t when $d = 90$, given that t is positive.

Model answer.

(a) $d = kt^2$ (M1)

$40 = k \times 4^2$, so $k = 2.5$ (M1)

$d = 2.5t^2$ (A1)

(b) $90 = 2.5t^2$, so $t^2 = 36$ (M1)

$t = 6$ (A1)

Work out the effect of a change in one variable

3 marks | Grade 8

1. Write the relationship with k.
2. Replace the variable by its changed value, e.g. 1.25d for a 25% increase, or 3d if it is tripled.
3. Simplify to see what the other variable is multiplied by.
4. Turn that multiplier into the answer asked for, e.g. a percentage change.

Example. The intensity, I, of the light from a lamp is inversely proportional to the square of the distance, d, from the lamp.
The distance from the lamp is increased by 25%.
Work out the percentage decrease in the intensity of the light.

Model answer.

$I = \dfrac{k}{d^2}$ and the distance is multiplied by 1.25 (M1)

I is multiplied by $\dfrac{1}{1.25^2} = 0.64$ (M1)

36% decrease (A1)

Don’t lose marks

- Treating an inverse problem as direct: more workers cannot take more days.

- Forgetting the power when you substitute: in $y = kx^2$ with x = 3, you need 9k, not 3k.
- Leaving k in the final formula: 'find a formula' needs the value of k, e.g. $y = 5x^2$
- Comparing best buys in mixed units (pounds and pence), or not saying which is better value.
- Giving the value of k when the question asks for y (or x).

More tips

Memory tricks

- Direct: both go up together at the same rate. Inverse: one goes up as the other goes down, and their product stays the same.
- Graph check: direct proportion must pass through (0, 0). A straight line that misses the origin is not direct proportion.
- Always write the equation with k first; it is often worth a mark on its own.
- Best buys: compare like with like, e.g. pence per tea bag for every pack.

Exam technique

- 'Find a formula for y in terms of x' means an equation starting y = ... with a number in place of k.
- When asked why a graph shows direct proportion, say it is a straight line through the origin.
- Read the wording carefully: 'the square of x' is x^2, 'the square root of x' is $\sqrt{x}$ and 'the cube of x' is x^3
- For best buys, show a value per unit for every option, then write a sentence naming the best value.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 3 — **Scale a recipe up or down**
Find the amount for one (or a scale factor) and multiply every ingredient by the same number.

Grade 4 — **Compare value for money using unit prices**
Work out the cost per item, or per 100 g, for each pack and compare.

Grade 4 — **Solve inverse proportion problems like workers and days**
Find the total work (workers × days), then divide by the new number of workers.

Grade 5 — **Recognise direct and inverse proportion graphs**
Direct proportion is a straight line through the origin; inverse proportion is a curve that gets closer and closer to both axes.

Grade 6 — **Form and use equations like y = kx**
Substitute a known pair of values to find k, then use the formula.

Grade 7 — **Form equations involving squares, cubes and roots**
Write, for example, $y = kx^2$ or $y = \frac{k}{\sqrt{x}}$ and find k from given values.

Grade 8 — **Work out the effect of changing a variable**
If $y = kx^2$ and x is tripled, y is multiplied by 9.

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