Primes, factors, HCF and LCM

N4 · Number · revision notes

Primes, factors and multiples, writing a number as a product of its prime factors, and finding the highest common factor (HCF) and lowest common multiple (LCM). Expect 2 to 3 mark questions on both papers, often a word problem about sharing into equal groups or things that happen together.

Key facts

- Prime: exactly two factors (1 and itself). 1 is not prime; 2 is the only even prime.
- HCF: primes in both numbers, lower power
- HCF × LCM = product of the two numbers
- Grade 6+ only Square number: every prime-factor power is even; cube number: every power is a multiple of 3
- Primes to 50: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47
- LCM: all the primes, higher power
- Sharing into equal groups (largest group size) → HCF; happening together again, or the smallest equal total from packs of different sizes → LCM

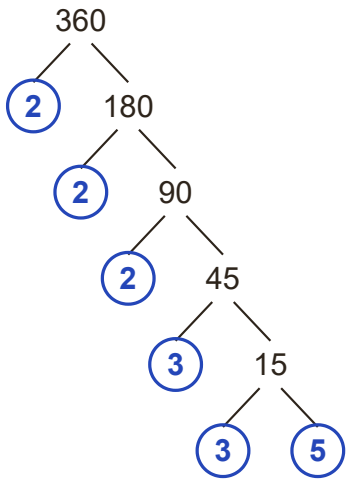
Notes

Factors, multiples and primes

- A **factor** of a number divides into it exactly. Find factors in pairs: for 36, 1 × 36, 2 × 18, 3 × 12, 4 × 9 and 6 × 6.
- A **multiple** of a number is in its times table: the multiples of 7 are 7, 14, 21, 28, ...
- A **prime number** has exactly two factors: 1 and itself. 1 is not prime (it has only one factor). 2 is the only even prime.
- Primes up to 50: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47.

Product of prime factors

- Every whole number greater than 1 can be written as a product of primes in exactly one way (apart from the order). This is the **unique factorisation theorem**.
- Use a factor tree (split into factor pairs until every branch ends in a prime) or repeated division: 360 ÷ 2 = 180, ÷ 2 = 90, ÷ 2 = 45, ÷ 3 = 15, ÷ 3 = 5.



360 = 2 × 2 × 2 × 3 × 3 × 5

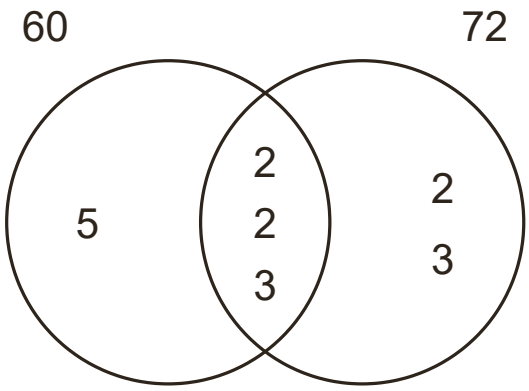
= $2^3 \times 3^2 \times 5$

Keep splitting until every branch ends in a prime (circled). Multiply the circled primes.

- Write the answer as a product (with × signs), usually in index form: $360 = 2^3 \times 3^2 \times 5$
- Check by multiplying back: 8 × 9 × 5 = 360.

HCF and LCM

- **HCF**: the highest number that is a factor of both. **LCM**: the lowest number that is a multiple of both.
- Example: $60 = 2^2 \times 3 \times 5$ and $72 = 2^3 \times 3^2$
- HCF: take each prime that is in **both**, with the **lower** power: $2^2 \times 3 = 12$
- LCM: take **every** prime, with the **higher** power: $2^3 \times 3^2 \times 5 = 360$
- Or use a Venn diagram: the shared primes (2, 2, 3) go in the overlap, the rest of 60's primes (5) on the left and the rest of 72's (2, 3) on the right. HCF = product of the overlap = 12; LCM = product of everything in the diagram = 360.



- Check: HCF × LCM = the two numbers multiplied together. 12 × 360 = 4320 = 60 × 72.

Using prime factor form Grade 6+ only

- In a square number, every power in the prime factorisation is even. In a cube number, every power is a multiple of 3.
- To make $360 = 2^3 \times 3^2 \times 5$ into a square number, multiply by 2 × 5 = 10: $3600 = 2^4 \times 3^2 \times 5^2$
- To find a square root from prime factors, halve every power: $\sqrt{2^4 \times 3^2 \times 5^2} = 2^2 \times 3 \times 5 = 60$

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, P = a process mark.

Write a number as a product of its prime factors 2 marks **Grade 4**

1. Use a factor tree or divide repeatedly by primes (2, 3, 5, 7, ...).
2. Stop only when every branch ends in a prime.
3. Write the primes multiplied together, not as a list with commas. Index form is fine.
4. Check by multiplying back.

Example. Write 252 as a product of its prime factors.

Model answer.

252 ÷ 2 = 126, 126 ÷ 2 = 63, 63 ÷ 3 = 21, 21 ÷ 3 = 7 (M1 for at least two correct steps in a factor tree or repeated division)

$$252 = 2 \times 2 \times 3 \times 3 \times 7$$
$$= 2^2 \times 3^2 \times 7$$ (A1)

Find the HCF and LCM of numbers in prime factor form 1 to 2 marks each **Grade 5**

1. Do not multiply the numbers out first; read the powers.
2. HCF: each prime that is in both, with the lower power.
3. LCM: every prime, with the higher power.
4. Give the answer in the form the question asks (index form or an ordinary number).

Example. $A = 2^3 \times 3 \times 5^2$ and $B = 2 \times 3^2 \times 5$.
(a) Find the highest common factor (HCF) of A and B.
(b) Find the lowest common multiple (LCM) of A and B.

Model answer.

(a) Primes in both with the lower power: $2 \times 3 \times 5 = 30$ (B1)

(b) Every prime with the higher power: $2^3 \times 3^2 \times 5^2 = 1800$ (B1)

HCF or LCM word problem

3 marks | Grade 5

1. Decide: are things happening together again (LCM), or being shared into equal groups (HCF)?
2. List multiples (or factors), or use prime factors.
3. Answer the question that was asked, e.g. a time of day or a number of packs, not just the LCM.

Example. Buses to the airport leave a station every 18 minutes. Buses to the zoo leave the same station every 24 minutes. Both buses leave at 9:00 am.
When will both buses next leave the station at the same time?

Model answer.

Multiples of 18: 18, 36, 54, 72, ... Multiples of 24: 24, 48, 72, ... (P1 for listing multiples of both, or $18 = 2 \times 3^2$ and $24 = 2^3 \times 3$)

LCM = 72 minutes (P1)

9:00 am + 72 minutes = 10:12 am (A1)

Make a square number using prime factors

2 marks | Grade 6

1. Write the number in prime factor form (it is often given).
2. For a square, every power must be even; for a cube, every power must be a multiple of 3.
3. Multiply by the primes needed to fix each power, and no more.

Example. $540 = 2^2 \times 3^3 \times 5$
Find the smallest positive integer n such that $540n$ is a square number.

Model answer.

Every power must be even (M1 for identifying that one more 3 and one more 5 are needed)

$n = 3 \times 5 = 15$ (A1)

Check: $540 \times 15 = 8100 = 90^2$

Work backwards from the HCF and LCM 2 to 3 marks **Grade 7**

1. Both numbers are multiples of the HCF, so write them as HCF × a and HCF × b.
2. Use HCF × LCM = product of the numbers to find a × b.
3. a and b must have no common factor; test the pairs against any condition in the question.

Example. The HCF of two numbers is 6. The LCM of the same two numbers is 72. Both numbers are greater than 6.
Find the two numbers.

Model answer.

The numbers are 6a and 6b, and 6a × 6b = 6 × 72 = 432, so ab = 12	(M1)
a and b have no common factor: 1 and 12 (gives 6, not allowed) or 3 and 4	(M1)
The numbers are 18 and 24	(A1)

Don't lose marks

- Including 1 as a prime number, or forgetting that 2 is prime.
- Stopping a factor tree too early, e.g. leaving 9 or 15 at the end of a branch (they are not prime).
- Writing the primes as a list (2, 2, 3, 3, 7) instead of a product (2 × 2 × 3 × 3 × 7).
- Mixing up HCF and LCM: things happening together again needs the LCM.
- Finding the LCM (72 minutes) but not answering the question (10:12 am).

More tips

Memory tricks

- Divisibility tests: by 2 if even; by 3 if the digits add up to a multiple of 3; by 5 if it ends in 0 or 5; by 9 if the digits add up to a multiple of 9.
- Sense check: the HCF can never be bigger than the smaller number, and the LCM can never be smaller than the bigger number.
- Check a prime factorisation by multiplying it back out.
- Many scientific calculators have a prime factor (FACT) function: use it to check on the calculator paper.

Exam technique

- 'Write as a product of its prime factors' needs × signs between the primes; index form is accepted.
- For word problems, decide HCF or LCM first, then finish with a sentence that answers the question, with units.
- If the numbers are given in prime factor form, don't multiply them out; read the HCF and LCM straight from the powers.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 1 **Find factors and multiples of a number**

Factors divide exactly into a number (factors of 12: 1, 2, 3, 4, 6, 12); multiples are in its times table (12, 24, 36, ...).

Grade 2 **Recognise prime numbers**

A prime has exactly two factors, 1 and itself: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ...

Grade 3 **Find HCF and LCM by listing**

List factors for the HCF and multiples for the LCM: for 12 and 18, the HCF is 6 and the LCM is 36.

Grade 4 **Write a number as a product of primes**

Use a factor tree or repeated division:

$$\begin{aligned} 360 &= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \\ &= 2^3 \times 3^2 \times 5 \end{aligned}$$

Grade 5 **Find HCF and LCM using prime factors**

HCF: primes in both numbers with the lower power; LCM: every prime with the higher power (or use a Venn diagram).

Grade 5 **Solve HCF and LCM word problems**

Sharing into equal groups or the largest size means HCF; things happening together again means LCM.

Grade 6 **Use prime factor form to reason**

E.g. find the smallest number to multiply by to make a square number, using even powers.

Grade 7 **Find numbers from a given HCF and LCM**

Work backwards from the HCF and LCM to find possible pairs of numbers.

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