Percentages and reverse percentages

R9 · Ratio, proportion and rates of change · revision notes

A percentage is a number of parts per hundred. You need to find percentages of amounts, increase or decrease by a percentage, work out percentage change, profit and simple interest, and work backwards to an original amount (reverse percentages). Expect these on both calculator and non-calculator papers, on both tiers.

Key facts

- Per cent = out of 100: 37% = $\frac{37}{100}$ = 0.37
- Increase by r%: multiplier $1 + \frac{r}{100}$
- New amount = original amount × multiplier
- Simple interest = original amount × rate as a decimal × number of years
- Percentage change = $\frac{\text{change}}{\text{original}} \times 100$
- Decrease by r%: multiplier $1 - \frac{r}{100}$
- Original amount = new amount ÷ multiplier
- Profit = selling price − cost price; percentage profit is out of the cost price

Notes

Finding percentages

- Per cent means 'out of 100': 37% = $\frac{37}{100}$ = 0.37.
- Without a calculator, build up from 10% (÷ 10), 5% (half of 10%) and 1% (÷ 100). 35% of £60 = £18 + £3 = £21.
- With a calculator, multiply by the decimal: 17.5% of 240 = 0.175 × 240 = 42.
- One quantity as a percentage of another: $\frac{\text{part}}{\text{whole}} \times 100$, with both in the same units. 18 out of 24 is 75%.
- To compare two quantities, change both to percentages: 18 out of 24 (75%) is a higher score than 28 out of 40 (70%).
- Percentages over 100% are fine: 150% of 40 = 1.5 × 40 = 60.

Multipliers

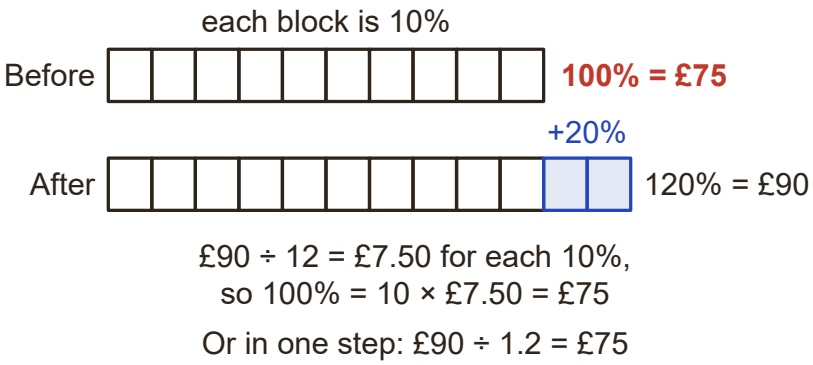
- Increase by r%: multiply by $1 + \frac{r}{100}$. An 8% rise is × 1.08; adding VAT at 20% is × 1.2.
- Decrease by r%: multiply by $1 - \frac{r}{100}$. An 8% fall is × 0.92; a 35% discount is × 0.65.
- Changes one after another: multiply the multipliers. A 10% rise then a 10% fall is × 1.1 × 0.9 = × 0.99, which is a 1% fall overall.
- The same idea works for areas: if a length and a width both increase by 10%, the area is multiplied by 1.1 × 1.1 = 1.21, a 21% increase. Grade 7+ only

Percentage change, profit and simple interest

- Percentage change = $\frac{\text{change}}{\text{original}} \times 100$. Always divide by the original amount.
- Profit or loss uses the cost price as the original: bought for £40 and sold for £52 is a profit of $\frac{12}{40} \times 100$ = 30%.
- Simple interest is paid on the original amount only, so it is the same every year. £600 at 4% for 3 years earns £24 a year, £72 in total.

Reverse percentages

- If you know the amount after a change, divide by the multiplier to find the original amount.
- After a 20% increase a price is £90, so the original price = 90 ÷ 1.2 = £75.
- Do not take 20% of £90 and subtract it: that gives £72, which is wrong.
- Without a calculator: £90 is 120% of the original, so 10% = £7.50 and 100% = £75.



The price after the rise is 120% of the original, not 100%.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Increase or decrease by a percentage 2 marks **Grade 4**

1. Non-calculator: find the percentage using 10%, 5% and 1%, then add it on or take it off.
2. Calculator: multiply by the multiplier in one step, e.g. × 0.85 for a 15% decrease.
3. Give money to 2 decimal places.

Example. A sofa normally costs £640.
In a sale, the price is reduced by 15%.
Work out the sale price of the sofa.

Model answer.

10% = £64 and 5% = £32, so 15% = £96 (M1)
640 − 96 = £544 (A1)
(Or 0.85 × 640 = £544)

Work out a percentage change, profit or loss

3 marks **Grade 5**

1. Find the change: new amount − original amount.
2. Divide the change by the ORIGINAL amount.
3. Multiply by 100 and say whether it is an increase or a decrease (profit or loss).

Example. Leah buys a bike for £250.
She sells it for £195.
Work out her percentage loss.

Model answer.

250 − 195 = £55 (M1)

$$\frac{55}{250} \times 100$$ (M1)

= 22% loss (A1)

Find the original amount (reverse percentage)

3 marks **Grade 5**

1. Write the multiplier for the change, e.g. a 30% decrease gives 0.7.
2. Divide the new amount by the multiplier (never multiply).
3. Check by applying the change forwards to your answer.

Example. In a sale, all prices are reduced by 30%.
The sale price of a coat is £59.50.
Work out the price of the coat before the sale.

Model answer.

100% − 30% = 70%, so the multiplier is 0.7 (M1)

59.50 ÷ 0.7 (M1)

= £85 (A1)

Check: 0.7 × 85 = 59.50

Work out simple interest

2 to 3 marks | Grade 4

1. Find one year's interest: the rate as a decimal × the amount invested.
2. Multiply by the number of years.
3. Add the amount invested only if the question asks for the total in the account.

Example. Ravi invests £2400 for 5 years in an account that pays 3.5% per year simple interest.
Work out the total amount of money in the account at the end of the 5 years.

Model answer.

One year's interest = 0.035 × 2400 = £84	(M1)
Interest for 5 years = 5 × 84 = £420	(M1)
Total = 2400 + 420 = £2820	(A1)

Find the overall percentage change from two changes

3 marks | Grade 7

1. Write a multiplier for each change.
2. Multiply the multipliers together.
3. Turn the result back into a percentage change: 1.04 is a 4% increase and 0.96 is a 4% decrease.

Example. The length of a rectangle is increased by 30%.
The width of the rectangle is decreased by 20%.
Work out the percentage change in the area of the rectangle.
State whether it is an increase or a decrease.

Model answer.

Multipliers 1.3 and 0.8	(M1)
1.3 × 0.8 = 1.04	(M1)
4% increase	(A1)

Don’t lose marks

- Dividing by the new amount in a percentage change question: always divide by the original.
- Reverse percentages: taking the percentage of the new amount and adding it on (or taking it off).
- Using the wrong decimal, e.g. 0.5 for 5% (it is 0.05) or 1.5 for a 5% increase (it is 1.05).
- Giving the interest or discount when the question asks for the final amount, or the other way round.
- Adding percentages for changes one after another: a 10% rise then a 10% fall is not 0% overall.
- Writing money with one decimal place: £4.5 should be £4.50.

More tips

Memory tricks

- Multiplier fast: work out 100 plus or minus the percentage, then divide by 100, e.g. a 12.5% decrease gives 87.5 ÷ 100 = 0.875.
- Percentages swap round: 8% of 25 = 25% of 8 = 2. Use whichever is easier.
- Check a reverse percentage by applying the change forwards: 0.7 × £85 = £59.50.
- Percentage change is 'difference over original'.
- Sense check: after a decrease the original amount must be bigger than the new amount.

Exam technique

- Underline what the question wants: the change (discount, profit, interest) or the new amount.
- On the non-calculator paper, write down each step (10%, 5%, 1%), as each one can earn method marks.
- For 'Is she correct?' questions, work it out fully, then give a clear yes or no with the figures that prove it.
- Say 'increase' or 'decrease' (or 'profit' or 'loss') with a percentage change when the question asks you to.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 2 — **Find percentages of amounts without a calculator**
Build up from 10%, 5% and 1%, e.g. 35% of £60 = £18 + £3 = £21.

Grade 3 — **Convert between fractions, decimals and percentages**
For example, $\frac{3}{8}$ = 0.375 = 37.5%, and 120% = 1.2.

Grade 3 — **Write one amount as a percentage of another**
Divide the part by the whole and multiply by 100, e.g. 18 out of 24 is 75%.

Grade 4 — **Increase or decrease by a percentage**
Use a multiplier, e.g. × 1.15 for a 15% increase or × 0.85 for a 15% decrease.

Grade 4 — **Calculate simple interest**
Find one year's interest on the original amount and multiply by the number of years.

Grade 5 — **Work out percentage change, profit and loss**
Divide the change by the original amount and multiply by 100.

Grade 5 — **Find the original amount (reverse percentages)**
Divide the new amount by the multiplier, e.g. £68 after a 15% reduction was 68 ÷ 0.85 = £80.

Grade 7 — **Combine percentage changes using multipliers**
Multiply the multipliers, e.g. if the length and width of a rectangle both go up by 10%, its area is multiplied by 1.1 × 1.1 = 1.21.

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