Algebraic notation, substitution and identities

A1–A3 · Algebra · revision notes

How to write and read algebra correctly, substitute numbers into expressions and formulae, and use the words term, expression, equation, formula, identity and inequality. It comes up as quick 1 to 3 mark questions on both tiers and underpins every other algebra topic.

Key facts

- $ab = a \times b$ and $\frac{a}{b} = a \div b$
- Like terms: same letters with the same powers
- Substituting negatives: use brackets, $(-3)^2 = 9$
- $a^2 = a \times a$, $a^3 = a \times a \times a$, $a^2b = a \times a \times b$
- Expression: no equals sign. Equation: can be solved. Formula: links quantities. Identity ($\equiv$): true for all values
- Order of operations: brackets, indices, × and ÷, + and −

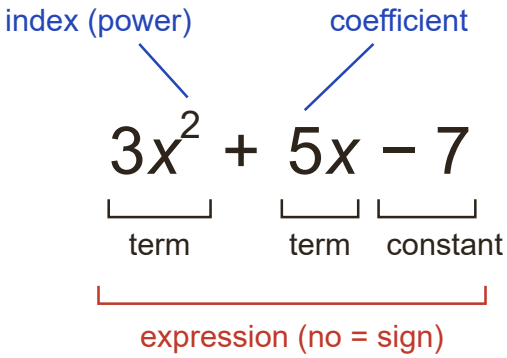
Notes

Writing algebra

- Leave out the × sign and write the number first: $a \times b = ab$ and $b \times 3 = 3b$
- $y + y + y = 3y$, but $y \times y \times y = y^3$
- Division is written as a fraction: $a \div b = \frac{a}{b}$. Write coefficients as fractions, not decimals: $\frac{1}{2}x$ or $\frac{x}{2}$, not $0.5x$
- $2a^2$ means $2 \times a \times a$: only the a is squared. $(2a)^2 = 4a^2$
- Like terms have exactly the same letters and powers: $3xy$ and $-5yx$ are like terms, but x^2 and x are not.

The vocabulary

- **Term**: a number, a letter, or numbers and letters multiplied together, such as 5, $4x$ or $-3ab^2$
- **Expression**: one or more terms with no equals sign, e.g. $4x - 7$



An expression with three terms. A coefficient is the number in front of a letter.

- **Equation**: has an equals sign and is true only for particular values, e.g. $4x - 7 = 5$ is true only when $x = 3$
- **Formula**: a rule linking two or more quantities, e.g. $A = lw$. You substitute values into it.
- **Identity**: true for every value of the letter, written with $\equiv$, e.g. $3(x + 2) \equiv 3x + 6$
- **Inequality**: uses $<$, $>$, $\leq$ or $\geq$, e.g. $2x + 1 > 9$
- **Factor**: something that divides exactly into a term, e.g. 3 and x are factors of $3x$

Substitution

- Replace each letter with its value in brackets, then use the order of operations.
- Put negative numbers in brackets: if $t = -2$, then $t^2 = (-2)^2 = 4$, but $-t^2 = -(-2)^2 = -4$
- Scientific formulae work the same way: in $v = u + at$ with $u = 3$, $a = 2$ and $t = 5$, $v = 3 + 2 \times 5 = 13$

Identities

- To show that an identity is true, expand and simplify one side until it is exactly the same as the other side.
- To find unknown constants, expand and simplify, then match coefficients: if $ax + 12 \equiv 5x + 4b$, then $a = 5$ and $4b = 12$, so $b = 3$ Grade 6+ only

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Simplify an expression

1 to 2 marks | Grade 2

1. Look at each term together with the sign in front of it.
2. Group like terms (same letters and powers).
3. Add or subtract the coefficients and write the answer with no like terms left.

Example. Simplify $5p + 3q - 2p - 7q + p$

Model answer.

$5p - 2p + p = 4p$ and $3q - 7q = -4q$

$4p - 4q$ B2

(B1 for $4p$ or $-4q$)

Choose the expression, equation or formula

1 mark each | Grade 3

1. An expression has no equals sign.
2. An equation is true only for certain values; a formula links different quantities.
3. An identity uses $\equiv$ and is true for all values.

Example. Here are four statements: $5x - 2 = 13$, $v = u + at$, $4(y + 1) \equiv 4y + 4$, $3p^2 - p$. From the list, write down (a) an expression (b) an equation (c) a formula.

Model answer.

(a) $3p^2 - p$ B1

(b) $5x - 2 = 13$ B1

(c) $v = u + at$ B1

Work out the value (substitution)

2 marks | Grade 4

1. Write the expression with each letter replaced by its value in brackets.
2. Work out powers first, then multiplication, then addition and subtraction.

Example. $P = 3m^2 - 4n$. Work out the value of P when $m = -2$ and $n = 5$

Model answer.

$P = 3 \times (-2)^2 - 4 \times 5$ M1

$= 12 - 20 = -8$ A1

Find the values of constants in an identity

3 marks | Grade 6

1. Expand every bracket and collect like terms.
2. Match the coefficient of x on both sides.
3. Match the number terms on both sides and solve.

Example. $4(2x + a) - 3(x - 1) \equiv bx + 23$. Find the values of a and b.

Model answer.

$8x + 4a - 3x + 3$	M1
$5x + 4a + 3$, so $b = 5$	A1
$4a + 3 = 23$, so $a = 5$	A1

Don't lose marks

- Writing $y + y + y$ as y^3 (it is $3y$).
- Squaring a negative without brackets: -4^2 on a calculator gives −16, but $(-4)^2 = 16$
- Collecting unlike terms, e.g. $3x + 2x^2 = 5x^3$. These cannot be combined.
- Losing the sign in front of a term: in $7a - 3b + 2a$, the b term is $-3b$
- Adding before multiplying: when $x = 4$, $2 + 3x = 2 + 12 = 14$, not 20.

More tips

Memory tricks

- Quick identity test: put $x = 0$ and $x = 1$ into both sides. If either value gives different answers, it is not an identity.
- On a calculator, type negative values in brackets: $(-3)^2$, not -3^2
- Sense check a substitution: if the sign or size of your answer looks odd, re-check your brackets and the order of operations.

Exam technique

- Write the substituted line before calculating: it earns the method mark even if you slip later.
- 'Simplify' means collect terms fully: no like terms should be left in your answer.
- In 'show that' questions, show every step and make your last line exactly match what you were asked to show.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 1 — **Collect like terms**
Add or subtract terms with exactly the same letters, e.g. $3a + 5b - a + 2b = 2a + 7b$

Grade 2 — **Write products in algebraic notation**
Write $a \times b$ as ab, $y \times y$ as y^2 and $3 \times a \times a \times b$ as $3a^2b$

Grade 3

Substitute positive numbers into expressions

Replace each letter by its value, e.g. when $a = 4$ and $b = 3$, $5a - 2b = 20 - 6 = 14$

Grade 3

Tell expressions, equations, formulae and identities apart

An expression has no equals sign, an equation can be solved, a formula links quantities and an identity is true for every value.

Grade 4

Substitute negative numbers into formulae

Put negative values in brackets, e.g. when $x = -3$, $x^2 = (-3)^2 = 9$

Grade 5

Use the identity symbol correctly

Write $\equiv$ when both sides are equal for every value of the letter, e.g. $2(x + 3) \equiv 2x + 6$

Grade 6

Find unknown constants in an identity

Expand and simplify one side, then match the coefficients of each power of x and the number terms.

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