# Straight-line graphs

A9, A10 · Algebra · revision notes

Plotting and interpreting straight-line graphs: gradient, y-intercept, $y = mx + c$, finding the equation of a line from two points or a point and a gradient, and parallel lines (both tiers). Higher also covers perpendicular lines.

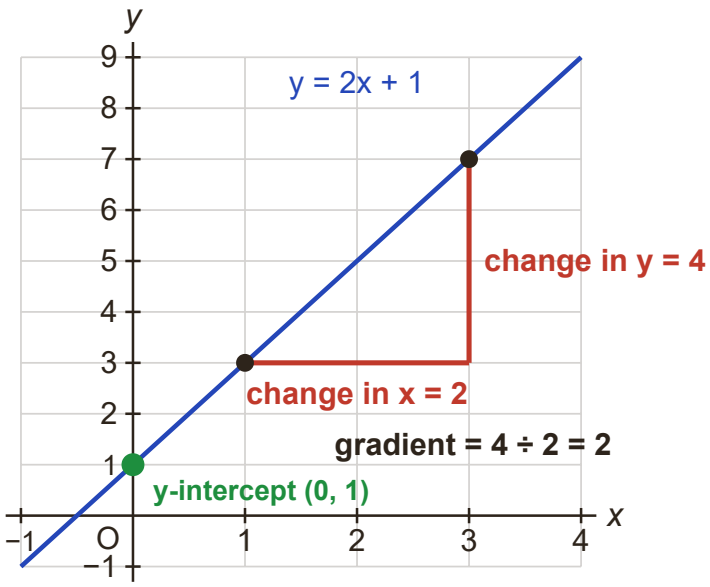
## Key facts

- $y = mx + c$: m is the gradient, c is the y-intercept
- Parallel lines: same gradient
- $x = a$ is vertical; $y = b$ is horizontal
- Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$
- Grade 7+ only Perpendicular lines: $m_1 \times m_2 = -1$, so $m_2 = -\dfrac{1}{m_1}$
- Midpoint of $(x_1, y_1)$ and $(x_2, y_2)$: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

## Notes

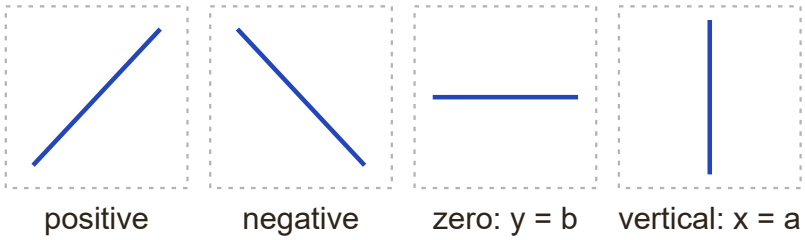
### Gradient and intercept

- The **gradient** m measures steepness: how far up the line goes for every 1 unit across.
- Gradient $= \dfrac{\text{change in } y}{\text{change in } x} = \dfrac{y_2 - y_1}{x_2 - x_1}$




Gradient = change in y ÷ change in x. The line crosses the y-axis at $c = 1$

- Lines going uphill from left to right have positive gradients; downhill lines have negative gradients; horizontal lines have gradient 0.
- The **y-intercept** c is where the line crosses the y-axis, at $(0, c)$
- $x = a$ is a vertical line and $y = b$ is a horizontal line.



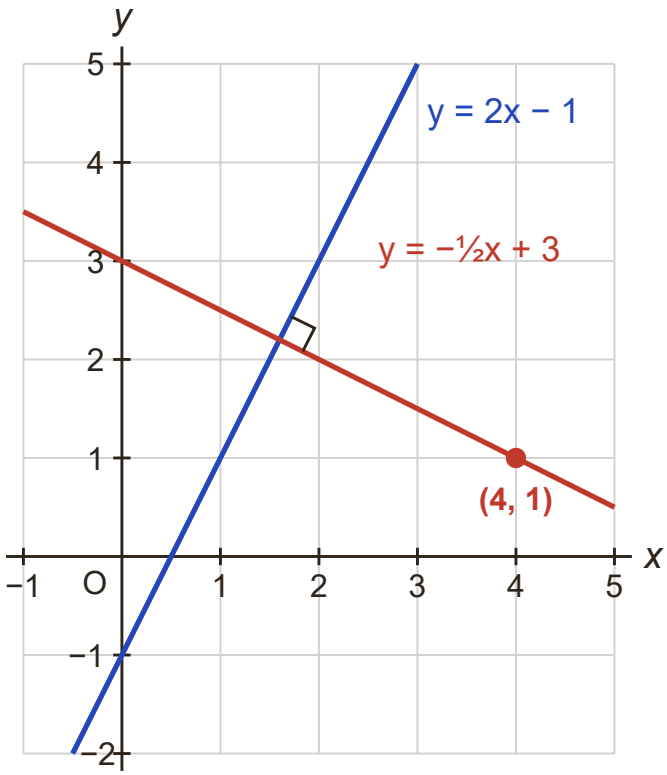

Uphill: positive. Downhill: negative. Horizontal: zero.

## Using $y = mx + c$

- Make y the subject before reading off m and c: $3x + 2y = 8$ gives $y = -\frac{3}{2}x + 4$
- From a point and a gradient: gradient 4 through (2, 5) gives $5 = 4 \times 2 + c$, so $c = -3$ and $y = 4x - 3$
- From two points: through (1, 6) and (3, 2), $m = \frac{2-6}{3-1} = -2$. Then $6 = -2 + c$, so $c = 8$ and $y = -2x + 8$
- Parallel lines have the same gradient.

## Perpendicular lines Grade 7+ only

- Perpendicular lines meet at right angles, and their gradients multiply to give −1.
- If one gradient is m, the perpendicular gradient is $-\frac{1}{m}$ (the negative reciprocal): for 3 it is $-\frac{1}{3}$; for $-\frac{2}{5}$ it is $\frac{5}{2}$
- Perpendicular to $y = 2x - 1$ through (4, 1): gradient $-\frac{1}{2}$, so $1 = -2 + c$, $c = 3$ and $y = -\frac{1}{2}x + 3$




Gradients 2 and $-\frac{1}{2}$ multiply to −1, so the lines meet at 90°.

## Straight lines in context

- The gradient is a rate of change, e.g. pounds per hour or litres per minute.
- The y-intercept is the starting value, e.g. a fixed charge before any hours are worked.

## How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, C = a communication mark.

### Find the equation of a line through two points

3 marks **Grade 5**

1. Work out the gradient from the two points.
2. Substitute one point into $y = mx + c$ to find c.
3. Write the equation and check it with the other point.

**Example.** Find the equation of the line that passes through (−1, 7) and (3, −5).

**Model answer.**

$m = \dfrac{-5-7}{3-(-1)} = -3$ M1

$7 = -3 \times (-1) + c$, so $c = 4$ M1

$y = -3x + 4$ A1

### Find a parallel line through a point

2 marks **Grade 5**

1. Use the same gradient as the given line.
2. Substitute the point to find c.

**Example.** Line L has equation $y = 4x - 7$. Find the equation of the line that is parallel to L and passes through (2, 3).

**Model answer.**

Gradient 4, so $3 = 4 \times 2 + c$ M1

$y = 4x - 5$ A1

### Show that two lines are parallel

2 marks **Grade 5**

1. Rearrange each equation into the form $y = mx + c$
2. Compare the gradients and write a conclusion.

**Example.** Show that the lines $2y = 6x + 5$ and $y - 3x = 1$ are parallel.

**Model answer.**

$y = 3x + 2.5$ and $y = 3x + 1$ M1

Both have gradient 3, so the lines are parallel C1

### Interpret the gradient and intercept

2 marks | Grade 5

1. The gradient is the rate: the change in y for each 1 unit of x.
2. The intercept is the value of y when x is 0.
3. Use the context and its units in your answer.

**Example.** The cost, £C, of hiring a van for d days is $C = 45d + 30$.
(a) What does the 45 represent?
(b) What does the 30 represent?

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**Model answer.**

(a) The cost per day: each extra day costs £45 B1

(b) A fixed charge of £30, paid whatever the number of days B1

### Find a perpendicular line

3 marks | Grade 7

1. Find the gradient of the given line.
2. Take the negative reciprocal.
3. Substitute the point to find c.

**Example.** A is the point (1, 2) and B is the point (5, 10). Find the equation of the line that is perpendicular to AB and passes through B.

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**Model answer.**

Gradient of AB $= \dfrac{10-2}{5-1} = 2$ M1

Perpendicular gradient $-\dfrac{1}{2}$, so $10 = -\dfrac{1}{2} \times 5 + c$ M1

$y = -\dfrac{1}{2}x + 12.5$ A1

(or $x + 2y = 25$)

## Don't lose marks

- Dividing the change in x by the change in y (the fraction upside down).
- Reading c from $2y = 6x + 5$ as 5: make y the subject first.
- Sign errors with negative coordinates: $3 - (-1) = 4$
- Higher: using $\dfrac{1}{m}$ or $-m$ instead of $-\dfrac{1}{m}$ for a perpendicular gradient.

## More tips

### Memory tricks

- Sketch the two points first: if the line goes downhill, your gradient must be negative.
- Check your equation by substituting both points; each should fit.
- When finding a gradient from a graph, choose two points far apart where the line crosses grid corners.
- Higher: negative reciprocal means flip the fraction and change the sign.

### Exam technique

- Give the equation in the form asked for, e.g. $y = mx + c$ or $ax + by = c$
- When working from a graph, draw the triangle you used to find the gradient.
- In context questions, include units in your interpretation, e.g. '£45 per day'.

## What each grade needs

What you need to be able to do, from the first marks up to the top grade.

**Grade 3** — **Plot a straight line from its equation**
Make a table of values, plot the points and join them with a ruler.

**Grade 4** — **Find the gradient between two points**
Gradient = change in y ÷ change in x, e.g. from (1, 2) to (4, 11): $9 \div 3 = 3$

**Grade 4** — **Identify gradient and intercept from an equation**
In $y = 5x - 2$, the gradient is 5 and the line crosses the y-axis at (0, −2).

**Grade 5** — **Find the equation of a line**
Work out m, then substitute a point to find c, e.g. through (2, 7) and (4, 11) gives $y = 2x + 3$

**Grade 5** — **Identify and find parallel lines**
Parallel lines have equal gradients: $y = 3x + 1$ is parallel to $y = 3x - 4$

**Grade 5** — **Rearrange an equation to find the gradient**
Make y the subject, e.g. $2y + 6x = 5$ gives $y = -3x + 2.5$, so the gradient is −3.

**Grade 5** — **Interpret gradient and intercept in context**
e.g. in $C = 45d + 30$, 45 is the cost per day and 30 is a fixed charge.

**Grade 7** — **Find perpendicular gradients and lines**
Perpendicular gradients multiply to −1, so a line perpendicular to $y = 2x + 1$ has gradient $-\frac{1}{2}$

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