Linear equations

A17, A21 · Algebra · revision notes

Solving equations such as $5x - 3 = 2x + 9$, including brackets and fractions, and forming your own equation from words or a diagram (angles, perimeters, ages, costs) and solving it. It is on both tiers, from 1-mark questions to multi-step problems.

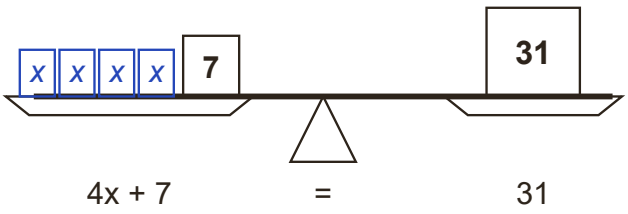
Key facts

- Do the same to both sides
- Collect x terms on one side and numbers on the other
- Angles on a straight line = 180°; in a triangle = 180°; in a quadrilateral = 360°
- Expand brackets first
- Clear fractions by multiplying by the denominator (or the LCM of the denominators)
- Check: substitute your answer into the original equation

Notes

Balancing

- An equation is like a balance: whatever you do to one side, do to the other.



Take 7 off both sides: $4x = 24$. Then share both sides between 4: $x = 6$

- Undo operations with inverses: + and −, × and ÷.
- Solve $4x + 7 = 31$: subtract 7 to get $4x = 24$, then divide by 4 to get $x = 6$
- Answers can be negative or fractions: $3x = 2$ gives $x = \frac{2}{3}$. Leave it as an exact fraction unless you are told to round.
- Using a graph: the solution of $2x + 1 = 6$ is the x-coordinate where the line $y = 2x + 1$ meets the line $y = 6$, so $x = 2.5$

Brackets and unknowns on both sides

- Expand brackets first: $3(2x - 1) = 21$ gives $6x - 3 = 21$, so $6x = 24$ and $x = 4$
- With x on both sides, subtract the smaller x term from both sides: $8x + 1 = 3x + 26$ gives $5x = 25$, so $x = 5$
- Take care with negatives: $10 - 2x = 4$ gives $-2x = -6$, so $x = 3$

Fractions

- Multiply both sides by the denominator: $\frac{x-2}{5}=3$ gives $x-2=15$, so $x=17$
- With more than one fraction, multiply every term by the LCM of the denominators: $\frac{x+1}{2}+\frac{x}{3}=8$, × 6 gives $3(x+1)+2x=48$, so $5x=45$ and $x=9$ Grade 6+ only

Forming equations

- Choose a letter for the unknown and say what it stands for.
- Use the facts in the question: angles on a straight line add to 180°, angles in a triangle add to 180°, the perimeter is the total of the sides.
- Solve, then answer the actual question: if it asks for the largest angle, substitute x back in.
- Check that the answer makes sense (a length cannot be negative).

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, P = a process mark.

Solve (brackets, unknown on both sides) 3 marks Grade 4

1. Expand any brackets.
2. Collect the x terms on one side and the numbers on the other.
3. Divide to find x.

Example. Solve $5(x-2)=2x+11$

Model answer.

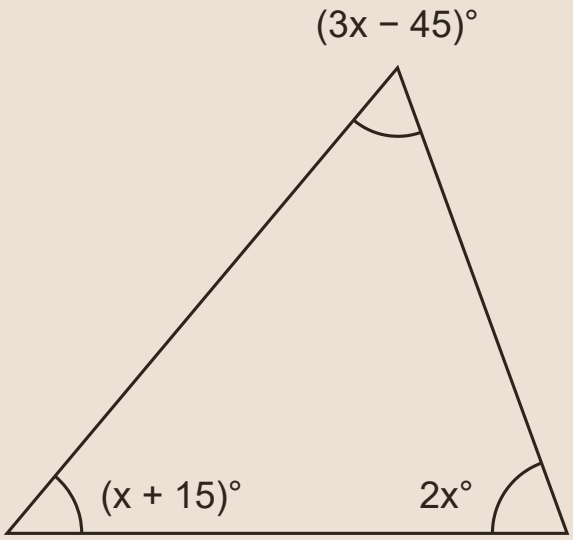
$5x-10=2x+11$	M1
$3x=21$	M1
$x=7$	A1

Form and solve an equation (geometry)

3 marks **Grade 5**

1. Use the angle or perimeter fact to write an equation.
2. Solve it.
3. Substitute back to answer the question asked.

Example. The angles of a triangle are $(x + 15)^\circ$, $2x^\circ$ and $(3x - 45)^\circ$. Work out the size of the largest angle.



Model answer.

$x + 15 + 2x + 3x - 45 = 180$	P1
$6x - 30 = 180$, so $x = 35$	P1
Angles are 50°, 70° and 60°, so the largest is 70°	A1

Form and solve an equation (words)

4 marks **Grade 5**

1. Write each unknown quantity in terms of one letter.
2. Add them (or use the given fact) to form an equation.
3. Solve, then give the value the question asks for.

Example. Ali is x years old. Ben is 4 years older than Ali. Cara is twice as old as Ben. Their ages add up to 60. How old is Cara?

Model answer.

Ben is $x + 4$ and Cara is $2(x + 4)$	P1
$x + x + 4 + 2x + 8 = 60$	P1
$4x = 48$, so $x = 12$	P1
Cara is 32	A1

Solve an equation with fractions

3 marks | Grade 6

1. Multiply every term by the LCM of the denominators, keeping numerators in brackets.
2. Expand and simplify.
3. Solve.

Example. Solve $\frac{2x-1}{3} - \frac{x+2}{4} = 1$

Model answer.

$4(2x-1) - 3(x+2) = 12$ M1

$8x - 4 - 3x - 6 = 12$ M1

$5x = 22$, so $x = \frac{22}{5}$ (= 4.4) A1

Don't lose marks

- Expanding brackets wrongly, e.g. $-3(x-2) = -3x - 6$ (it is $-3x + 6$).
- When multiplying by a denominator, forgetting to multiply the other terms too.
- Dividing the wrong way: $4x = 20$ gives $x = 5$, not $x = \frac{4}{20}$
- Stopping at the value of x when the question asks for an angle, a length or an age.

More tips

Memory tricks

- Check by substituting: for $5(x-2) = 2x + 11$ with $x = 7$, both sides equal 25.
- Move the smaller x term so that you are left with a positive number of x.
- With fractions, clear them in the first line; after that it is an ordinary equation.

Exam technique

- When the question says 'You must show all your working', an answer found by trial and improvement may get no marks.
- Write the equation you formed clearly (with its total, e.g. '= 180') before solving it.
- Give answers that are not whole numbers as exact fractions or decimals unless you are told to round.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 2 — **Solve one-step equations**
Use the inverse operation, e.g. $x + 8 = 15$ gives $x = 7$, and $4x = 28$ gives $x = 7$

Grade 3 — **Solve two-step equations**
e.g. $3x - 4 = 11$, so $3x = 15$ and $x = 5$

Grade 4

Solve equations with brackets

Expand first, e.g. $2(x+5)=18$ gives $2x+10=18$, so $x=4$

Grade 4

Solve with the unknown on both sides

Collect the x terms on one side, e.g. $7x-3=4x+12$ gives $3x=15$, so $x=5$

Grade 5

Solve equations with a fraction

Multiply both sides by the denominator, e.g. $\frac{x+4}{3}=5$ gives $x=11$

Grade 5

Form and solve an equation from context

Write an equation for an angle sum, perimeter, age or cost problem, solve it and answer the question.

Grade 6

Solve equations with several fractions

Multiply every term by the LCM of the denominators, e.g. $\frac{x}{2}+\frac{x}{3}=10$ gives $x=12$

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