Inequalities

A22 · Algebra · revision notes

Solving inequalities such as $3x - 4 < 11$, showing solutions on a number line and listing integer values (both tiers). Higher also solves quadratic inequalities, uses set notation and shows regions on a graph.

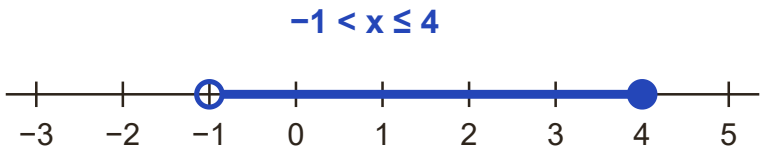
Key facts

- $<$ and $>$: open circle. $\leq$ and $\geq$: filled circle
- Double inequality: do the same to all three parts
- Grade 6+ only Region: solid line for $\leq$ and $\geq$, dashed line for $<$ and $>$
- Multiply or divide by a negative: reverse the sign
- Integers include 0 and negative whole numbers
- Grade 7+ only Quadratic inequality (positive x^2): < 0 is between the roots, > 0 is outside them

Notes

Symbols and number lines

- $<$ less than, $>$ greater than, $\leq$ less than or equal to, $\geq$ greater than or equal to.
- On a number line, an open (empty) circle means the value is not included; a filled circle means it is included.
- $-1 < x \leq 4$: open circle at −1, filled circle at 4, and a line joining them.



Open circle at −1 (not included), filled circle at 4 (included).

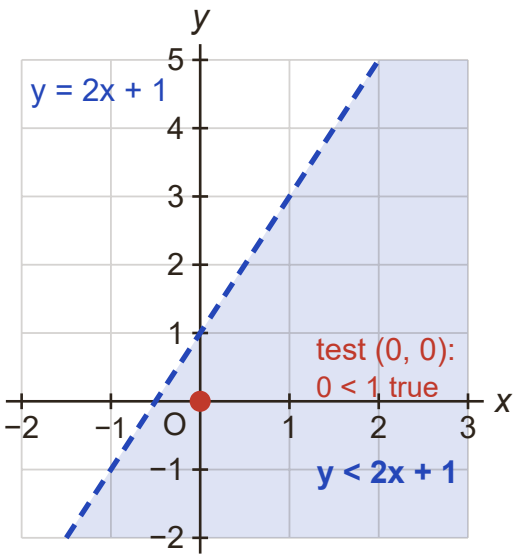
- Integers are whole numbers, including 0 and negative whole numbers. For $-1 < n \leq 4$, $n = 0, 1, 2, 3, 4$

Solving linear inequalities

- Solve like an equation: add, subtract, multiply or divide both sides by the same number.
- If you multiply or divide by a **negative** number, reverse the inequality sign: $-3x < 12$ gives $x > -4$
- You can avoid dividing by a negative by moving the x term: $7 - 2x \geq 1$ gives $6 \geq 2x$, so $x \leq 3$
- Double inequalities: do the same to all three parts. $5 < 3x - 1 \leq 14$ gives $6 < 3x \leq 15$, so $2 < x \leq 5$

Regions on a graph Grade 6+ only

- Draw each boundary as a line: $y < 2x + 1$ has boundary $y = 2x + 1$
- Use a solid line for $\leq$ or $\geq$ and a dashed line for $<$ or $>$
- Test a point that is not on the line, such as (0, 0), to decide which side you need.

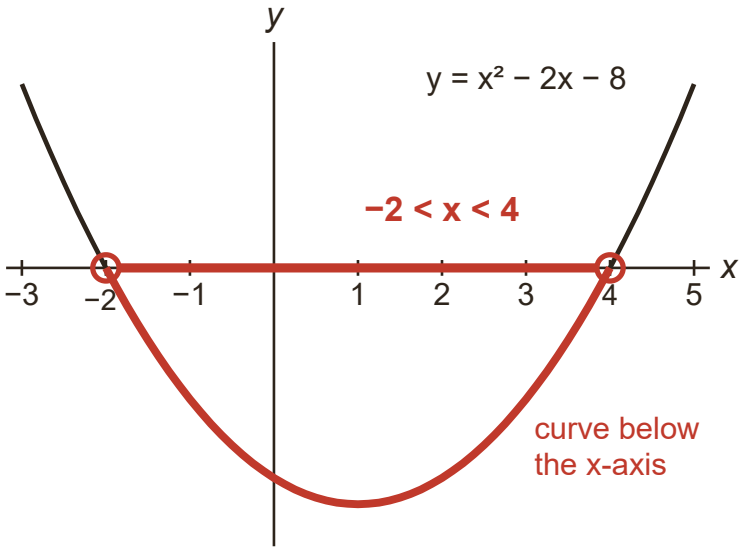


Dashed line because $<$ does not include the line. (0, 0) works, so shade its side.

- Shade and label the region that satisfies all the inequalities, as the question asks.

Quadratic inequalities Grade 7+ only

- Rearrange so one side is 0, solve the equation to find the critical values, and sketch the curve.
- $x^2 - 9 > 0$: the roots are −3 and 3. The curve is above the x-axis outside the roots, so $x < -3$ or $x > 3$
- For < 0, take the part below the axis (between the roots): $x^2 - 2x - 8 < 0$ gives $-2 < x < 4$



$x^2 - 2x - 8 < 0$ where the curve is below the x-axis.

- In set notation: $\{x : -2 < x < 4\}$, and $\{x : x < -3\} \cup \{x : x > 3\}$

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Solve and show on a number line

3 marks **Grade 4**

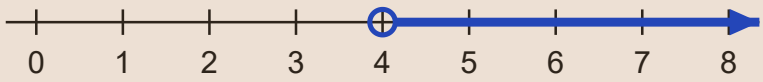
1. Solve as you would an equation, keeping the inequality sign.
2. Draw the correct circle (open or filled) at the boundary value.
3. Draw the line or arrow in the right direction.

Example. (a) Solve $4x + 3 > 19$.
(b) Show your answer on a number line.

Model answer.

(a) $4x > 16$ M1, $x > 4$ A1

(b) Open circle at 4 with an arrow pointing to the right B1



List the integers

2 marks **Grade 5**

1. Solve the double inequality for n.
2. List every integer in the range, checking whether each end value is included.

Example. n is an integer and $-3 < 2n \leq 8$. Write down all the possible values of n.

Model answer.

$-1.5 < n \leq 4$ M1

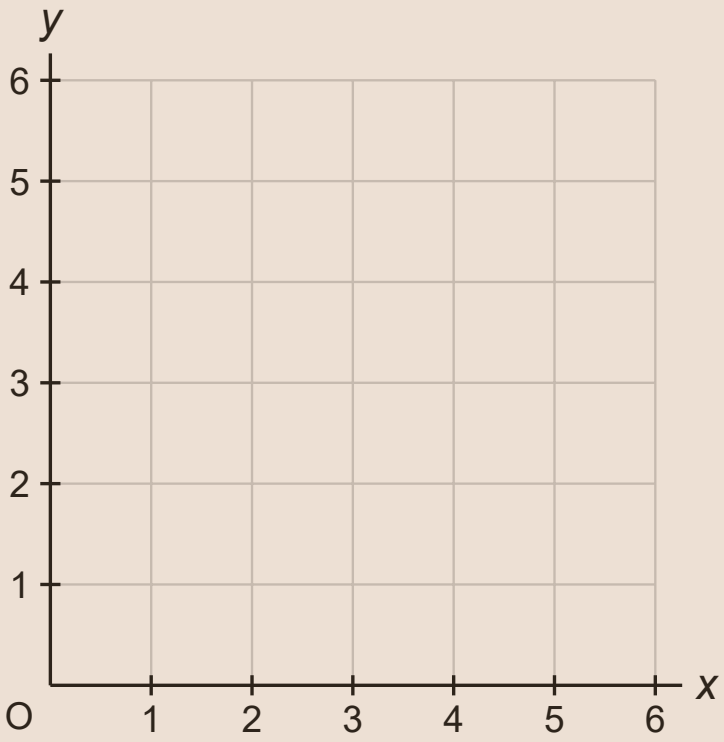
−1, 0, 1, 2, 3, 4 A1

Show a region by shading

4 marks | **Grade 6**

1. Draw each boundary line, choosing solid or dashed.
2. Test a point to find the correct side of each line.
3. Shade and label the region that satisfies all the inequalities.

Example. On a coordinate grid with x and y from 0 to 6, show by shading the region that satisfies all of $y \geq 1$, $x + y < 6$ and $y \leq 2x$. Label the region R.



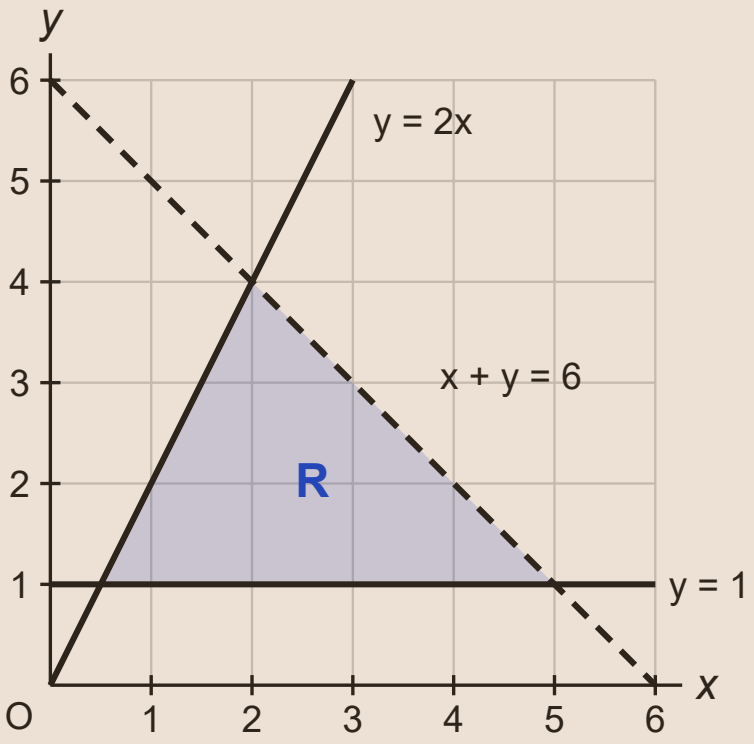
Model answer.

Solid line $y = 1$ — B1

Dashed line $x + y = 6$, through (0, 6) and (6, 0) — B1

Solid line $y = 2x$, through (0, 0) and (3, 6) — B1

R: the triangle with vertices (0.5, 1), (5, 1) and (2, 4), shaded and labelled — B1



Solve a quadratic inequality

3 marks **Grade 7**

1. Factorise (or use the formula) to find the critical values.
2. Sketch the curve through these points.
3. Write the solution: between the roots for < 0, outside them for > 0

Example. Solve $x^2 + x - 12 \geq 0$

Model answer.

$(x + 4)(x - 3)$: critical values −4 and 3	M1
The curve is above the x-axis outside the roots	M1
$x \leq -4$ or $x \geq 3$	A1

Don’t lose marks

- Writing an equals sign in the final answer: the answer to an inequality is an inequality, e.g. $x > 4$
- Forgetting to reverse the sign when multiplying or dividing by a negative number.
- Missing 0 or the negative values when listing integers, or including an end value that is not allowed.
- Higher: writing 'outside' solutions as one double inequality such as $-4 \geq x \geq 3$; write $x \leq -4$ or $x \geq 3$

More tips

Memory tricks

- Check with a number: for $x > 4$, try $x = 5$ in the original inequality (it should work) and $x = 3$ (it should not).
- The wide, open side of $<$ or $>$ always faces the bigger value.
- Higher: for a quadratic inequality, always sketch the parabola; it shows at once which part you need.

Exam technique

- Read whether the question wants integers or all possible values of x.
- Keep the inequality sign on every line of working rather than switching to = and back.
- On number lines, make open and filled circles clearly different.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 3 — **Write an inequality from a number line**
An open circle means the value is not included ($<$ or $>$); a filled circle means it is included ($\leq$ or $\geq$).

Grade 4 — **List integers that satisfy an inequality**
e.g. $-2 < n \leq 3$ gives $n = -1, 0, 1, 2, 3$

Grade 4 — **Solve a linear inequality**
Solve it like an equation, e.g. $4x + 1 > 13$ gives $x > 3$

Grade 5 — **Solve a double inequality**
Do the same to all three parts, e.g. $-3 \leq 2x + 1 < 9$ gives $-2 \leq x < 4$

Grade 5 — **Reverse the sign when dividing by a negative**
e.g. $-2x > -4$ gives $x < 2$

Grade 6 — **Show inequalities as a region on a graph**
Draw each boundary line (dashed for $<$ or $>$, solid for $\leq$ or $\geq$) and shade the region that fits all of them.

Grade 7 — **Solve a quadratic inequality**
Find the roots, sketch the graph and choose the part above or below the x-axis, e.g. $x^2 - x - 6 < 0$ gives $-2 < x < 3$

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