Indices and roots

N6, N7 · Number · revision notes

Squares, cubes and roots, powers of numbers, and the laws of indices, including zero and negative indices and, on the Higher tier, fractional indices. Expect short non-calculator questions such as 'write down the value of' or 'write as a single power', worth 1 to 3 marks.

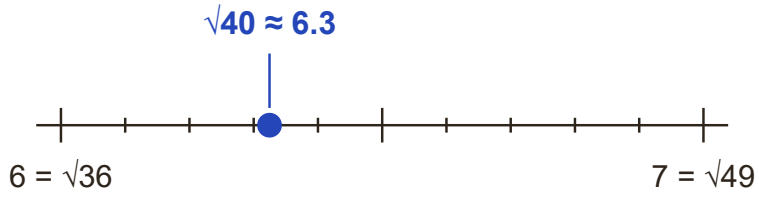
Key facts

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$ (for $a \neq 0$) and $a^1 = a$
- $a^{-n} = \dfrac{1}{a^n}$
- Grade 6+ only $a^{\frac{1}{n}} = \sqrt[n]{a}$
- Grade 7+ only $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$
- Squares up to $15^2 = 225$; cubes 1, 8, 27, 64, 125, 1000

Notes

Powers and roots

- a^n means n lots of a multiplied together: $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$
- Squares to learn: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225. Cubes: 1, 8, 27, 64, 125 and 1000.
- Powers to recognise: 2, 4, 8, 16, 32, 64 (powers of 2); 3, 9, 27, 81, 243 (powers of 3); 4, 16, 64, 256 (powers of 4); 5, 25, 125, 625 (powers of 5).
- A positive number has two square roots: 7 and −7 both square to 49. The $\sqrt{\ }$ sign means the positive one.
- Cube roots of negative numbers are negative: $\sqrt[3]{-27} = -3$
- Estimating: $\sqrt{40}$ lies between $\sqrt{36} = 6$ and $\sqrt{49} = 7$, and is nearer 6, so $\sqrt{40} \approx 6.3$. For a power, round the base: $2.9^3 \approx 3^3 = 27$ Grade 6+ only

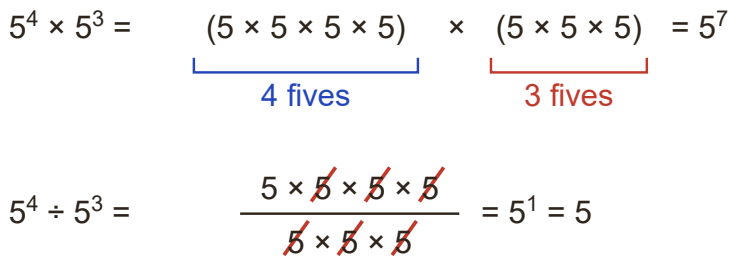


40 is much nearer 36 than 49, so √40 is nearer 6 than 7.

Grade 6+ only

Laws of indices

- Multiplying: add the indices: $5^4 \times 5^3 = 5^7$
- Dividing: subtract the indices: $5^4 \div 5^3 = 5^1 = 5$



Write the powers out: multiplying adds the number of fives; dividing cancels three fives from the top and bottom.

- Power of a power: multiply the indices: $(5^4)^3 = 5^{12}$
- The laws only work when the base is the same: $2^3 \times 3^4$ cannot be written as a single power.
- Rewrite numbers as powers of the same base first: $8 = 2^3$, $9 = 3^2$, $27 = 3^3$

Zero and negative indices

- Any non-zero number to the power 0 is 1: $9^0 = 1$ (because $9^3 \div 9^3 = 9^0$, and anything divided by itself is 1).
- A negative index means 'one over': $a^{-n} = \frac{1}{a^n}$. So $4^{-2} = \frac{1}{16}$ and $10^{-3} = \frac{1}{1000} = 0.001$
- A fraction to a negative power: flip it, then use the positive power:

$$\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^{2} = \frac{9}{4}$$

Grade 6+ only

Fractional indices Grade 6+ only

- $a^{\frac{1}{2}} = \sqrt{a}$, $a^{\frac{1}{3}} = \sqrt[3]{a}$ and in general $a^{\frac{1}{n}} = \sqrt[n]{a}$. So $25^{\frac{1}{2}} = 5$ and $81^{\frac{1}{4}} = 3$
- $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$: the denominator is the root and the numerator is the power (roots are at the bottom of a tree). Take the root first to keep the numbers small: $27^{\frac{2}{3}} = 3^2 = 9$ Grade 7+ only
- Negative fractional index: root, power, then 'one over': $16^{-\frac{3}{4}} = \frac{1}{2^3} = \frac{1}{8}$, and

$$\left(\frac{8}{27}\right)^{-\frac{2}{3}} = \left(\frac{27}{8}\right)^{\frac{2}{3}} = \frac{9}{4}$$

Grade 7+ only

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Write as a single power, or work out the value 1 to 2 marks **Grade 4**

1. Check the bases are the same.
2. Multiply → add the indices; divide → subtract; power of a bracket → multiply.
3. For a fraction, simplify the top first, then divide by the bottom.
4. Only work out the value if the question asks for it.

Example. (a) Write $7^5 \times 7^3$ as a power of 7.
(b) Work out the value of $\frac{3^6 \times 3^2}{3^5}$

Model answer.

(a) 7^8 (B1)

(b) $\frac{3^8}{3^5} = 3^3$ (M1 for 3^8 or 3^3)
$= 27$ (A1)

Write down the value: zero and negative indices

1 mark each | Grade 5

1. Power 0: the answer is 1.
2. Negative power: write 'one over' the positive power, then work it out.
3. Leave the answer as an exact fraction; it stays positive.

Example. Write down the value of

(a) 8^0

(b) 5^{-2}

Model answer.

(a) 1 (B1)

(b) $\frac{1}{5^2} = \frac{1}{25}$ (B1)

Work out the value: fractional indices

1 to 2 marks each | Grade 7

1. Negative index: flip the number (take the reciprocal) and make the index positive.
2. Denominator of the index: take that root.
3. Numerator of the index: raise to that power.

Example. Work out the value of

(a) $49^{\frac{1}{2}}$

(b) $32^{\frac{3}{5}}$

(c) $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$

Model answer.

(a) 7 (B1)

(b) $\sqrt[5]{32} = 2$ (M1 for 2 or 2^3 seen), $2^3 = 8$ (A1)

(c) $\left(\frac{16}{9}\right)^{\frac{1}{2}}$ (M1 for $\frac{16}{9}$ or $\frac{3}{4}$ seen) $= \frac{4}{3}$ (A1)

Find the missing power

2 marks | Grade 7

1. Write every number as a power of the same prime, including roots as fractional powers.
2. Use the laws of indices to combine them into a single power.
3. Compare the indices to find the unknown.

Example. Given that $\frac{9^2 \times \sqrt{3}}{27} = 3^n$, find the value of n

Model answer.

$9^2 = 3^4$, $\sqrt{3} = 3^{\frac{1}{2}}$, $27 = 3^3$ (M1 for at least two written as powers of 3)

$3^{4+\frac{1}{2}-3} = 3^{\frac{3}{2}}$, so $n = \frac{3}{2}$ (A1)

Don't lose marks

- Multiplying the bases: $3^4 \times 3^2 = 3^6$, not 9^6
- Multiplying the indices instead of adding: $5^3 \times 5^4 = 5^7$, not 5^{12}
- Thinking a negative index makes the number negative: $2^{-3} = \frac{1}{8}$, not −8.
- Writing $7^0 = 0$ or $7^0 = 7$. It is 1.
- Working out 2×3^2 as $6^2 = 36$. The power comes first: 2 × 9 = 18.

More tips

Memory tricks

- Multiply → Add, Divide → Subtract, Bracket power → Multiply.
- Negative index? 'One over': $3^{-2} = \frac{1}{9}$. The answer stays positive.
- Anything (except 0) to the power 0 is 1, not 0.
- Learn the powers of 2 up to $2^{10} = 1024$: they turn up in many index questions.

Exam technique

- 'Write down' means you can give the answer without working, but it must be exact.
- If asked to write as a power (e.g. 'as a power of 7'), leave the answer as 7^8; don't work out its value.
- Rewrite every number as a power of the same prime before using the laws, e.g. $8 = 2^3$ and $27 = 3^3$

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 1 — **Recall square numbers and square roots**
Know the squares up to $15^2 = 225$ and their square roots, e.g. $\sqrt{144} = 12$

Grade 2 — **Recall cube numbers and cube roots**

Know 1, 8, 27, 64, 125 and 1000, e.g. $\sqrt[3]{64} = 4$

Grade 3

Recognise powers of 2, 3, 4 and 5

E.g. $2^5 = 32$, $3^4 = 81$, $4^3 = 64$ and $5^3 = 125$

Grade 4

Use the laws of indices with numbers

Add indices to multiply, subtract to divide, multiply for a power of a power: $(3^4)^2 = 3^8$

Grade 5

Use zero and negative indices

$a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$, e.g. $2^{-3} = \frac{1}{8}$

Grade 6

Estimate powers and roots

Use nearby squares or cubes: $\sqrt{40}$ is between 6 and 7 (36 and 49), about 6.3.

Grade 6

Evaluate unit fractional indices

$a^{\frac{1}{n}}$ is the nth root of a, so $64^{\frac{1}{3}} = 4$

Grade 7

Evaluate any fractional or negative fractional index

Root, then power, then flip if negative:

$$\begin{aligned} & 8^{-\frac{2}{3}} \\ &= \frac{1}{2^2} \\ &= \frac{1}{4} \end{aligned}$$

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