Histograms

S3 · Statistics · revision notes

Histograms show grouped continuous data, often with classes of different widths. You need to work out frequency densities, draw histograms, find frequencies from the areas of the bars, and estimate the number in part of a class or the median. This subtopic is Higher tier only.

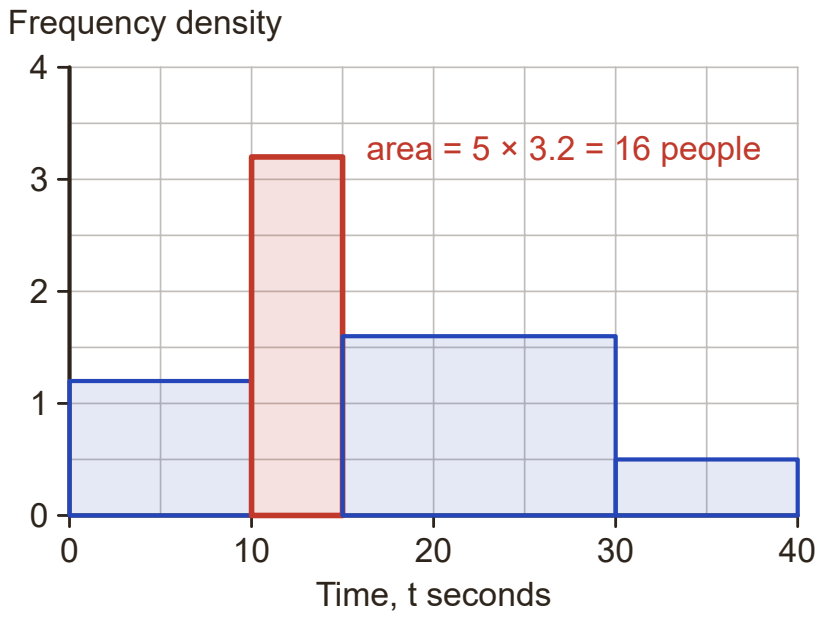
Key facts

- Frequency density = frequency ÷ class width
- Class width = upper class boundary − lower class boundary
- Part of a class: frequency ≈ frequency density × width of the part
- Frequency = frequency density × class width (the area of the bar)
- Vertical axis: frequency density; bars touch; horizontal scale is continuous
- Median: the $\frac{n}{2}$th value, assuming values are spread evenly through its class

Notes

Frequency density

- In a histogram, the **area** of each bar represents the frequency, not the height.
- The height of each bar is the **frequency density**: frequency ÷ class width.



The frequency is the area of the bar, not its height.

- Class width = upper boundary − lower boundary, e.g. $10 < t \leq 25$ has a width of 15.
- This keeps wide classes fair: a class twice as wide with the same frequency has a bar half as tall.

Drawing a histogram

- Add a frequency density column to the table: frequency ÷ class width for each class.
- Label the vertical axis 'Frequency density' and use a continuous scale on the horizontal axis.
- Draw each bar from the lower boundary to the upper boundary of its class. The bars touch: there are no gaps.

Reading a histogram

- Frequency = frequency density × class width (the area of the bar).
- Total frequency = the sum of the frequencies of all the bars.
- If the vertical axis has no numbers, use a bar whose frequency you know. Work out its frequency density (or how many items one square of area stands for), then use that scale for the other bars.

Estimates from a histogram

- Assume the values in each class are spread evenly across it.
- For part of a class, frequency ≈ frequency density × the width of that part. Add on any whole classes in between.
- For the median, find the $\frac{n}{2}$th value. Use a running total of the frequencies to find the class it is in, then work out what fraction of the way through that class it lies.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Work out frequency densities and draw a histogram

3 marks **Grade 6**

1. Work out each class width.
2. Frequency density = frequency ÷ class width, for each class.
3. Choose a sensible scale for the frequency density axis.
4. Draw touching bars with the correct widths and heights.

Example. The table shows information about the masses, m kg, of 60 parcels.

Mass (m kg)	Frequency
$0 < m \leq 2$	8
$2 < m \leq 5$	18
$5 < m \leq 10$	20
$10 < m \leq 20$	14

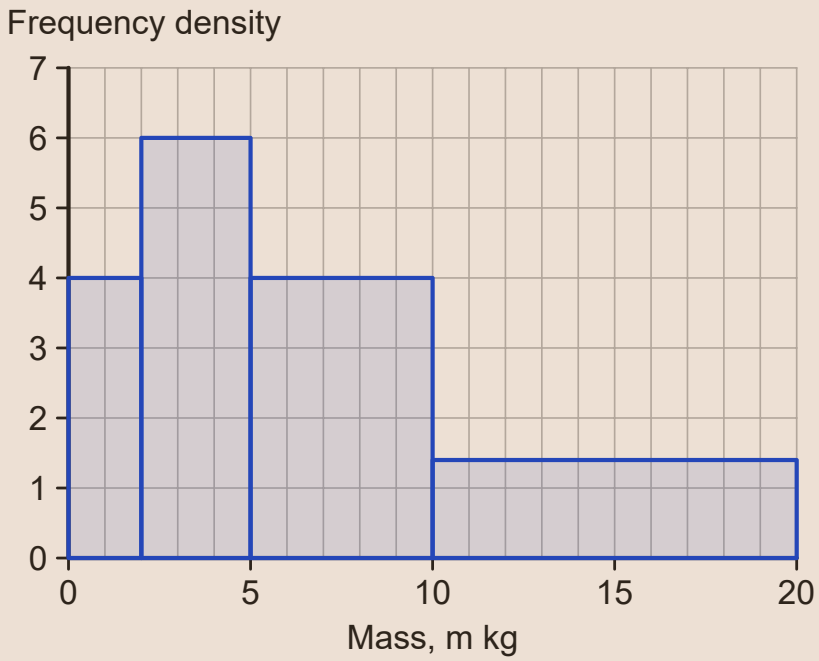
Draw a histogram for this information.

Model answer.

Mass (m kg)	Frequency	Class width	Frequency density
$0 < m \leq 2$	8	2	8 ÷ 2 = 4
$2 < m \leq 5$	18	3	18 ÷ 3 = 6
$5 < m \leq 10$	20	5	20 ÷ 5 = 4
$10 < m \leq 20$	14	10	14 ÷ 10 = 1.4

B3 for a fully correct histogram with bars of these heights and the correct widths

(B2 for three correct bars; B1 for two correct bars, or for frequency ÷ class width worked out for at least three classes)



Find frequencies from a histogram

2 marks **Grade 7**

1. Read the frequency density (the height) of the bar.
2. Multiply by the class width to get the frequency.
3. Add the frequencies if the question asks about more than one class.

Example. A histogram shows the times, t seconds, that some people took to solve a puzzle.
The bar for $10 < t \leq 15$ has a frequency density of 3.2
The bar for $15 < t \leq 30$ has a frequency density of 1.6
Work out the number of people who took more than 10 seconds but no more than 30 seconds.

Model answer.

3.2 × 5 = 16 or 1.6 × 15 = 24 (M1)
16 + 24 = 40 people (A1)

Work out a missing frequency density scale

3 marks | Grade 8

1. Use the bar whose frequency is given: frequency density = frequency ÷ class width.
2. Work out what one unit of height (or one square of area) stands for.
3. Use this to find the frequency density, then the frequency, of the bar you need.

Example. A histogram shows the heights, h cm, of some plants. There are no numbers on the frequency density axis.
The bar for $20 < h \leq 30$ is 3 cm tall and represents 12 plants.
The bar for $30 < h \leq 35$ is 7 cm tall.
Work out the number of plants in the class $30 < h \leq 35$

Model answer.

Frequency density for $20 < h \leq 30$ = 12 ÷ 10 = 1.2, so 1 cm stands for 1.2 ÷ 3 = 0.4 (M1)

Frequency density for $30 < h \leq 35$ = 7 × 0.4 = 2.8 (M1)

Frequency = 2.8 × 5 = 14 plants (A1)

Estimate the number in part of a class

3 marks | Grade 8

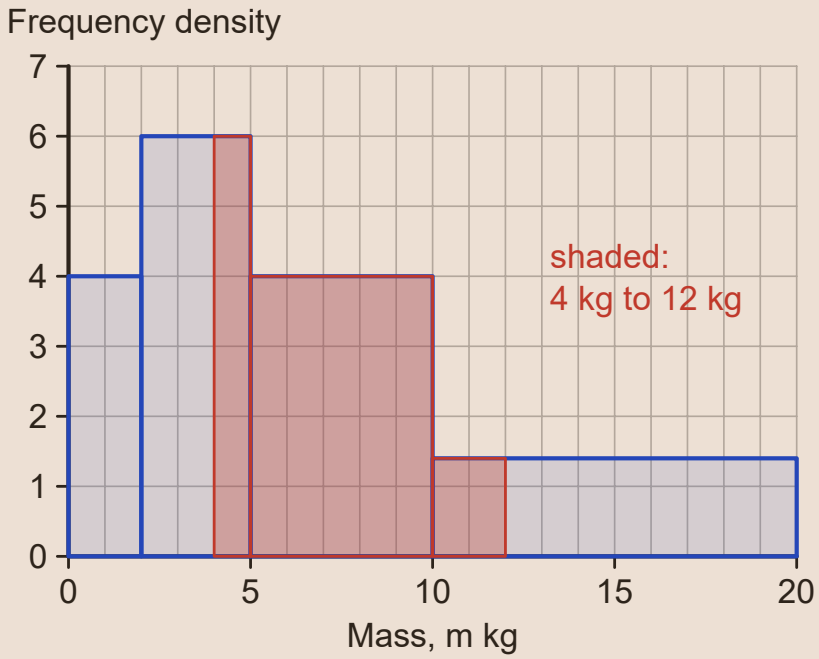
1. Split the interval into whole classes and parts of classes.
2. For each part, multiply its frequency density by the width of the part.
3. Add the frequencies of the parts and of the whole classes.

Example. A histogram of the masses, m kg, of 60 parcels has these frequency densities.

Mass (m kg)	Frequency density
$0 < m \leq 2$	4
$2 < m \leq 5$	6
$5 < m \leq 10$	4
$10 < m \leq 20$	1.4

Work out an estimate for the number of parcels with a mass between 4 kg and 12 kg.

Model answer.



4 kg to 5 kg: 6 × 1 = 6, or 10 kg to 12 kg: 1.4 × 2 = 2.8 (M1)

The whole class $5 < m \leq 10$: 4 × 5 = 20, so 6 + 20 + 2.8 (M1)

= 28.8, so about 29 parcels (A1)

Estimate the median from a histogram

3 marks | Grade 9

1. Find the frequency of every class (frequency density × class width) and the total, n.
2. Find $\frac{n}{2}$ and use a running total to find the class it lies in.
3. Median ≈ lower boundary + (number still needed ÷ frequency of the class) × class width.

Example. Use the histogram of the masses of the 60 parcels in the previous example.
Work out an estimate for the median mass.

Model answer.

Frequencies 8, 18, 20, 14, with running totals 8, 26, 46, 60. The median is the 30th value, which is in the class $5 < m \leq 10$ (M1)

It is 30 − 26 = 4 values into a class of 20 values, so $5 + \frac{4}{20} \times 5$ (M1)

= 6 kg (A1)

Don't lose marks

- Plotting frequency instead of frequency density on the vertical axis.
- Getting a class width wrong, e.g. using 20 for $10 < m \leq 20$ instead of 10.
- Leaving gaps between the bars, or drawing a bar at the wrong boundaries.
- Reading the height of a bar as its frequency.
- Using the whole bar when the question asks about only part of a class.

More tips

Memory tricks

- Three words to remember: area = frequency.
- Formula triangle: frequency on top, frequency density × class width underneath.
- Sense check: the frequencies you find from all the bars must add up to the total you were given.
- Grid trick: once you know how many items one square of area stands for, count squares to find any frequency.

Exam technique

- Write a frequency density column next to the table and show each division. It earns method marks.
- Label the vertical axis 'Frequency density' and choose a scale that uses most of the grid.
- For a missing scale, find the scale from the bar you know first, and write down what one unit or one square stands for.
- When an estimate of a number of items is not a whole number, show the exact value and then round it, e.g. 28.8, so about 29.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 6

Calculate frequency density for each class

Frequency density = frequency ÷ class width, so a wider class needs a lower bar for the same frequency.

Grade 6

Draw a histogram with unequal class widths

Draw touching bars whose heights are the frequency densities, against a continuous scale.

Grade 7

Find frequencies from a histogram

Frequency = frequency density × class width, which is the area of the bar.

Grade 8

Work out a missing frequency density scale

Use a bar whose frequency you know to find what each unit of height, or each square, represents.

Grade 8

Estimate a frequency within part of a class

Assume the values are spread evenly through the class, and use frequency density × the width of the part you need.

Grade 9

Estimate the median from a histogram

Find the class containing the $\frac{n}{2}$th value and work out how far through that class it lies.

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