Compound interest, growth and decay

R16 · Ratio, proportion and rates of change · revision notes

Compound interest, growth and decay are repeated percentage changes: each year's change is worked out on the new amount, so you multiply by the same multiplier again and again. Questions ask for the value after a number of years, the interest earned or when a target is passed and, at Higher tier, you may have to work backwards, find the rate or use an iterative formula.

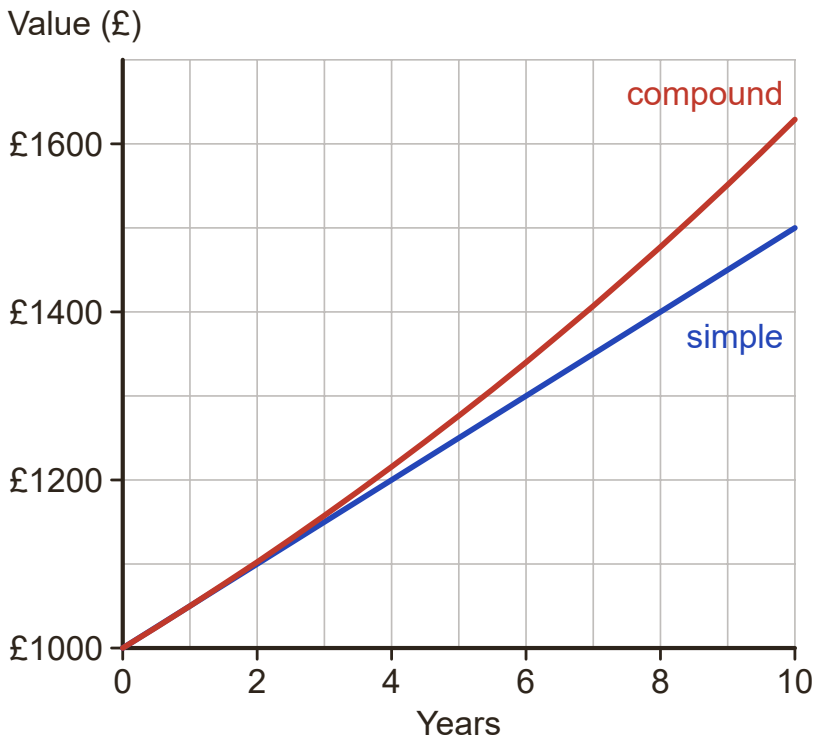
Key facts

- Growth: value after n years = start × $\left(1+\frac{r}{100}\right)^n$
- Interest earned = final amount − starting amount
- Grade 6+ only Overall percentage change = (multiplier^n − 1) × 100 (a negative answer is a decrease)
- Grade 7+ only $u_{n+1} = ku_n$: multiply by k at each step
- Decay: value after n years = start × $\left(1-\frac{r}{100}\right)^n$
- Simple interest is the same every year; compound interest also earns interest on the interest
- Grade 7+ only Original value = final value ÷ multiplier^n
- Grade 8+ only Multiplier = $\sqrt[n]{\text{final} \div \text{start}}$

Notes

Simple and compound interest

- Simple interest is worked out on the original amount only, so you get the same interest every year.
- Compound interest is added on each year, and the next year's interest is worked out on the new total, so it grows faster.
- £1000 at 5% a year: simple interest pays £50 every year. Compound interest pays £50, then £52.50, then £55.13, giving £1157.63 after 3 years.



£1000 at 5% a year: simple interest grows in a straight line; compound interest curves upwards.

Using a multiplier and a power

- Value after n years = starting value × multiplier^n.
- Growth of r%: the multiplier is $1 + \frac{r}{100}$, so 3% gives 1.03. Decay or depreciation of r%: the multiplier is $1 - \frac{r}{100}$, so 12% gives 0.88.
- £2500 at 3% compound interest for 4 years: $2500 \times 1.03^4 = £2813.77$.
- Interest earned = final amount − starting amount.
- The same method works for anything that changes by the same percentage in each time period: savings, populations, the value of a car, bacteria.
- Check the time period: a rate per month for 2 years means n = 24.
- Overall percentage change: work out multiplier^n. A 10% fall each year for 3 years gives $0.9^3 = 0.729$, which is a 27.1% decrease overall. Grade 6+ only

How many years?

- Type the starting value and press =, then type × 1.04 and press = again and again: each press is one more year (the calculator uses Ans).
- Stop at the first value that passes the target, and write down the values for the year before and the year after.

Working backwards, rates and iteration Grade 7+ only

- Original value = final value ÷ multiplier^n. A car worth £9216 after losing 20% of its value each year for 2 years was worth 9216 ÷ 0.8^2 = £14 400.
- Finding the rate: if £800 grows to £882 in 2 years, multiplier^2 = 882 ÷ 800 = 1.1025, so the multiplier is $\sqrt{1.1025} = 1.05$, which is 5% a year. Grade 8+ only
- An iterative formula such as $u_{n+1} = 1.04u_n$ with $u_0 = 500$ means multiply by 1.04 at each step: $u_1 = 520$, $u_2 = 540.8$
- Some iterative formulas also add or subtract an amount, e.g. $u_{n+1} = 0.9u_n + 50$: multiply, then add, at each step.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Work out the value after compound interest 3 marks Grade 5

1. Write the multiplier: 1 + the rate as a decimal (2.5% gives 1.025).
2. Multiply the amount invested by the multiplier to the power of the number of years.
3. Round only at the end, to the nearest penny unless told otherwise.
4. Subtract the amount invested if the question asks for the interest.

Example. Maya invests £4000 in a savings account for 3 years.
The account pays compound interest at a rate of 2.5% per year.
Work out the value of Maya's investment at the end of the 3 years.

Model answer.

Multiplier = 1.025 (M1)

4000×1.025^3 (M1)

= £4307.56 (A1)

Work out a value after depreciation or decay

3 marks **Grade 5**

1. Write the multiplier: 1 − the rate as a decimal (15% gives 0.85).
2. Multiply the starting value by the multiplier to the power of the number of years.
3. Round as the question says.

Example. A van cost £18 000 when it was new.
The value of the van depreciates by 15% each year.
Work out the value of the van after 4 years.
Give your answer to the nearest pound.

Model answer.

Multiplier = 0.85 (M1)

18000×0.85^4 = 9396.11... (M1)

= £9396 (A1)

Find when a value first passes a target

3 marks **Grade 5**

1. Work out the value after 1, 2, 3, ... years (use Ans × multiplier on the calculator).
2. Stop at the first year that passes the target.
3. Write the values for the year before and the year after the target, then give the number of years.

Example. Josh invests £3000 in an account paying 4% per year compound interest.
After how many whole years will the investment first be worth more than £3500?
You must show your working.

Model answer.

3000 × 1.04 = £3120, then £3244.80, ... (M1)

After 3 years: £3374.59; after 4 years: £3509.58 (M1)

4 years (A1)

Work back to the original value

3 marks | Grade 7

1. Write the multiplier for one year.
2. Original value × multiplier^n = final value, so divide the final value by multiplier^n.
3. Check by working forwards from your answer.

Example. The value of a painting increased by 6% each year for 5 years.
The painting is now worth £13 382.26.
Work out the value of the painting 5 years ago.

Model answer.

Multiplier = 1.06, so original × 1.06^5 = 13 382.26 (M1)

13 382.26 ÷ 1.06^5 (M1)

= £10 000 (A1)

Find the rate of compound interest

3 marks | Grade 8

1. Divide the final value by the starting value to get multiplier^n.
2. Take the nth root (square root for 2 years) to find the multiplier.
3. Turn the multiplier into a percentage: 1.05 means 5%.

Example. £6000 is invested for 2 years at r% per year compound interest.
At the end of the 2 years the investment is worth £6615.
Work out the value of r

Model answer.

6615 ÷ 6000 = 1.1025 (M1)

$\sqrt{1.1025} = 1.05$ (M1)

$r = 5$ (A1)

Don’t lose marks

- Using simple interest (the same amount each year) when the question says compound interest.
- Multiplying by 0.15 instead of 0.85 for a 15% decrease: 0.15 is the part that is lost.
- Writing the multiplier for 3% as 1.3 instead of 1.03.
- Rounding in the middle of the calculation: keep the full calculator value until the end.
- Working backwards by multiplying, or by adding the percentage back on: divide by multiplier^n.
- Giving the total when the question asks for the interest, or the other way round.

More tips

Memory tricks

- Calculator trick: type the start value and press =, then × 1.04 =, and every extra press of = moves on one more year.
- Sense check: a multiplier above 1 must make the value bigger; a multiplier below 1 must make it smaller.
- The power is the number of time periods; the rate only goes into the multiplier.
- At the same rate, compound interest earns more than simple interest after the first year, because it earns interest on the interest.

Exam technique

- Write down the calculation you type in, e.g. 4000×1.025^3: it earns method marks even if you mistype.
- Round money to the nearest penny unless the question says otherwise (e.g. 'to the nearest pound').
- 'Per annum' means per year.
- For 'how many years' questions, show the value just before and just after the target.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 4 — **Work out compound interest year by year**
Add each year's interest before working out the next, e.g. £500 at 4% becomes £520, then £540.80.

Grade 5 — **Use a multiplier raised to a power**
Value after n years = start × multiplier^n, e.g. £2500 at 3% for 4 years is 2500 × 1.03^4.

Grade 5 — **Calculate depreciation and decay**
Use a multiplier less than 1, e.g. × 0.85^n for a 15% fall each year.

Grade 5 — **Find when a value first passes a target**
Work out the value year by year and show the values either side of the target.

Grade 6 — **Find the overall percentage change over time**
Work out multiplier^n, e.g. $0.9^3 = 0.729$ is a 27.1% decrease.

Grade 7 — **Work backwards to find the original value**
Divide the final value by multiplier^n.

Grade 7 — **Use an iterative formula for growth or decay**
Apply a formula such as $u_{n+1} = 1.04u_n$ one step at a time.

Grade 8 — **Find the rate from start and end values**
Work out final ÷ start, take the nth root to get the multiplier, then turn it into a percentage.

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