Graphs of functions

A12, A14 · Algebra · revision notes

Recognising, sketching, plotting and interpreting graphs of quadratic, cubic and reciprocal functions (both tiers), and on Higher exponential and trigonometric graphs. Includes reading approximate solutions from graphs and interpreting real-life graphs such as distance–time graphs.

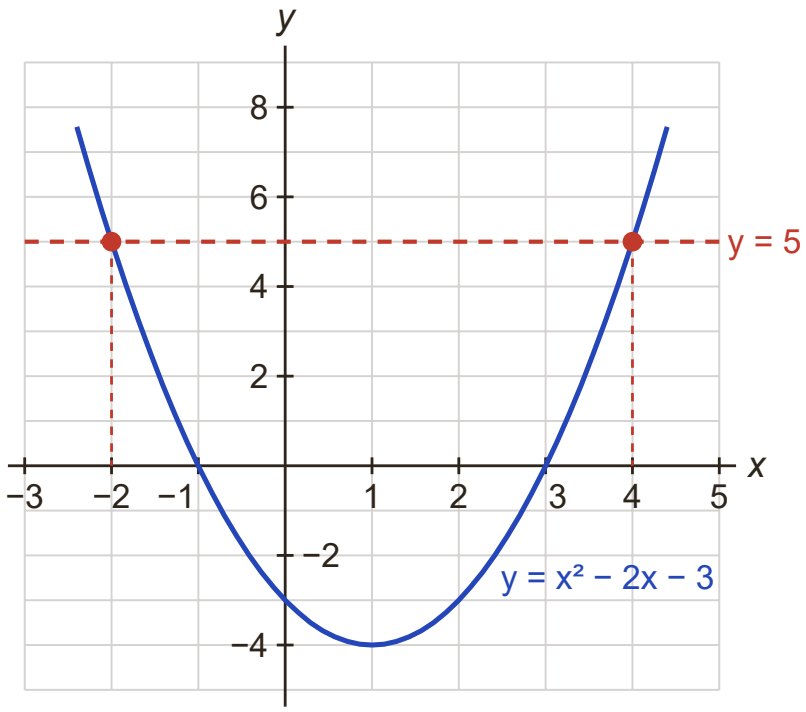
Key facts

- Quadratic: U shape (a positive) or ∩ shape (a negative), symmetrical
- Reciprocal $y = \frac{k}{x}$: two curves that never touch the axes
- Grade 7+ only $\sin x$ and $\cos x$: between −1 and 1, repeat every 360°; $\tan x$: repeats every 180°
- Distance–time gradient = speed; velocity–time gradient = acceleration
- Cubic: S shape
- Grade 6+ only Exponential $y = k^x$: through (0, 1), always above the x-axis
- Roots: x-coordinates where the curve crosses the x-axis (y = 0)

Notes

Plotting graphs

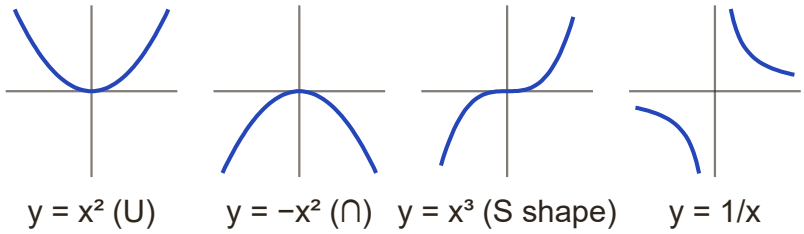
- Make a table of values, substitute each x (negatives in brackets) and plot the points accurately.
- Join the points of a curve with a smooth curve, not straight line segments. Never join across a gap (reciprocal graphs).
- To solve $x^2 - 2x - 3 = 5$ using the graph of $y = x^2 - 2x - 3$, draw $y = 5$ and read the x-coordinates where it crosses the curve.



The line $y = 5$ crosses the curve at $x = -2$ and $x = 4$: the solutions of $x^2 - 2x - 3 = 5$

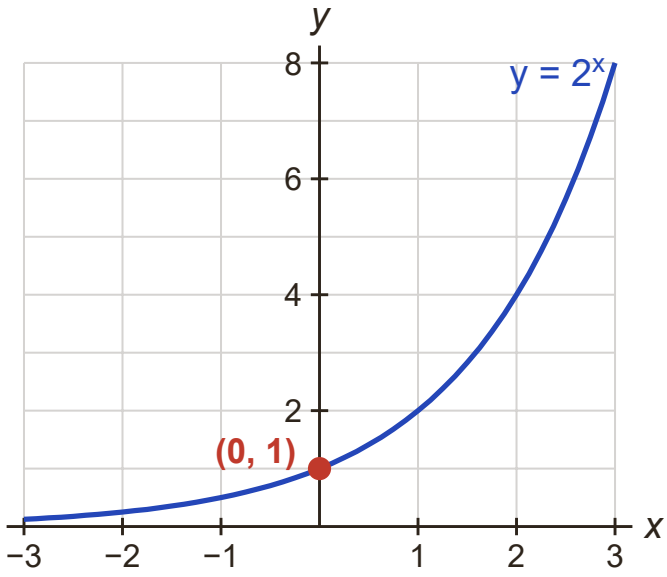
Shapes to recognise

- Quadratic $y = ax^2 + bx + c$: a U shape if a is positive, a ∩ shape if a is negative. It is symmetrical about a vertical line through its turning point.
- Cubic, e.g. $y = x^3$: an S-shaped curve. $y = x^3$ passes through the origin and goes from bottom left to top right; $y = -x^3$ goes from top left to bottom right.
- Reciprocal $y = \frac{1}{x}$: two separate curves in opposite quadrants, getting closer and closer to the axes without touching them. x cannot be 0.



Learn these shapes. The reciprocal graph never touches either axis.

- Exponential $y = k^x$ (k positive): passes through (0, 1) and is always above the x-axis. For $k > 1$ it rises more and more steeply; for $0 < k < 1$ it falls towards the x-axis. Grade 6+ only

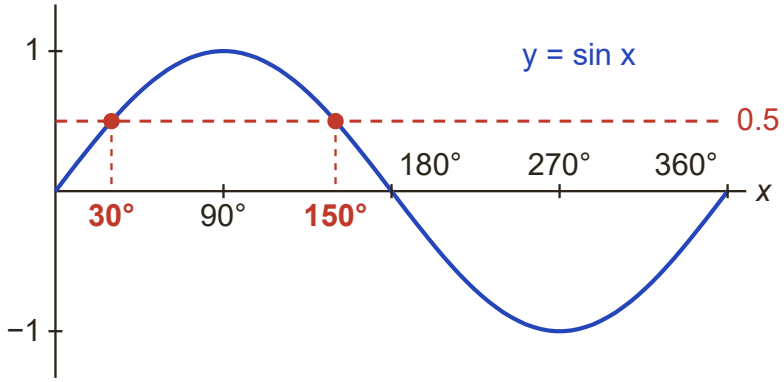


Exponential growth: through (0, 1), always above the x-axis.

Grade 6+ only

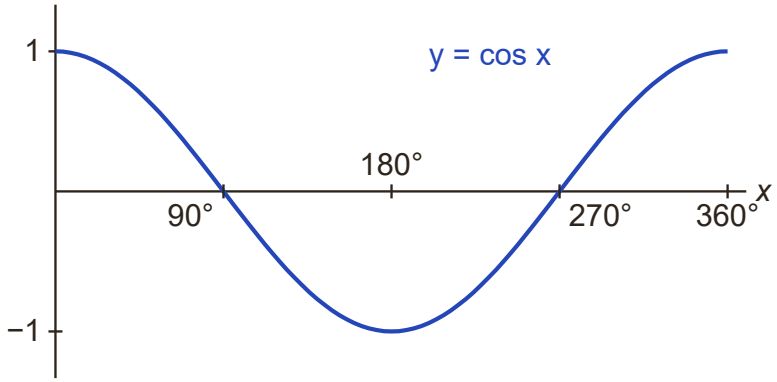
Trigonometric graphs Grade 7+ only

- $y = \sin x$: starts at (0, 0), maximum 1 at 90°, 0 at 180°, minimum −1 at 270°, and repeats every 360°.



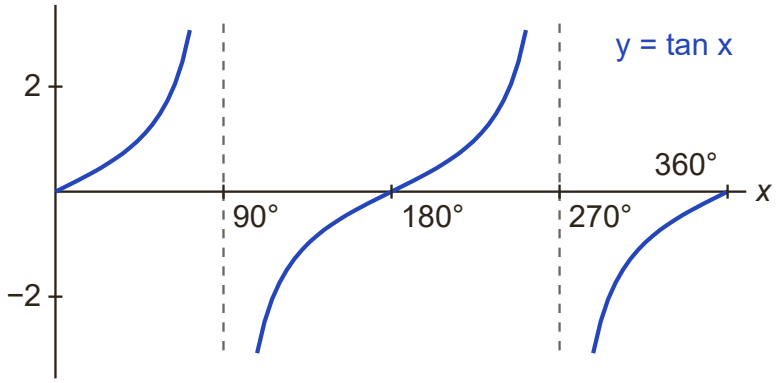
$\sin x = 0.5$ at 30° and, by symmetry about 90°, at 180° − 30° = 150°.

- $y = \cos x$: starts at (0, 1), 0 at 90°, minimum −1 at 180°, and repeats every 360°.



The cos graph is the sin graph moved 90° to the left.

- $y = \tan x$: passes through (0, 0), repeats every 180°, and is undefined at 90°, 270°, … where the graph has asymptotes.

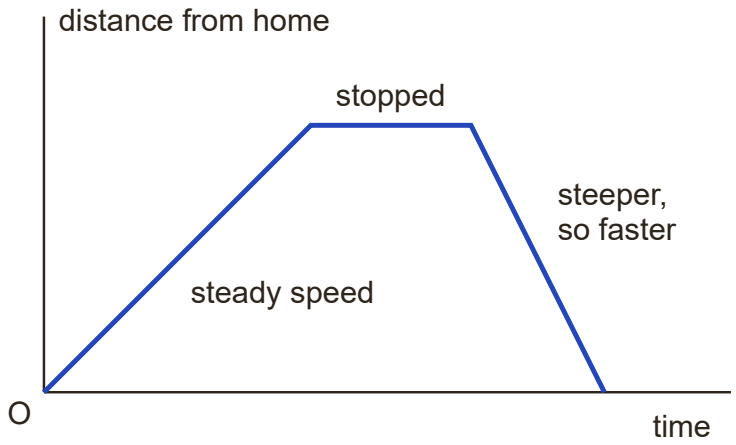


Dashed lines: asymptotes at 90° and 270°, where tan x is undefined.

- Use symmetry to find other angles: if $\sin x = 0.5$, then $x = 30^\circ$ or $x = 150^\circ$ between 0° and 360°.

Real-life graphs

- Distance–time graph: the gradient is the speed; a horizontal line means stopped; steeper means faster.



Gradient = speed; flat = stopped; a line back down means returning.

- Velocity–time graph: the gradient is the acceleration.
- Read scales carefully: check what one small square represents on each axis.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Complete a table of values and draw the graph

4 marks | Grade 4

1. Substitute each x value, using brackets for negatives.
2. Plot every point accurately.
3. Join with one smooth curve.

Example. (a) Complete the table of values for $y = x^2 - 2x - 3$.

x	−2	−1	0	1	2	3	4
y			−3			0	

(b) Draw the graph of $y = x^2 - 2x - 3$ for values of x from −2 to 4.

Model answer.

(a)

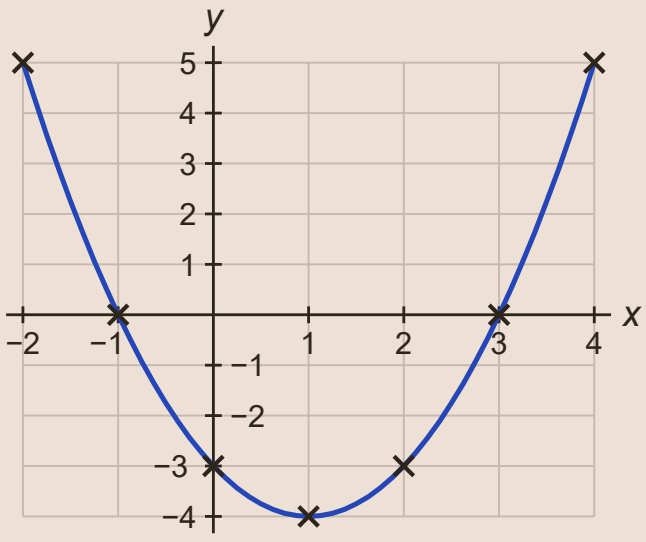
x	−2	−1	0	1	2	3	4
y	5	0	−3	−4	−3	0	5

B2

(B1 for at least 3 of the 5 missing values correct)

(b) At least 6 points plotted correctly M1

A smooth U-shaped curve through all 7 points A1

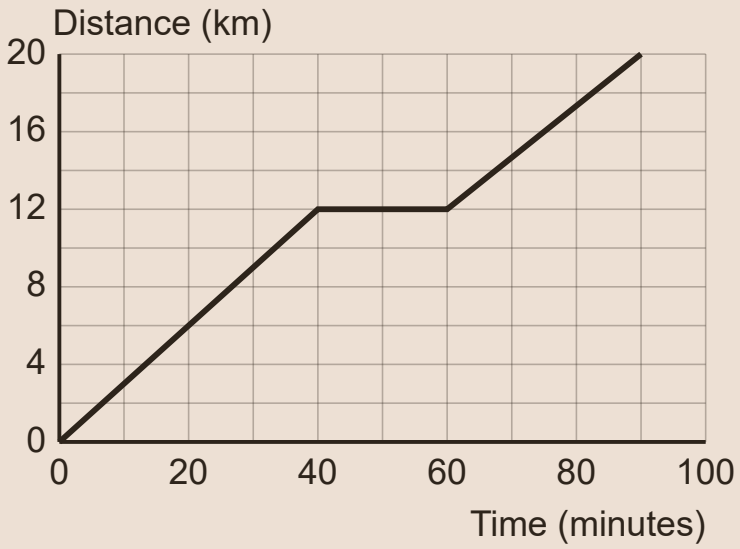


Interpret a distance–time graph

3 marks | Grade 4

1. A horizontal section means no movement.
2. Speed = distance ÷ time, using the right units.

Example. The distance–time graph shows a cyclist's journey.



(a) Describe the cyclist's journey between 40 and 60 minutes.

(b) Work out the cyclist's speed, in km/h, for the first 40 minutes.

Model answer.

(a) The graph is horizontal, so the cyclist is stationary (stopped) B1

(b) 12 km in 40 minutes: $12 \div \frac{40}{60}$ M1

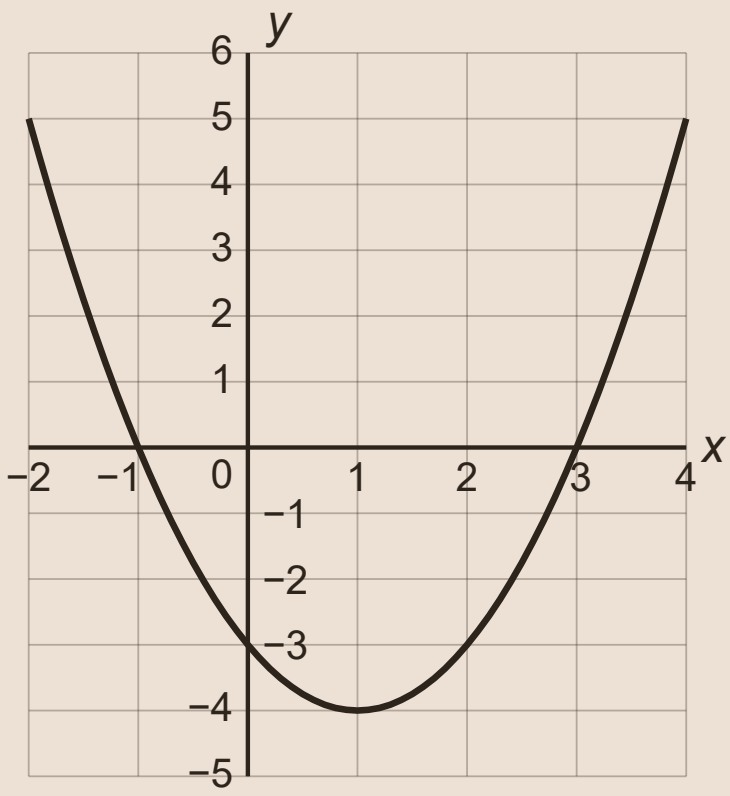
= 18 km/h A1

Use a graph to solve an equation

2 marks | Grade 5

1. Rearrange so one side is the equation of the graph you have.
2. Draw the line for the other side.
3. Read off the x-coordinates of the crossing points.

Example. Here is the graph of $y = x^2 - 2x - 3$.



Use the graph to estimate the solutions of $x^2 - 2x - 3 = 2$

Model answer.

Draw $y = 2$ and read off where it meets the curve M1

$x \approx -1.4$ and $x \approx 3.4$ A1

(a small range is accepted)

Match equations to graphs

1 mark each | Grade 5

1. Identify the type of each equation (quadratic, cubic, reciprocal, linear).
2. Use the sign of the leading term and the y-intercept to decide.

Example. Match each equation to its description.

(a) $y = x^3 - 2$ (b) $y = \dfrac{4}{x}$ (c) $y = 3 - x^2$

P: a ∩-shaped curve crossing the y-axis at 3. Q: two separate curves, in the first and third quadrants. R: an S-shaped curve crossing the y-axis at −2.

Model answer.

(a) R B1
(b) Q B1
(c) P B1

Use a trigonometric graph to find an angle

2 marks | Grade 7

1. Sketch the graph from 0° to 360°.
2. Use its symmetry to find the other angle with the same value.

Example. Given that $\cos 60^\circ = 0.5$, find the other value of x between 0° and 360° for which $\cos x = 0.5$

Model answer.

The cos graph is symmetrical about $x = 180^\circ$, so $360^\circ - 60^\circ$ M1

$x = 300^\circ$ A1

Don't lose marks

- Substituting negatives without brackets: for $x = -2$,

$$\begin{aligned} x^2 - 2x &= (-2)^2 - 2 \times (-2) \\ &= 4 + 4 \\ &= 8 \end{aligned}$$

- Joining points with straight lines instead of a smooth curve.
- Drawing a flat bottom on a U-shaped curve: the turning point is a smooth point.
- Joining the two parts of a reciprocal graph across $x = 0$

More tips

Memory tricks

- Quadratic tables are symmetrical, e.g. 5, 0, −3, −4, −3, 0, 5. A value that breaks the pattern is probably wrong.
- If your calculator has a table mode, use it to fill in tables of values, and check one value by hand.
- Higher: before finding angles, sketch sin or cos on axes marked 0°, 90°, 180°, 270° and 360°.

Exam technique

- Draw curves in one smooth stroke with a sharp pencil, and plot points with small crosses.
- When reading solutions from a graph, give answers as accurately as the scale allows; examiners accept a small range.
- Show the line you drew (e.g. $y = 2$) on the graph: it is part of your method.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 3 — **Complete a table of values and plot**
Substitute each x (with brackets for negatives), plot the points and join them with a smooth curve.

Grade 4 — **Recognise the shapes of standard graphs**
Linear, quadratic (U or ∩ shape), cubic (S shape) and reciprocal $y = \frac{1}{x}$ (two separate curves).

Grade 4 — **Interpret distance–time graphs**
The gradient is the speed and a horizontal section means the object is stationary.

Grade 5 — **Read roots and turning points from graphs**
Roots are where the curve crosses the x-axis; the turning point is the minimum or maximum point.

Grade 5 — **Solve equations using a graph**
Draw a horizontal line such as $y = 3$ and read the x-coordinates where it meets the curve.

Grade 6 — **Sketch and interpret exponential graphs**
$y = k^x$ with $k > 1$ passes through (0, 1), rises more and more steeply and never touches the x-axis.

Grade 7 — **Sketch and use trigonometric graphs**
Know $y = \sin x$ and $y = \cos x$ (repeat every 360°) and $y = \tan x$ (repeats every 180°), and use symmetry to find angles.

Grade 8 — **Solve equations by drawing a suitable line**
e.g. use the graph of $y = x^2 - 3x$ and the line $y = x + 2$ to solve $x^2 - 4x - 2 = 0$

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