Gradients and areas under graphs

A15, R14, R15 · Algebra · revision notes

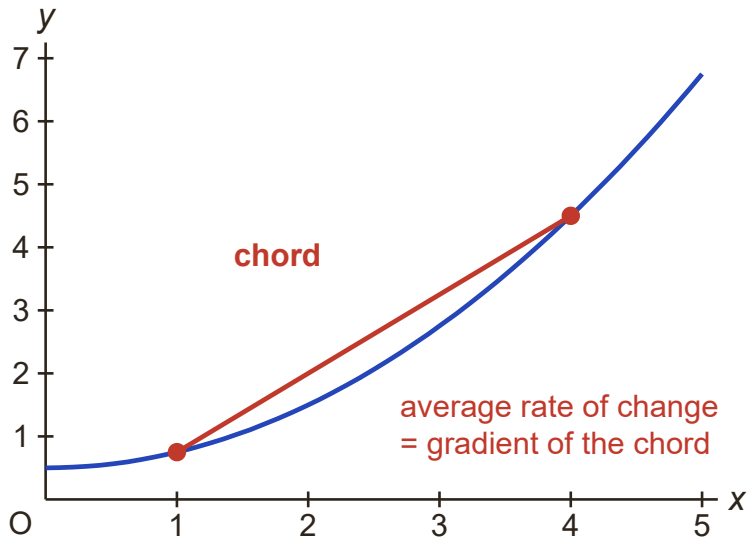
Higher only. Interpreting the gradient of a straight line as a rate of change, estimating the gradient of a curve by drawing a tangent, and finding or estimating the area under a graph. Most questions use distance–time, velocity–time or money graphs.

Key facts

- Gradient $= \dfrac{\text{change in } y}{\text{change in } x}$
- Velocity–time: gradient = acceleration; area = distance
- Chord between two points: average rate of change
- Curve bending upwards: trapezia overestimate. Bending downwards: underestimate
- Distance–time: gradient = speed
- Tangent at a point: instantaneous rate of change
- Area of a trapezium $= \dfrac{1}{2}(a+b)h$

Gradient as a rate of change

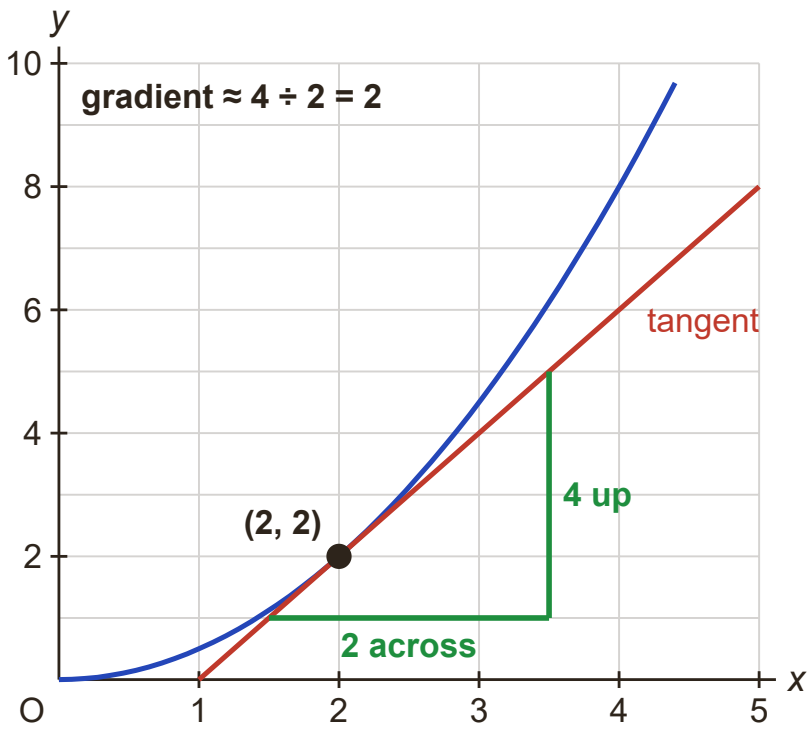
- The gradient of a straight-line graph is the rate of change of y with respect to x. Its units are y units per x unit.
- Distance–time graph: gradient = speed. Velocity–time graph: gradient = acceleration.
- A negative gradient on a velocity–time graph means deceleration (slowing down).
- The average rate of change between two points on a curve is the gradient of the chord (the straight line joining them).



The chord joins two points on the curve.

The gradient of a curve

- The gradient at a point on a curve is the instantaneous rate of change. Estimate it by drawing a **tangent** at that point.
- Place your ruler so that it touches the curve at the point without cutting through it there, and draw a long line.
- Choose two points on the tangent far apart, then gradient = change in y ÷ change in x.

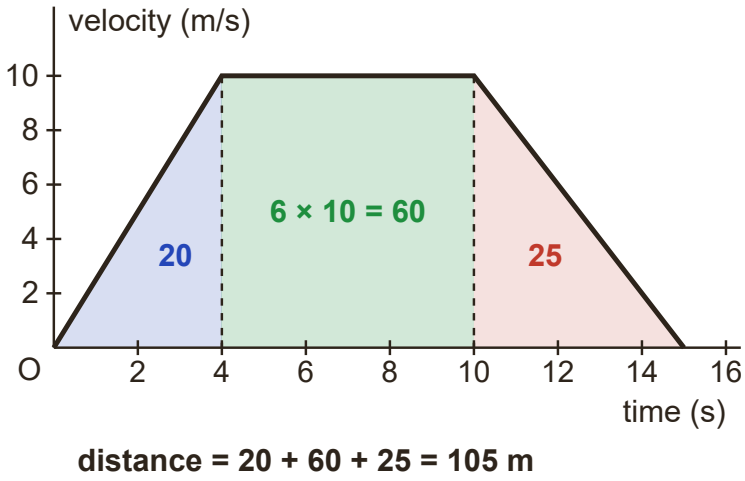


Make the triangle on the tangent as big as you can for accuracy.

- The answer is an estimate, so a range of answers is accepted.

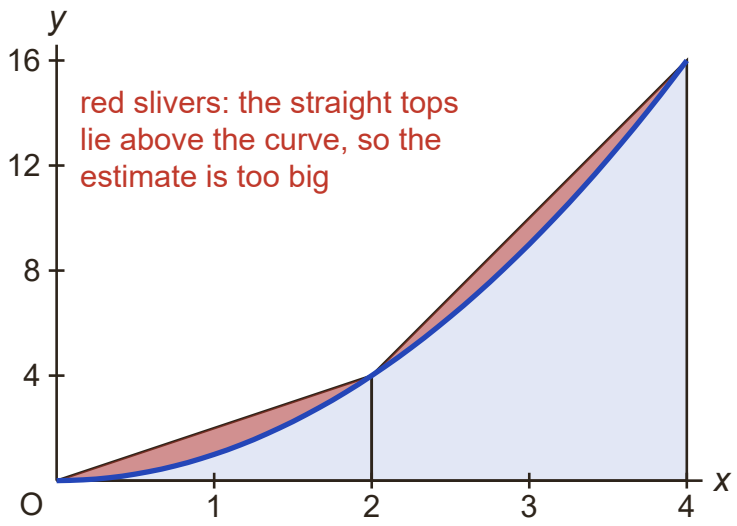
The area under a graph

- For a velocity–time graph, the area under the graph is the distance travelled.
- Straight-line graphs: split the area into rectangles, triangles and trapezia. Area of a trapezium $= \frac{1}{2}(a+b)h$



Area under a velocity–time graph = distance: split it into simple shapes.

- Curves: split into strips of equal width, join the tops with straight lines, and add the areas of the trapezia.
- If the curve bends upwards (like $y = x^2$), the straight tops lie above it, so the estimate is too big. If it bends downwards, the estimate is too small.



Curve bending upwards: the trapezia overestimate the area.

- Other contexts: the area under a rate graph gives a total, e.g. litres from a graph of litres per minute.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, C = a communication mark.

Acceleration and distance from a velocity–time graph 5 marks **Grade 6**

1. Acceleration: gradient of the line, change in velocity ÷ time.
2. Distance: split the area into triangles, rectangles and trapezia.
3. Add the areas and give units.

Example. A car starts from rest and accelerates steadily to 18 m/s in 12 seconds. It then travels at 18 m/s for 20 seconds, then decelerates steadily to rest in 6 seconds.
(a) Work out the acceleration in the first 12 seconds.
(b) Work out the total distance travelled.

Model answer.

(a) $\frac{18}{12}$ M1, = 1.5 m/s^2 A1

(b) $\frac{1}{2} \times 12 \times 18 + 20 \times 18 + \frac{1}{2} \times 6 \times 18$ M1 M1

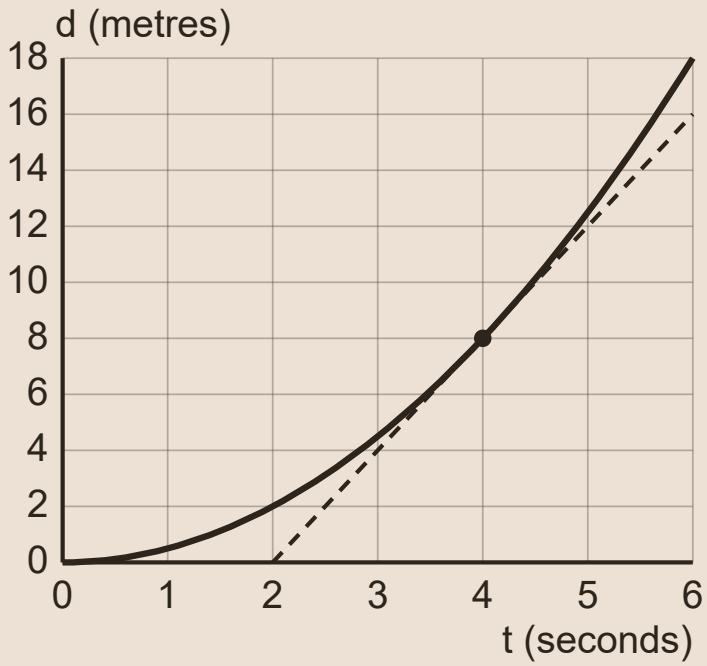
$= 108 + 360 + 54 = 522$ m A1

Estimate the gradient at a point

3 marks | Grade 7

1. Draw a tangent to the curve at the point.
2. Pick two points on the tangent far apart.
3. Work out change in y ÷ change in x and give units.

Example. The graph shows the distance, d metres, travelled by a ball after t seconds. A tangent to the curve (dashed) has been drawn at the point where $t = 4$.



Use the tangent to estimate the speed of the ball when $t = 4$

Model answer.

Two points on the tangent, e.g. (2, 0) and (6, 16) M1

$\frac{16 - 0}{6 - 2}$ M1

= 4 m/s A1

(a range of answers is accepted)

Estimate the area under a curve and interpret it

4 marks | Grade 7

1. Split into the stated number of equal strips.
2. Work out the area of each trapezium and add them.
3. Say what the area represents and whether it is an overestimate or underestimate.

Example. The table gives a cyclist's velocity, v m/s, at time t seconds.

t (s)	0	5	10	15	20
v (m/s)	0	6	9	10	10.5

The velocity–time graph is a smooth curve through these points that gets less steep as t increases. Use 4 strips of equal width to estimate the distance travelled in the first 20 seconds. Is your estimate an overestimate or an underestimate? Give a reason.

Model answer.

$$\frac{1}{2}(0+6)\times 5+\frac{1}{2}(6+9)\times 5$$
$$+\frac{1}{2}(9+10)\times 5+\frac{1}{2}(10+10.5)\times 5 \qquad \text{M1 M1}$$
$$=15+37.5+47.5+51.25$$
$$=151.25 \text{ m} \qquad \text{A1}$$

Underestimate: the curve bends downwards, so the straight tops of the trapezia lie below it C1

Average rate of change

2 marks | Grade 7

1. Read the values at the two times.
2. Work out the gradient of the chord: change in value ÷ change in time, with units.

Example. The value of an investment is £2000 at t = 0 years and £2550 at t = 5 years. Work out the average rate of increase in its value over the 5 years.

Model answer.

$$\frac{2550-2000}{5-0} \qquad \text{M1}$$

£110 per year A1

Don't lose marks

- Drawing a chord that cuts the curve instead of a tangent when asked for the gradient at a point.
- Misreading the scales, e.g. assuming each small square is 1 unit.
- Using strips of different widths, or the wrong number of strips, in an area estimate.

- Leaving out units, or giving a speed when the question asks for an acceleration.

More tips

Memory tricks

- Units check: gradient units are y units per x unit; area units are y units × x units (m/s × s = m).
- Make the triangle under your tangent as large as possible to reduce reading errors.
- Trapezia and curves: bends up means over, bends down means under.

Exam technique

- Draw the tangent or the strips on the graph: they are part of your method.
- When asked to 'interpret', say what the value represents, with units.
- When asked if an estimate is too big or too small, give a reason based on the shape of the curve.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 5 — **Interpret a straight-line gradient as a rate**
The gradient shows how fast y changes as x increases, e.g. litres per minute.

Grade 6 — **Find acceleration from a velocity–time graph**
Acceleration is the gradient; a negative gradient is a deceleration.

Grade 6 — **Find distance from a velocity–time graph**
Split the area under the graph into triangles, rectangles and trapezia and add them.

Grade 7 — **Estimate a gradient by drawing a tangent**
Draw a tangent at the point, make a large right-angled triangle and work out rise ÷ run.

Grade 7 — **Find an average rate of change**
Work out the gradient of the chord joining two points on the curve.

Grade 7 — **Estimate the area under a curve**
Split it into strips of equal width and add the areas of the trapezia.

Grade 8 — **Tell whether trapezia overestimate or underestimate**
Trapezia overestimate if the curve bends upwards, and underestimate if it bends downwards.

Grade 8 — **Interpret gradients and areas in context**
Say what the value means, with units, e.g. an area under a velocity–time graph is a distance in metres.

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