Composite and inverse functions

A7 · Algebra · revision notes

Higher only. Using function notation f(x), working out composite functions such as fg(x), and finding inverse functions $f^{-1}(x)$. Questions are usually 2 to 4 marks and link closely to substitution, rearranging and solving equations.

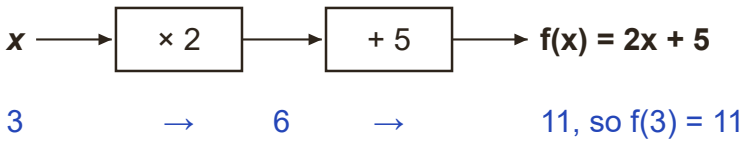
Key facts

- $f(a)$: replace every x with a
- $fg(x)$ is usually not the same as $gf(x)$
- $ff^{-1}(x) = x$
- $fg(x) = f(g(x))$: g first, then f
- Inverse: write $y = f(x)$, make x the subject, then write it in terms of x
- $f^{-1}(x) \neq \frac{1}{f(x)}$

Notes

Function notation

- $f(x) = 2x + 5$ means 'the function f doubles x and adds 5'. x is the input and f(x) is the output.

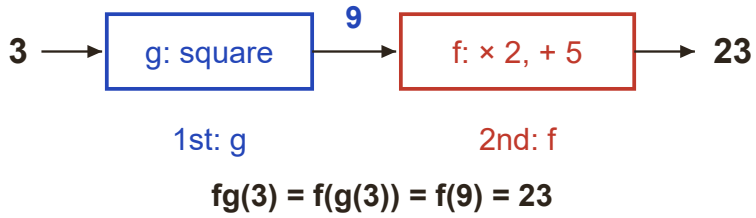


A function is a machine: input x, output f(x).

- $f(3)$ means substitute $x = 3$: $f(3) = 2 \times 3 + 5 = 11$
- $f(a + 1)$ means replace every x with $(a + 1)$: $2(a + 1) + 5 = 2a + 7$
- 'Solve $f(x) = 9$' means solve $2x + 5 = 9$, so $x = 2$

Composite functions

- $fg(x)$ means apply g first, then apply f to the result: $fg(x) = f(g(x))$. Work from the inside out.
- With $f(x) = 2x + 5$ and $g(x) = x^2$: $fg(x) = 2x^2 + 5$, but $gf(x) = (2x + 5)^2$. The order matters.
- For a number, e.g. $fg(3)$: $g(3) = 9$, then $f(9) = 23$



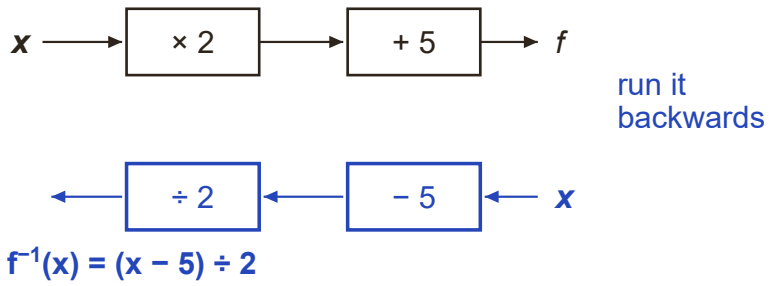
In $fg(x)$, the function next to x (g) acts first.

- $ff(x)$ means apply f twice:

$$f(2x + 5) = 2(2x + 5) + 5$$
$$= 4x + 15$$

Inverse functions

- $f^{-1}(x)$ undoes f: if $f(3) = 11$, then $f^{-1}(11) = 3$
- Method: write $y = f(x)$, rearrange to make x the subject, then write the answer as $f^{-1}(x) = \ldots$ using x.
- $y = 2x + 5$ gives $x = \frac{y - 5}{2}$, so $f^{-1}(x) = \frac{x - 5}{2}$



The inverse undoes each step in reverse order.

- $f^{-1}(x)$ does not mean $\frac{1}{f(x)}$
- $ff^{-1}(x) = f^{-1}f(x) = x$: a function and its inverse cancel each other out. Use this to check an inverse. Grade 8+ only

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Evaluate a function and solve f(x) = k

3 marks | Grade 6

1. For f(number), substitute the number for x.
2. For f(x) = k, write the equation and solve it.

Example. $f(x) = 4x - 3$
(a) Find $f(-2)$.
(b) Solve $f(x) = 19$

Model answer.

(a) $4 \times (-2) - 3 = -11$ B1

(b) $4x - 3 = 19$ M1

$x = 5.5$ A1

Find a composite function

4 marks | Grade 7

1. Work out the inner function first.
2. Substitute the result into the outer function, using brackets.
3. Expand and simplify if asked.

Example. $f(x) = 3x + 1$ and $g(x) = x^2 - 2$.
(a) Find $fg(4)$.
(b) Find $gf(x)$. Simplify your answer.

Model answer.

(a) $g(4) = 14$ M1, $f(14) = 43$ A1

(b) $(3x + 1)^2 - 2$ M1

$= 9x^2 + 6x - 1$ A1

Find an inverse function

2 marks | Grade 7

1. Write $y =$ the function.
2. Rearrange to make x the subject.
3. Write $f^{-1}(x) = \ldots$ with x in place of y.

Example. $h(x) = \dfrac{2x - 7}{5}$. Find $h^{-1}(x)$

Model answer.

$y = \dfrac{2x - 7}{5}$, so $5y + 7 = 2x$ M1

$h^{-1}(x) = \dfrac{5x + 7}{2}$ A1

Solve an equation involving a composite function

3 marks | Grade 8

1. Write the composite function as an expression in x.
2. Set it equal to the value and solve (a square gives ±).
3. Give all solutions.

Example. $f(x) = 2x - 1$ and $g(x) = x^2$. Solve $gf(x) = 25$

Model answer.

$gf(x) = (2x - 1)^2$ M1

$(2x - 1)^2 = 25$, so $2x - 1 = \pm 5$ M1

$x = 3$ or $x = -2$ A1

Find the inverse of a fractional function

4 marks | Grade 9

1. Write $y =$ the function and multiply both sides by the denominator.
2. Expand, then collect the x terms on one side.
3. Factorise out x and divide.

Example. $f(x) = \dfrac{3x+2}{x-4}$, $x \neq 4$. Find $f^{-1}(x)$

Model answer.

$y(x-4) = 3x+2$ M1

$xy - 3x = 4y + 2$ M1

$x(y-3) = 4y+2$ M1

$f^{-1}(x) = \dfrac{4x+2}{x-3}$ A1

Don't lose marks

- Doing composite functions in the wrong order: fg(x) means g first.
- Multiplying the functions: fg(x) is not $f(x) \times g(x)$
- Leaving an inverse in terms of y, or writing $f^{-1}(x) = \dfrac{1}{f(x)}$
- Forgetting brackets when substituting an expression: with $g(x) = x^2 - 2$, $g(x+1) = (x+1)^2 - 2$

More tips

Memory tricks

- Read fg(x) as 'f of g of x': the function next to x acts first.
- Check an inverse with a number: work out f(2), then put the answer into f^{-1}; you should get 2.
- For simple functions, find the inverse with a function machine: reverse the order and use inverse operations.

Exam technique

- Write the middle step (e.g. $g(4) = 14$) so the method mark can be seen.
- Give an inverse as $f^{-1}(x) = \ldots$ with x as the variable.
- In 'show that' questions, expand and simplify fully and end with the given expression.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 6 **Evaluate a function for a given input**

Substitute the value for x, e.g. $f(x) = 3x^2 - 1$ gives $f(-2) = 11$

Grade 6

Solve an equation such as f(x) = k

Set the function equal to the value and solve, e.g. $2x + 7 = 15$ gives $x = 4$

Grade 7

Find a composite function

fg(x) means apply g first, then f: substitute g(x) into f.

Grade 7

Find an inverse function

Write $y = f(x)$, make x the subject, then write $f^{-1}(x)$ in terms of x.

Grade 8

Solve equations involving composite functions

e.g. form and solve an equation from $gf(x) = 25$

Grade 9

Find inverses of fractional functions

When x appears twice after rearranging, collect the x terms and factorise, e.g. for $f(x) = \dfrac{3x + 2}{x - 4}$

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