Factorising

A4b · Algebra · revision notes

Factorising is the reverse of expanding: taking out common factors, factorising quadratics $x^2 + bx + c$ and differences of two squares (both tiers) and, on Higher, quadratics $ax^2 + bx + c$. You need it to solve quadratics, simplify algebraic fractions and write proofs.

Key facts

- Factorising is the reverse of expanding
- Common factor: HCF outside, divide each term by it to fill the bracket
- $x^2 + bx + c = (x + p)(x + q)$ where $pq = c$ and $p + q = b$
- $a^2 - b^2 = (a + b)(a - b)$
- Grade 6+ only $ax^2 + bx + c$: find two numbers with product ac and sum b, then split the middle term
- Check any factorisation by expanding it

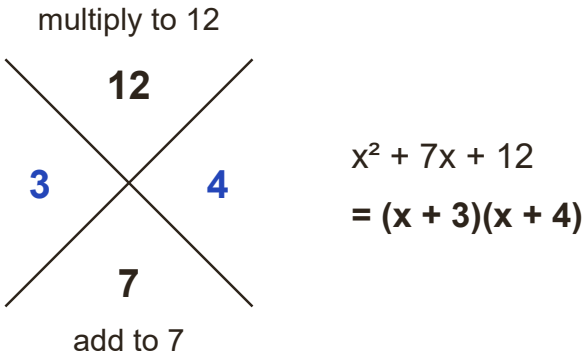
Notes

Common factors

- Find the HCF of the numbers and the lowest power of each letter that is in every term.
- Write the HCF outside the bracket and divide each term by it to find what goes inside: $12a^2 - 18ab = 6a(2a - 3b)$
- 'Factorise fully' means nothing else can be taken out: $3a(4a - 6b)$ is correct but not fully factorised.
- Check by expanding your answer.

Quadratics $x^2 + bx + c$

- Find two numbers that **multiply** to give c and **add** to give b.
- $x^2 + 7x + 12$: $3 \times 4 = 12$ and $3 + 4 = 7$, so it is $(x + 3)(x + 4)$

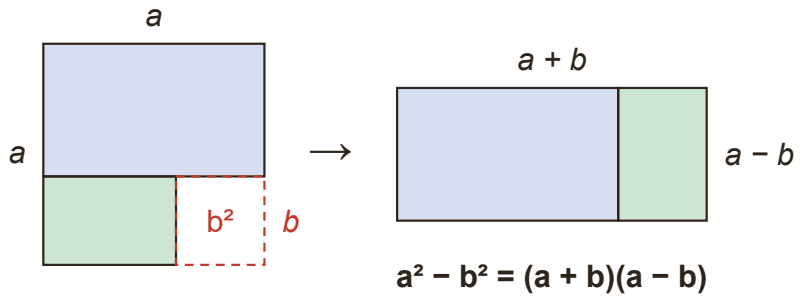


Find the pair that multiplies to c (top) and adds to b (bottom).

- If c is negative, the numbers have different signs: $x^2 - 2x - 24 = (x - 6)(x + 4)$
- If c is positive and b is negative, both numbers are negative: $x^2 - 9x + 20 = (x - 4)(x - 5)$

Difference of two squares

- $a^2 - b^2 = (a + b)(a - b)$. Look for two square terms with a minus sign between them.



Cut b^2 out of a^2 and move the green piece: the rectangle is $(a + b)$ by $(a - b)$

- $x^2 - 36 = (x + 6)(x - 6)$ and $25y^2 - 4 = (5y + 2)(5y - 2)$
- A sum of two squares, such as $x^2 + 36$, does not factorise.

Quadratics $ax^2 + bx + c$ Grade 6+ only

- Work out $a \times c$. Find two numbers that multiply to ac and add to b.
- Split the middle term using these two numbers, then factorise in pairs.
- $2x^2 + 7x + 3$: $ac = 6$, and $6 \times 1 = 6$, $6 + 1 = 7$. So:

$$\begin{aligned} &2x^2 + 6x + x + 3 \\ &= 2x(x + 3) + 1(x + 3) \\ &= (2x + 1)(x + 3) \end{aligned}$$

- Look for a common factor first:

$$\begin{aligned} 4x^2 - 36 &= 4(x^2 - 9) \\ &= 4(x + 3)(x - 3) \end{aligned}$$

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Factorise fully (common factor) 2 marks **Grade 5**

1. Find the HCF of the numbers.
2. Find the lowest power of each letter that appears in every term.
3. Divide each term by the HCF to fill the bracket, then check nothing else divides both terms.

Example. Factorise fully $10p^2q + 15pq^2$

Model answer. $5pq(2p + 3q)$ B2 (B1 for a correct but incomplete factorisation, e.g. $5p(2pq + 3q^2)$)

Factorise a quadratic $x^2 + bx + c$ 2 marks **Grade 5**

1. List factor pairs of c, with signs.
2. Choose the pair that adds to b.
3. Write the two brackets and expand to check.

Example. Factorise $x^2 - 3x - 28$

Model answer.

$-7 \times 4 = -28$ and $-7 + 4 = -3$

$(x - 7)(x + 4)$ B2

(B1 for $(x \pm 7)(x \pm 4)$)

Difference of two squares, then 'hence' 3 marks **Grade 5**

1. Write each term as a square: $x^2 - 64 = x^2 - 8^2$
2. Use $(a + b)(a - b)$
3. For 'hence', use the same pattern with the numbers given.

Example. (a) Factorise $x^2 - 64$.
(b) Hence, without a calculator, work out $58^2 - 42^2$

Model answer.

(a) $(x + 8)(x - 8)$ B1

(b) $(58 + 42)(58 - 42)$ M1

$= 100 \times 16 = 1600$ A1

Factorise $ax^2 + bx + c$ 2 marks **Grade 6**

1. Multiply a by c.
2. Find two numbers that multiply to ac and add to b, and split the middle term.
3. Factorise each pair; the brackets should match.

Example. Factorise $3x^2 - 11x - 4$

Model answer.

$ac = -12$; $-12 \times 1 = -12$ and $-12 + 1 = -11$

$3x^2 - 12x + x - 4$

$= 3x(x - 4) + 1(x - 4)$ M1

$(3x + 1)(x - 4)$ A1

Don't lose marks

- Not factorising fully, e.g. leaving a common factor of 2 inside the bracket.
- Getting the signs the wrong way round, e.g. $(x + 7)(x - 4)$ for $x^2 - 3x - 28$ (this expands to $x^2 + 3x - 28$).
- Trying to factorise a sum of two squares such as $x^2 + 25$
- Solving instead of factorising: the answer to 'factorise' is brackets, not values of x.

More tips

Memory tricks

- Signs: if c is positive, both numbers have the same sign as b; if c is negative, one is positive and one is negative, and the one with the larger size takes the sign of b.
- Learn the squares 1, 4, 9, …, 144 so you spot differences of two squares quickly.
- Expanding your answer to check takes a few seconds and catches sign errors.
- If a question says 'fully', expect more than one step.

Exam technique

- 'Factorise' wants brackets; 'solve' wants values of x. Read the command word.
- If part (a) says 'factorise' and part (b) says 'hence solve', use your brackets from part (a).
- Write down the pair of numbers you found as working.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 4 — **Take out a single common factor**
Find a number or letter that divides every term, e.g. $6x + 15 = 3(2x + 5)$

Grade 5 — **Factorise fully using the HCF**
Take out the highest common factor of the numbers and letters, e.g. $8x^2y - 12xy = 4xy(2x - 3)$

Grade 5 — **Factorise quadratics with x² coefficient 1**
Find two numbers that multiply to c and add to b, e.g. $x^2 + 2x - 15 = (x + 5)(x - 3)$

Grade 5 — **Factorise a difference of two squares**
Use $a^2 - b^2 = (a + b)(a - b)$, e.g. $x^2 - 49 = (x + 7)(x - 7)$

Grade 6 — **Factorise quadratics with x² coefficient above 1**
Split the middle term using two numbers with product ac, e.g. $3x^2 + 10x - 8 = (3x - 2)(x + 4)$

Grade 6 — **Factorise fully in two stages**
Take out a common factor, then factorise the bracket, e.g. $3x^2 - 12 = 3(x + 2)(x - 2)$

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