Expanding brackets

A4a · Algebra · revision notes

Multiplying out brackets: a single term over a bracket and the product of two binomials (both tiers), and three binomials (Higher). It appears as 1 to 3 mark questions and inside equations, proofs and area problems.

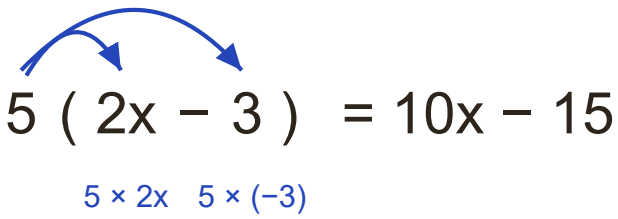
Key facts

- $a(b+c) = ab + ac$
- $(a+b)(c+d) = ac + ad + bc + bd$
- $(x+a)^2 = x^2 + 2ax + a^2$
- $(x-a)^2 = x^2 - 2ax + a^2$
- $(x+a)(x-a) = x^2 - a^2$
- $(x+a)(x+b) = x^2 + (a+b)x + ab$
- Grade 7+ only Three brackets: expand two, then multiply the result by the third

Notes

Single brackets

- Multiply the term outside by **every** term inside: $5(2x-3) = 10x - 15$



The number outside multiplies every term inside, signs included.

- A negative outside changes the sign of every term inside: $-3(x-4) = -3x + 12$
- Letters outside multiply too: $2y(y+5) = 2y^2 + 10y$
- 'Expand and simplify' means multiply out, then collect like terms:

$$\begin{aligned} &4(x+3) - 2(x-1) \\ &= 4x + 12 - 2x + 2 \\ &= 2x + 14 \end{aligned}$$

Two brackets

- Every term in the first bracket multiplies every term in the second, so two binomials give 4 products.
- Use a grid, or FOIL (First, Outer, Inner, Last):

$$\begin{aligned}(x+3)(x-7) &= x^2 - 7x + 3x - 21 \\ &= x^2 - 4x - 21\end{aligned}$$

	x	**−7**
x	x^2	$-7x$
+3	$3x$	-21

$x^2 - 7x + 3x - 21$

$= x^2 - 4x - 21$

Grid method: multiply each row by each column, then collect the x terms (blue).

- For a squared bracket, write the bracket twice:

$$\begin{aligned}(x+5)^2 &= (x+5)(x+5) \\ &= x^2 + 10x + 25\end{aligned}$$ never $x^2 + 25$

- With coefficients:

$$\begin{aligned}&(2x-3)(x+4) \\ &= 2x^2 + 8x - 3x - 12 \\ &= 2x^2 + 5x - 12\end{aligned}$$

- Difference of two squares: $(a+b)(a-b) = a^2 - b^2$, because the middle terms cancel.

Three brackets Grade 7+ only

- Expand any two of the brackets and simplify.
- Multiply each term of that answer by each term of the third bracket (6 products for three binomials), then collect like terms.
- Example: $(x+2)(x-1) = x^2 + x - 2$, so multiply this by $(x+3)$:

$$
\begin{aligned}
&(x^2 + x - 2)(x + 3) \\
&= x^3 + 3x^2 + x^2 + 3x - 2x - 6 \\
&= x^3 + 4x^2 + x - 6
\end{aligned}
$$

	x^2	$+x$	-2
x	x^3	x^2	$-2x$
$+3$	$3x^2$	$3x$	-6

$$= x^3 + 4x^2 + x - 6$$

Six products; collect the like terms (same colour).

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Expand and simplify two single brackets

2 marks | Grade 4

1. Multiply out each bracket, taking care with the sign in front of the second one.
2. Collect the letter terms and the number terms.

Example. Expand and simplify $3(2y - 5) - 4(y - 2)$

Model answer.

$6y - 15 - 4y + 8$ M1

(at least 3 terms correct)

$2y - 7$ A1

Expand and simplify two binomials

2 marks | Grade 5

1. Write down all four products (grid or FOIL).
2. Collect the two x terms.

Example. Expand and simplify $(x-6)(x+2)$

Model answer.

$x^2+2x-6x-12$ M1

(at least 3 of the 4 terms correct)

$x^2-4x-12$ A1

Show that an expression can be written in a given form

3 marks | Grade 6

1. Expand each bracket separately, writing squared brackets out twice.
2. Put a subtracted expansion in brackets before removing them.
3. Simplify and make the last line match the given expression exactly.

Example. Show that $(2x+1)^2-(x-3)^2$ can be written as $3x^2+10x-8$

Model answer.

$(2x+1)^2=4x^2+4x+1$ M1

$(x-3)^2=x^2-6x+9$ M1

$4x^2+4x+1-(x^2-6x+9)$

$=3x^2+10x-8$ A1

Expand three binomials

3 marks | Grade 7

1. Expand two of the brackets and simplify.
2. Multiply each term of the result by each term of the third bracket.
3. Collect like terms, ordering by powers of x.

Example. Expand and simplify $(x+4)(x-2)(2x-1)$

Model answer.

$(x+4)(x-2)=x^2+2x-8$ M1

$(x^2+2x-8)(2x-1)$

$=2x^3-x^2+4x^2-2x-16x+8$ M1

$=2x^3+3x^2-18x+8$ A1

Don’t lose marks

- Writing $(x+3)^2 = x^2 + 9$: the middle term $6x$ is missing.
- Only multiplying the first term: $3(x+4) = 3x + 4$ is wrong; it is $3x + 12$
- Sign errors with a negative outside: $-2(x-5)$ is $-2x + 10$, not $-2x - 10$
- Stopping before collecting like terms when the question says 'simplify'.

More tips

Memory tricks

- Check by substituting $x = 1$ into the question and your answer: $(1-6)(1+2) = -15$ and $1 - 4 - 12 = -15$, so they match.
- For $(x+a)(x+b)$: the x term is $a + b$ and the number term is $a \times b$
- Squared bracket: square the first term, double the product of the two terms, square the last term.
- Count your products: two binomials give 4 products before simplifying.

Exam technique

- 'Expand' means multiply out; 'expand and simplify' also means collect like terms.
- Show all the products before simplifying so the method mark can be seen.
- In 'show that' questions, write the final line so it matches the given expression exactly.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 3

Expand a single bracket

Multiply every term inside by the term outside, e.g. $4(2x-3) = 8x - 12$

Grade 4

Expand and simplify two single brackets

Expand each bracket, then collect like terms, e.g. $3(x+2) - 2(x-5) = x + 16$

Grade 4

Expand with a letter outside the bracket

Multiply letters as well as numbers, e.g. $2a(3a+b) = 6a^2 + 2ab$

Grade 5

Expand the product of two binomials

Multiply each term in the first bracket by each term in the second, e.g. $(x+3)(x-7) = x^2 - 4x - 21$

Grade 5

Expand a squared bracket

Write it out twice:

$$\begin{aligned}(x-4)^2 &= (x-4)(x-4) \\ &= x^2 - 8x + 16\end{aligned}$$

Grade 7

Expand the product of three binomials

Expand two brackets first, then multiply by the third, e.g. $(x+1)(x+2)(x-3) = x^3 - 7x - 6$

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