Listing and the product rule

N5 · Number · revision notes

Listing all the possible combinations in a systematic way, and using the product rule to count them without listing. Expect 2 to 3 mark questions about meals, codes or arrangements; Higher papers add restrictions such as no repeats, pairs, or conditions on certain positions.

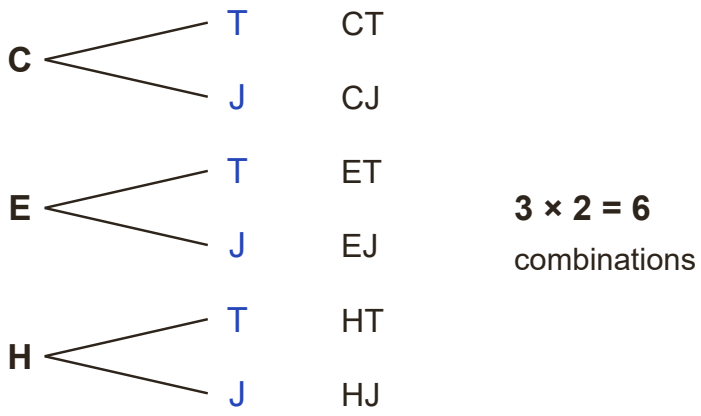
Key facts

- Product rule: m ways and n ways → $m \times n$ ways
- Grade 6+ only No repeats: 10 × 9 × 8 = 720
- Grade 7+ only Pairs from n (order doesn't matter): $\frac{n(n-1)}{2}$
- Repeats allowed: a 3-digit code from 0 to 9 has 10 × 10 × 10 = 1000 options
- Grade 6+ only Fill restricted positions first
- List systematically: fix the first item, and change the last item fastest

Notes

Systematic listing

- Work in a fixed order so you don't miss or repeat an outcome.
- For combinations from two lists, fix the first item and pair it with every item in the second list, then move on to the next first item.
- Example: 3 sandwiches (cheese C, egg E, ham H) and 2 drinks (tea T, juice J) give CT, CJ, ET, EJ, HT, HJ: 6 combinations.



Fix the first choice, pair it with every second choice, then move on.

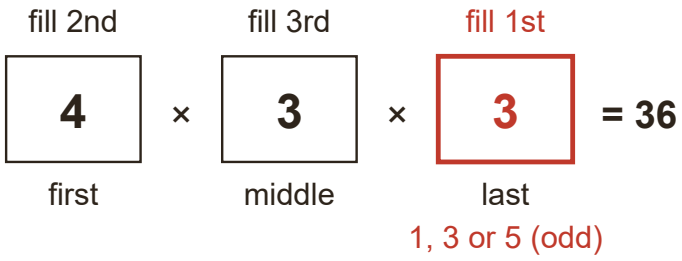
- If order doesn't matter (e.g. choosing two flavours), AB and BA are the same choice: list each pair only once.

The product rule

- If one choice can be made in m ways and a second, separate choice in n ways, both choices can be made in $m \times n$ ways.
- It works for any number of choices: 4 starters, 6 mains and 3 desserts give 4 × 6 × 3 = 72 different meals.
- A 4-digit PIN using the digits 0 to 9, with repeats allowed: 10 × 10 × 10 × 10 = 10,000 PINs.
- Use the product rule to check a list: 3 sandwiches × 2 drinks = 6 combinations.

Restrictions Grade 6+ only

- No repeats: each choice has one fewer option. Three different letters chosen in order from A, B, C, D, E: 5 × 4 × 3 = 60 ways.
- Fill a restricted position first. Odd 3-digit numbers from 1, 2, 3, 4, 5 with no repeats: last digit 3 ways (1, 3 or 5), first digit 4 ways, middle digit 3 ways: 3 × 4 × 3 = 36.



Fill the restricted box first, then the others with what is left.

- Two separate groups: choosing 1 boy from 12 and 1 girl from 15 gives 12 × 15 = 180 pairs.
- Pairs from one group, where order doesn't matter: n choices for the first person, $n - 1$ for the second, then halve because each pair was counted twice. 10 teams all playing each other once: $\frac{10 \times 9}{2} = 45$ matches. Grade 7+ only

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

List all the possible combinations 2 marks Grade 3

1. Use letters or short labels for each item.
2. Fix the first choice and go through every option for the second, then move on.
3. Check the number of items in your list with the product rule.

Example. Asha chooses one base and one topping for a pizza.
Bases: thin (T), deep (D). Toppings: ham (H), mushroom (M), pepper (P), olive (O).
List all the possible combinations she could choose.

Model answer. TH, TM, TP, TO, DH, DM, DP, DO (B2 for all 8 with no extras or repeats; B1 for at least 4 correct)

Use the product rule: how many different ways?

2 marks | Grade 4

1. Count the number of options for each choice.
2. Multiply them together (don't add).
3. Write the multiplication down, then the answer.

Example. A menu has 5 starters, 7 main courses and 4 desserts. Ben chooses one of each.
How many different meals could Ben choose?

Model answer.

5 × 7 × 4 (M1)
= 140 (A1)

Codes with restrictions

2 to 3 marks | Grade 6

1. Draw a dash for each position and write the number of options on each dash.
2. Reduce the options by 1 each time if repeats are not allowed.
3. Apply any other rule (e.g. 'cannot start with 0'), then multiply.

Example. A code is 2 different letters from A to Z, followed by 3 digits from 0 to 9. Digits can repeat, but the first digit cannot be 0.
How many different codes are possible?

Model answer.

Letters: 26 × 25; digits: 9 × 10 × 10 (M1 for either)
26 × 25 × 9 × 10 × 10 (M1)
= 585,000 (A1)

Counting pairs

3 marks | Grade 7

1. One from each of two groups: multiply the group sizes.
2. Two from the same group, order not mattering: $n \times (n-1)$, then divide by 2.
3. Read carefully: 'captain and vice-captain' is ordered, so don't halve.

Example. A club has 8 boys and 10 girls.
(a) One boy and one girl are chosen. How many different pairs are possible?
(b) Any two members of the club are chosen. How many different pairs are possible?

Model answer.

(a) 8 × 10 = 80 (B1)
(b) 18 × 17 = 306 (M1)
Each pair has been counted twice, so 306 ÷ 2 = 153 (A1)

How many numbers can be made? (conditions on positions)

3 marks | Grade 8

1. Fill the most restricted positions first.
2. If one restriction affects another, split into cases.
3. Multiply within each case, then add the cases.

Example. Four-digit numbers are made using the digits 1, 2, 3, 4, 5 and 6. Each digit can be used at most once.
How many of these numbers are even and between 3000 and 5000?

Model answer.

First digit 3: last digit 2, 4 or 6 (3 ways); middle two digits 4 × 3, so 3 × 4 × 3 = 36	(M1)
First digit 4: last digit 2 or 6 (2 ways), so 2 × 4 × 3 = 24	(M1)
36 + 24 = 60	(A1)

Don’t lose marks

- Adding instead of multiplying: 5 starters and 7 mains give 35 meals, not 12.
- Listing without a system, and missing or repeating combinations.
- Allowing repeats when the question says the letters or digits are different.
- Ignoring a condition such as 'the first digit cannot be 0' or 'the number must be even'.
- Counting the same pair twice (AB and BA) when the order doesn't matter.

More tips

Memory tricks

- Draw a dash for each position (_ _ _ _), write the number of options on each dash, then multiply.
- Check a list with the product rule: 2 bases × 4 toppings should give 8 combinations.
- 'Different' or 'no repeats' means the numbers go down by 1 each time.
- Work in a pattern: keep the first item fixed and change the last item fastest.

Exam technique

- When asked to list, set out the list clearly in order; one missing or repeated item usually costs a mark.
- Show the multiplication (e.g. 26 × 25 × 9 × 10 × 10), not just the answer, to earn the method mark.
- Underline words such as 'different', 'no repeats', 'even', 'greater than' and 'cannot start with'.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 2 — **List outcomes in a systematic order**
E.g. the two-digit numbers made from 3, 5 and 8 without repeats: 35, 38, 53, 58, 83, 85.

Grade 3 — **List combinations from two or three sets**

Fix one item and run through every option for the others, then check you have them all.

Grade 4

Use the product rule for two choices

If there are m ways to do one thing and n ways to do another, there are $m \times n$ ways to do both.

Grade 5

Use the product rule for several choices

E.g. 3 letters then 2 digits: 26 × 26 × 26 × 10 × 10 = 1,757,600 codes.

Grade 6

Count when repeats are not allowed

Each choice has one fewer option: 4-digit codes with different digits: 10 × 9 × 8 × 7 = 5040.

Grade 7

Count pairs when order doesn't matter

From n people, the number of pairs is $\frac{n(n-1)}{2}$, e.g. 8 people make 28 pairs.

Grade 8

Count with restrictions on positions

Fill the restricted places first and split into cases if needed, e.g. even numbers greater than 3000.

Free notes, Full packs, practice papers and mock exams at chhetriacademy.com. Written for this site; not taken from past papers. Stuck? 1-to-1 tutoring with a free 30-minute trial lesson.