Congruence and geometric proof

G5, G6 · Geometry and measures · revision notes

Congruent shapes are exactly the same shape and size. You need the four conditions for congruent triangles (SSS, SAS, ASA and RHS), and you use them, with angle facts and the properties of shapes, to write geometric proofs.

Key facts

- Congruent = same shape and same size
- SAS: two sides and the included angle
- RHS: right angle, hypotenuse and one other side
- Common reasons: given; common side; radii are equal; vertically opposite angles are equal; alternate angles are equal
- SSS: three pairs of equal sides
- ASA (or AAS): two angles and a corresponding side
- AAA shows similar only, not congruent

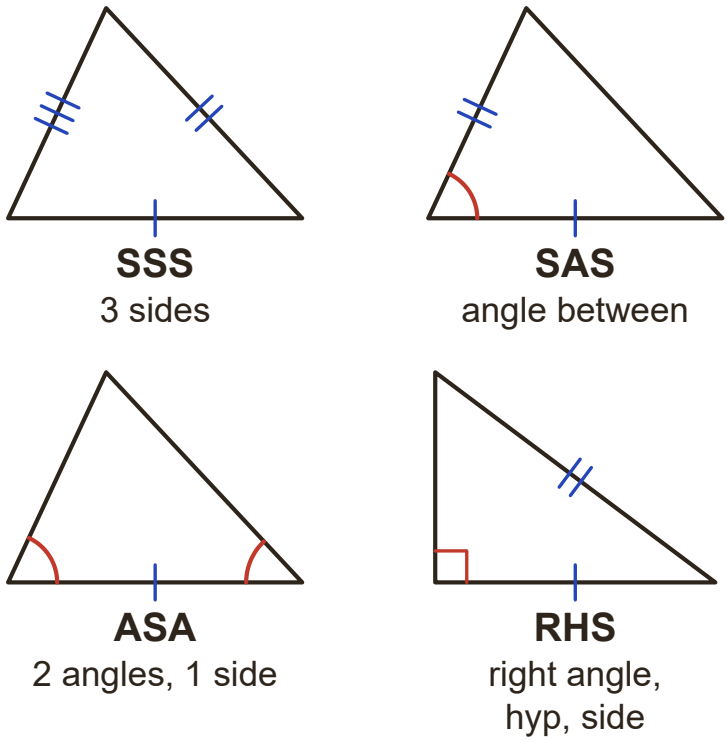
Notes

Congruent shapes

- Congruent shapes are identical: same shape and same size, so all corresponding sides and angles are equal.
- One shape may be a rotation, reflection or translation of the other.
- Similar shapes have equal angles but can be different sizes. Congruent shapes are similar with scale factor 1.

Conditions for congruent triangles

- **SSS**: all three pairs of sides are equal.
- **SAS**: two pairs of sides and the angle **between** them (the included angle) are equal.
- **ASA**: two pairs of angles and a pair of corresponding sides are equal (the side may be between the angles or not, sometimes written AAS).
- **RHS**: both have a right angle, equal hypotenuses and one other pair of equal sides.



The marks show which parts must be equal to the matching parts of the other triangle.

- Three equal angles (AAA) only shows the triangles are similar. Two sides and an angle that is not between them do not prove congruence.

Writing a congruence proof

- Name the triangles with corresponding vertices in the same order, e.g. triangle ABC and triangle CDA.
- Write three pairs of equal sides or angles, one per line, each with a reason: given, common side, radii are equal, vertically opposite angles are equal, alternate angles are equal, opposite sides of a parallelogram are equal.
- Finish with the conclusion and the condition, e.g. 'so triangle ABC is congruent to triangle CDA (SSS)'.
- Once triangles are congruent, all their corresponding sides and angles are equal, which you can use to prove further results.

Geometric proof

- Start from facts you know, give a reason for every step and end with exactly what you were asked to show.
- Use letters (e.g. let angle BAC = x) to prove a result in general; a numerical example is not a proof.
- Base angles of an isosceles triangle are equal: the line from the apex to the midpoint of the base makes two triangles congruent by SSS, so the base angles are equal.
- Pythagoras: four copies of a right-angled triangle (legs a and b, hypotenuse c) fit round a square of side c inside a square of side a + b.
- So $(a+b)^2 = 4 \times \frac{1}{2}ab + c^2$, which gives $a^2 + 2ab + b^2 = 2ab + c^2$, so $a^2 + b^2 = c^2$

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, C = a communication mark.

Decide whether triangles are congruent and give the condition 2 marks **Grade 5**

1. Match up the equal sides and angles in the two triangles.
2. Check whether an angle is between the two sides you are using.
3. Name the condition (SSS, SAS, ASA or RHS).

Example. Triangle PQR has PQ = 7 cm, angle PQR = 48° and QR = 10 cm. Triangle XYZ has XY = 10 cm, angle XYZ = 48° and YZ = 7 cm.
Are the two triangles congruent? Give a reason for your answer.

Model answer.

Yes (B1)

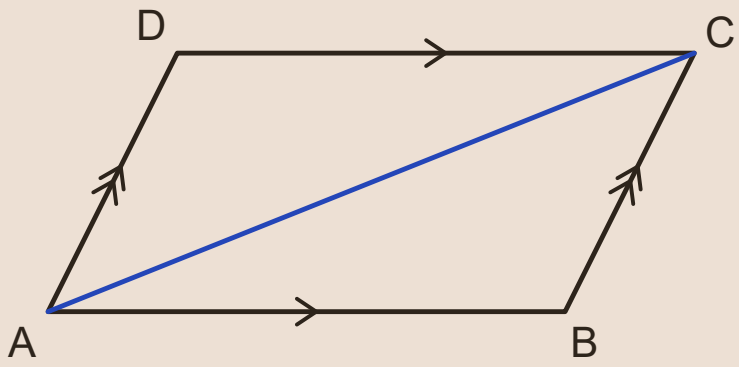
Both have sides of 7 cm and 10 cm with the 48° angle between them, so they are congruent by SAS (B1)

Prove that two triangles are congruent

3 marks **Grade 5**

1. Write the triangles with matching vertices in the same order.
2. List three pairs of equal sides or angles, each with a reason.
3. State the conclusion and the condition.

Example. ABCD is a parallelogram. The diagonal AC is drawn.



Prove that triangle ABC is congruent to triangle CDA.

Model answer.

AB = CD because opposite sides of a parallelogram are equal (M1)

BC = DA because opposite sides of a parallelogram are equal

AC = CA because it is a common side (M1)

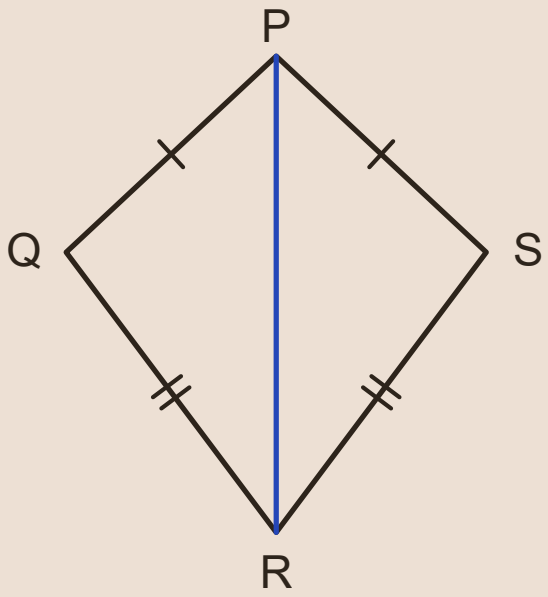
So triangle ABC is congruent to triangle CDA (SSS) (C1)

Use congruence to prove a further result

4 marks | Grade 7

1. Pick two triangles that contain the sides or angles you want to show are equal.
2. Prove the triangles are congruent, with reasons.
3. Conclude that the corresponding sides or angles are equal.

Example. PQRS is a kite with PQ = PS and RQ = RS. The diagonal PR is drawn.



Prove that angle PQR = angle PSR.

Model answer.

PQ = PS (given) and RQ = RS (given)	(M1)
PR is a common side	(M1)
So triangles PQR and PSR are congruent (SSS)	(A1)
So angle PQR = angle PSR, because corresponding angles of congruent triangles are equal	(C1)

Don’t lose marks

- Using AAA as a reason for congruence (it only shows similarity).
- Using SAS when the angle is not between the two sides.
- Stating facts without reasons: 'AB = CD' on its own does not score.
- Forgetting to write the condition (SSS, SAS, ASA or RHS) at the end.
- Using the result you are trying to prove as one of your facts.

More tips

Memory tricks

- There are only four tests: SSS, SAS, ASA, RHS. If yours is not one of these, it is not a valid reason.
- For SAS, point along side, angle, side in order: the angle must be sandwiched between the two sides.
- A side shared by both triangles is a free pair: write 'common side'.
- Writing the vertices in matching order makes it easy to read off corresponding sides.

Exam technique

- Set out a proof as three lines of 'fact because reason', then a conclusion with the condition; this earns the communication mark.
- Use only information that is given or that you have proved, not what the diagram looks like.
- If you are asked to prove two lengths or angles are equal, look for congruent triangles that contain them.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 3 — **Identify congruent shapes**
Congruent shapes are identical in shape and size, even if one is turned over or rotated.

Grade 5 — **Know the four congruence conditions**
SSS, SAS, ASA (or AAS) and RHS; three equal angles is not enough.

Grade 5 — **Choose the condition for two triangles**
Match the equal sides and angles, then name the condition that applies.

Grade 5 — **Prove that two triangles are congruent**
Give three pairs of equal sides or angles with reasons, then state the condition.

Grade 7 — **Use congruence to prove further results**
After proving triangles congruent, deduce that other sides or angles are equal.

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