Conditional probability

P9 · Probability · revision notes

A conditional probability is the probability of an event when you already know that another event has happened. It is Higher tier only (grades 6 to 9) and is tested using two-way tables, Venn diagrams, tree diagrams and expected frequencies, usually in questions worth 2 to 5 marks.

Key facts

- Conditional probability: the probability of an event given that another event has happened
- With frequencies: P(B given A) = number in both ÷ number in A
- Venn diagram: 'given A' means the denominator is everything in circle A
- Grade 9+ only Given the second event: P(first and second) ÷ P(second), where P(second) adds every path that ends in it
- P(B given A) = P(A and B) ÷ P(A)
- Two-way table: 'given that' means the denominator is that row or column total
- Tree diagram: the second-stage branches are conditional probabilities
- Grade 8+ only Independent events: P(A and B) = P(A) × P(B)

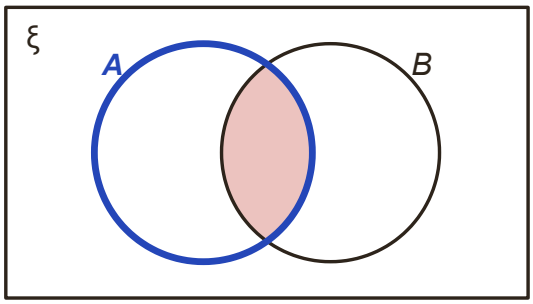
Notes

What conditional probability means Grade 6+ only

- A conditional probability is the probability of one event when you know that another has already happened.
- Look for the words 'given that', 'if' or 'of those who'.
- The extra information cuts down the possible outcomes: you only count the outcomes where the given event happened.
- P(B given A) = P(A and B) ÷ P(A). With frequencies, it is the number in both ÷ the number in A
- Some books write P(B given A) as $\mathrm{P}(B \mid A)$

Two-way tables, Venn diagrams and sample spaces Grade 6+ only

- Two-way table: 'given that' tells you which row or column to use. The denominator is that row or column total, not the grand total.
- Example: of 60 students, 25 are in Year 10 and 15 of those walk to school. P(walks given Year 10) = $\frac{15}{25} = \frac{3}{5}$, not $\frac{15}{60}$
- Venn diagram: 'given A' means the denominator is everything inside circle A



Given A: only look inside circle A.

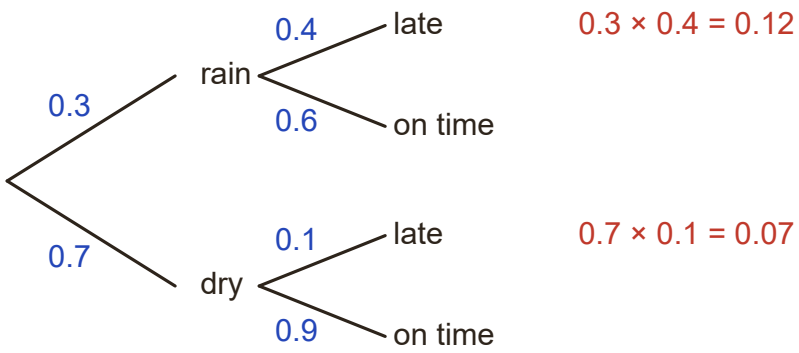
P(B given A) = overlap ÷ all of A

The whole of circle A becomes the new total.

- Sample space: given that the total on two fair dice is 10, the outcomes are (4, 6), (5, 5) and (6, 4). So P(at least one 6 given a total of 10) = $\frac{2}{3}$

Tree diagrams Grade 7+ only

- The second set of branches shows conditional probabilities: the probability of the second event given the first branch.
- Example: P(rain) = 0.3. If it rains, P(Ben is late) = 0.4; if it does not rain, P(Ben is late) = 0.1.
- P(Ben is late) = 0.3 × 0.4 + 0.7 × 0.1 = 0.12 + 0.07 = 0.19.

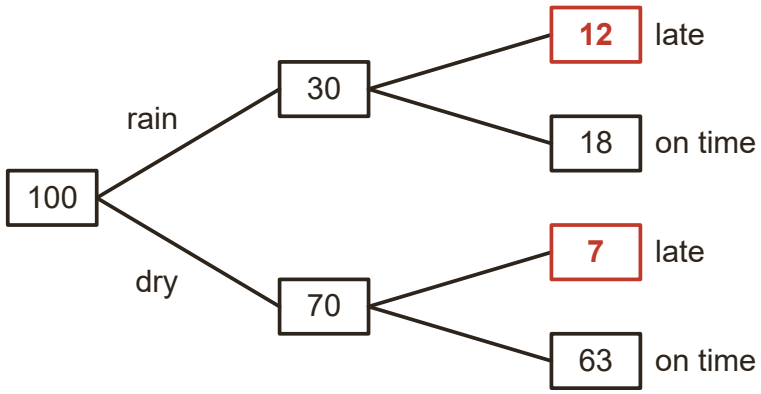


The second branches depend on the first: P(late) = 0.12 + 0.07 = 0.19.

- Given that Ben is late, P(it rained) = P(rain and late) ÷ P(late) = 0.12 ÷ 0.19 = $\frac{12}{19}$ Grade 9+ only

Expected frequencies Grade 7+ only

- Turning probabilities into numbers makes conditional questions easier. Choose a total such as 100 or 1000 and build a frequency tree or two-way table.
- Ben over 100 days: 30 rainy days, on 12 of which he is late; 70 dry days, on 7 of which he is late.



Ben is late on 12 + 7 = 19 of the 100 days.

- He is late on 12 + 7 = 19 days and 12 of those were rainy, so given that he is late, P(it rained) = $\frac{12}{19}$, the same answer as the tree method. Grade 9+ only

Independent events Grade 8+ only

- A and B are independent if P(A and B) = P(A) × P(B).
- Equivalently, P(A given B) = P(A): knowing that B happened does not change the chance of A
- To test, work out both sides and compare. If they are not equal, the events are not independent.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, C = a communication mark.

'Given that …' from a two-way table

2 marks per part | **Grade 6**

1. Complete the table first if you need to: every row and column must add up to its total.
2. Find the row or column named after 'given that': its total is your denominator.
3. Find the cell in that row or column that matches the event you want: this is your numerator.

Example. The table shows how 60 students travel to school.

	Walk	Bus	Car	Total
Year 10	15	6	4	25
Year 11	12	18	5	35

A student is chosen at random.
(a) Given that the student travels by bus, work out the probability that the student is in Year 11.
(2 marks)
(b) Given that the student is in Year 10, work out the probability that the student does not walk.
(2 marks)

Model answer.

(a) Students who travel by bus: 6 + 18 = 24 (M1)

P(Year 11 given bus) = $\frac{18}{24} = \frac{3}{4}$ (A1)

(b) Year 10 students who do not walk: 6 + 4 = 10 (M1)

P(does not walk given Year 10) = $\frac{10}{25} = \frac{2}{5}$ (A1)

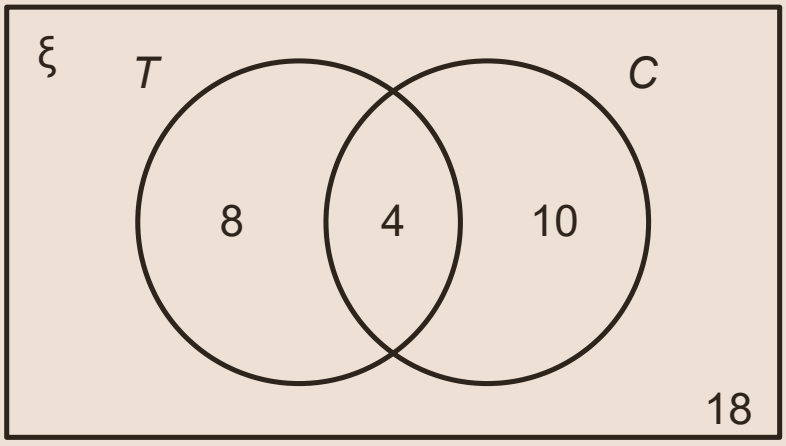
'Given that …' from a Venn diagram

3 marks | Grade 7

1. Complete the Venn diagram first, starting with the overlap.
2. Identify the given set and add up everything inside it: this is the denominator.
3. The numerator is the part of the given set that is also in the event you want.

Example. 40 people were asked whether they like tea and whether they like coffee. 12 like tea, 14 like coffee and 4 like both.
One of the 40 people is chosen at random. Given that this person likes at least one of the drinks, work out the probability that they like both.

Model answer.



Tea only: 12 − 4 = 8 and coffee only: 14 − 4 = 10 (M1)

People who like at least one drink: 8 + 4 + 10 = 22 (M1)

P(both given at least one) = $\frac{4}{22} = \frac{2}{11}$ (A1)

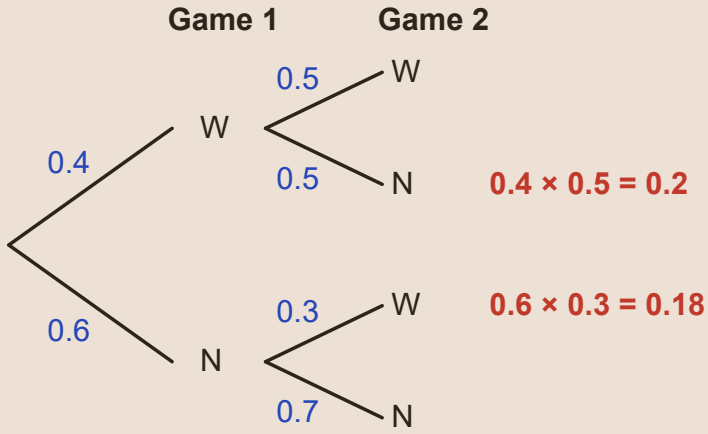
Tree diagram where the second event depends on the first

3 marks | Grade 7

1. Draw the tree: the second set of branches depends on the first, so write the probabilities for each branch separately.
2. Each pair of branches still adds up to 1.
3. Multiply along each path you need, then add the paths.

Example. Ali plays two games of chess. The probability that he wins the first game is 0.4. If he wins a game, the probability that he wins the next game is 0.5. If he does not win a game, the probability that he wins the next game is 0.3.
Work out the probability that Ali wins exactly one of the two games.

Model answer.



W = win, N = not win. Win then not win: 0.4 × 0.5 = 0.2 (M1 for one correct product)

Not win then win: 0.6 × 0.3 = 0.18

0.2 + 0.18 (M1 for adding both products) = 0.38 (A1)

Are the events independent?

3 marks | Grade 8

1. Work out P(A), P(B) and P(A and B) as fractions or decimals of the same total.
2. Multiply P(A) × P(B) and compare it with P(A and B). (Or compare P(A given B) with P(A).)
3. Write a conclusion: equal means independent, different means not independent.

Example. In a group of 50 students, 20 wear glasses and 30 are girls. 9 of the girls wear glasses.
Are the events 'wears glasses' and 'is a girl' independent? Show how you decide.

Model answer.

P(glasses) = $\frac{20}{50}$ = 0.4 and P(girl) = $\frac{30}{50}$ = 0.6, so P(glasses) × P(girl) = 0.24 (M1)

P(girl and glasses) = $\frac{9}{50}$ = 0.18 (M1)

0.18 ≠ 0.24, so the events are not independent (C1)

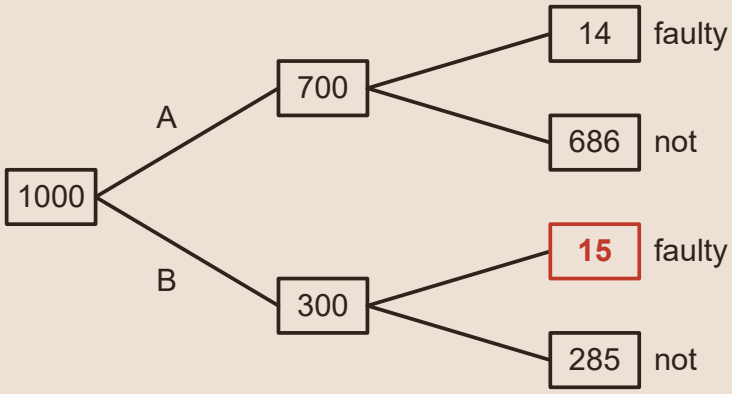
(Or: P(glasses given girl) = $\frac{9}{30}$ = 0.3, which is not equal to P(glasses) = 0.4.)

Given the outcome, find the probability of the cause 3 to 4 marks **Grade 9**

1. Imagine a convenient number of items, e.g. 1000, and split them using the first probabilities.
2. Split each group using the conditional probabilities.
3. Add the groups that match the given outcome: this is the denominator.
4. The numerator is the part of that total that also matches the event asked about.

Example. A factory has two machines. Machine A makes 70% of the bolts and machine B makes the rest. 2% of the bolts made by machine A are faulty and 5% of the bolts made by machine B are faulty. A bolt is chosen at random and is found to be faulty. Work out the probability that it was made by machine B.

Model answer.



Out of 1000 bolts, A makes 700 and B makes 300 (M1)

Faulty: 2% of 700 = 14 and 5% of 300 = 15, so 14 + 15 = 29 faulty bolts (M1)

P(machine B given faulty) = $\frac{15}{29}$ (A1)

Don’t lose marks

- Dividing by the grand total instead of the total of the given group.
- Mixing up P(A given B) with P(B given A): in the table example, P(Year 11 given bus) = $\frac{18}{24}$ but P(bus given Year 11) = $\frac{18}{35}$
- Copying the first-stage probabilities onto the second stage when the events are dependent.
- In a 'given the outcome' question, using only one path for the denominator instead of adding every path that ends in that outcome.
- Saying events are independent because they seem unrelated, instead of checking with numbers.

More tips

Memory tricks

- 'Given that' means shrink your total: the denominator is the given group only.
- Stuck with probabilities? Imagine 100 or 1000 people and turn every probability into a number.
- Check a tree: every pair of branches adds up to 1, even when the second-stage branches differ.
- For a 'given the outcome' question, the denominator is the total of every path that ends in that outcome.

Exam technique

- Underline the 'given that' part of the question and write down the group you are restricting to before you calculate.
- Write the fraction with its numerator and denominator (e.g. $\frac{18}{24}$) before simplifying, so the method mark is clear.
- For independence questions, show both calculations and finish with a clear conclusion in words.
- Keep answers as exact fractions: a result like $\frac{15}{29}$ does not give a neat decimal.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 6

Find 'given that' probabilities from two-way tables

Use only the row or column you are given, e.g. P(walks given Year 10) = Year 10 walkers ÷ all Year 10 students.

Grade 7

Find conditional probabilities from a Venn diagram

Divide the number in the part you need by the number in the whole of the given set, not by the grand total.

Grade 7

Use conditional probabilities on tree diagrams

The second set of branches gives the probability of the second event given what happened first, so the branches can differ.

Grade 7

Use expected frequencies to find conditional probabilities

Turn the probabilities into numbers out of a convenient total, such as 100 or 1000, then read off the fraction you need.

Grade 8

Test whether two events are independent

Compare P(A and B) with P(A) × P(B), or P(A given B) with P(A): if they are equal, the events are independent.

Grade 9

Find the first event given the second

Divide P(first and second) by the total P(second), e.g. P(rain given late) = P(rain and late) ÷ P(late).

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