Circle theorems

G10 · Geometry and measures · revision notes

Higher tier only. Circle theorems are facts about angles and lengths in circles. You use them with other angle facts to work out angles, giving a reason for each step, and you may be asked to prove a theorem or a related result.

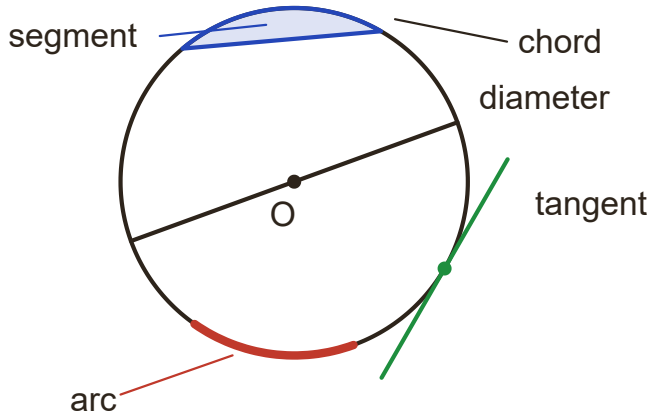
Key facts

- The angle at the centre is twice the angle at the circumference
- Angles in the same segment are equal
- A tangent is perpendicular to the radius at the point of contact
- The perpendicular from the centre to a chord bisects the chord
- Radii are equal, so a triangle with two radii as sides is isosceles
- The angle in a semicircle is 90°
- Opposite angles of a cyclic quadrilateral add up to 180°
- Tangents from an external point are equal in length
- Alternate segment theorem: angle between tangent and chord = angle in the alternate segment

Notes

Words you need

- **Chord**: a straight line joining two points on the circle. A **diameter** is a chord through the centre.
- **Tangent**: a straight line that touches the circle at exactly one point.
- **Arc**: part of the circumference. **Segment**: the region between a chord and an arc.
- A **cyclic quadrilateral** has all four vertices on the circle.



Chord, diameter, segment, arc and tangent.

- An angle is **subtended** by an arc or chord when its two arms go to the ends of that arc or chord.

The theorems

- The angle at the centre is **twice** the angle at the circumference subtended by the same arc.
- The angle in a **semicircle** is 90° (the angle at the circumference subtended by a diameter).
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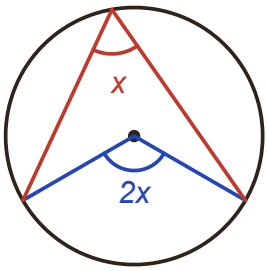
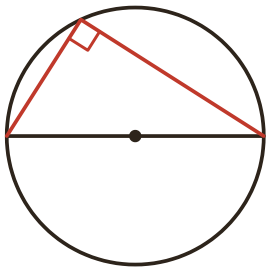


Angle at centre
= 2 × angle at circumference



Angle in a semicircle
= 90°

- Angles in the **same segment** are equal (angles at the circumference subtended by the same arc).
- Opposite angles of a **cyclic quadrilateral** add up to 180°.
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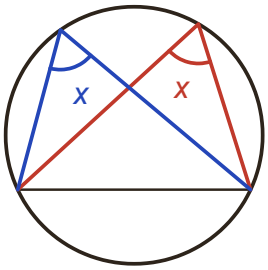
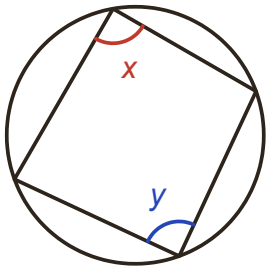


Angles in the same segment
are equal



Cyclic quadrilateral
x + y = 180°

- A tangent is **perpendicular** to the radius at the point of contact.
- The two tangents from a point outside a circle are **equal in length**.
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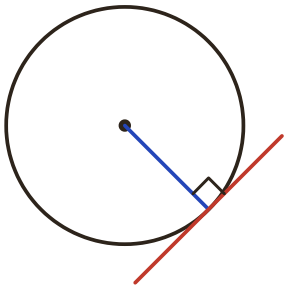
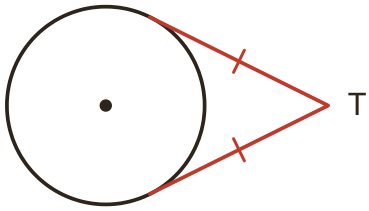
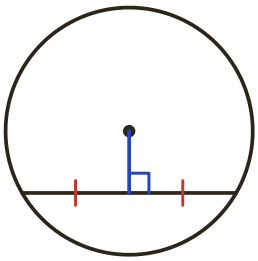


Tangent and radius
meet at 90°

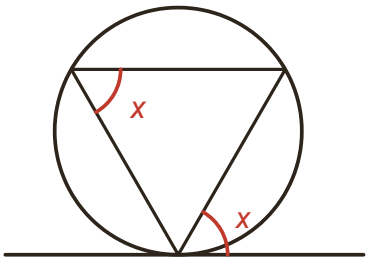


Tangents from a point
are equal

- The perpendicular from the centre to a chord **bisects** the chord.
- **Alternate segment theorem**: the angle between a tangent and a chord equals the angle subtended by the chord in the alternate segment.

- 

Perpendicular from centre bisects the chord



Alternate segment theorem

Finding angles

- Two radii make an isosceles triangle, so mark its equal base angles straight away.
- Look for the tangent–radius right angle and the angle in a semicircle first.
- Use ordinary angle facts as well: angles in a triangle add up to 180°, on a straight line 180°, in a quadrilateral 360°.

Proofs Grade 9+ only

- Give an unknown angle a letter (e.g. let angle BAD = x) and work in terms of it, with a reason for every step.
- Draw in radii to make isosceles triangles; use the base angles and the angle sum or exterior angle of a triangle.
- End with a clear statement of exactly what you were asked to prove.

How to answer each type of question

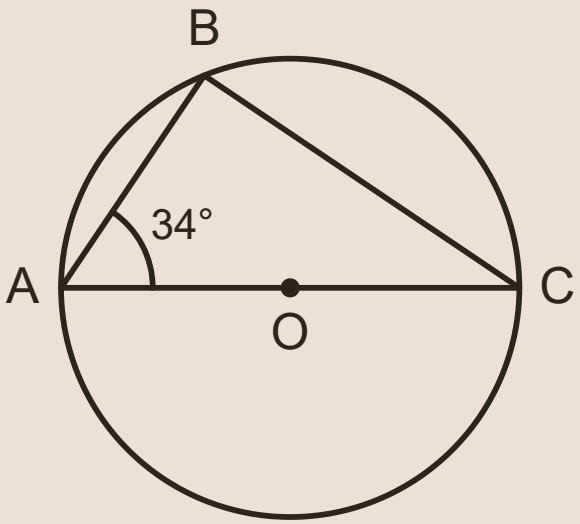
M = method mark, A = answer mark, B = a mark on its own, C = a communication mark.

Work out an angle and give reasons (one theorem)

3 marks | **Grade 6**

1. Spot which theorem the diagram uses (diameter, tangent, centre, cyclic quadrilateral).
2. Write the angle it gives you, with the theorem as the reason.
3. Finish with ordinary angle facts, again with reasons.

Example. A, B and C are points on a circle with centre O. AC is a diameter. Angle BAC = 34°.



Work out the size of angle BCA. Give reasons for your answer.

Model answer.

Angle ABC = 90° (B1)

Angle BCA = 180 − 90 − 34 = 56° (B1)

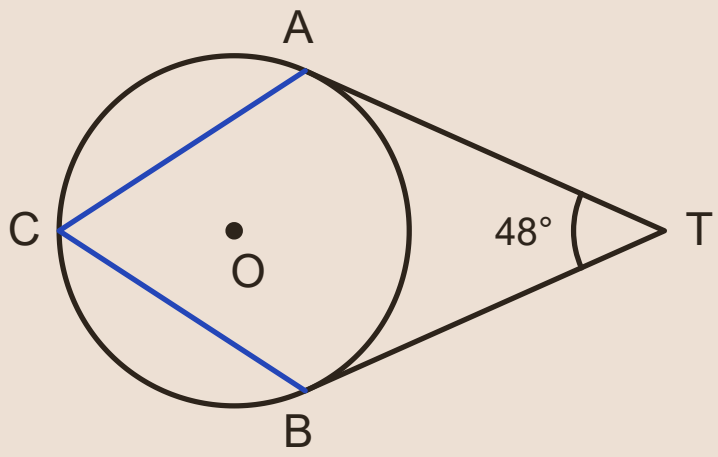
Reasons: the angle in a semicircle is 90°; angles in a triangle add up to 180° (C1)

Multi-step problem with tangents

4 marks | Grade 7

1. Mark the 90° angles where the tangents meet the radii.
2. Use the angle sum of a quadrilateral or triangle to find the angle at the centre.
3. Use the angle at the centre theorem to reach the angle you want, with a reason for every step.

Example. A and B are points on a circle with centre O. The tangents to the circle at A and B meet at the point T. Angle ATB = 48°. C is a point on the major arc AB.



Work out the size of angle ACB. Give reasons for each stage of your working.

Model answer.

Angle OAT = angle OBT = 90° (M1)

Angle AOB = 360 − 90 − 90 − 48 = 132° (M1)

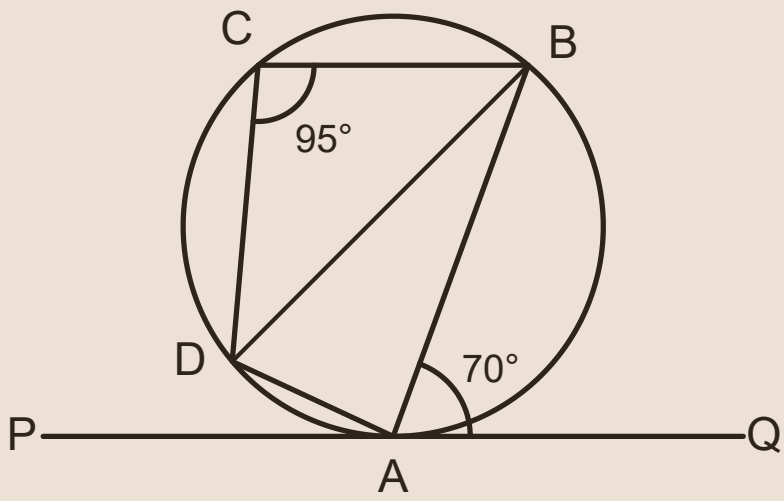
Angle ACB = 132 ÷ 2 = 66° (A1)

Reasons: a tangent is perpendicular to the radius; angles in a quadrilateral add up to 360°; the angle at the centre is twice the angle at the circumference (C1)

Alternate segment theorem and cyclic quadrilaterals 4 to 5 marks **Grade 8**

1. Find the angle between the tangent and a chord, and look across the chord to the alternate segment.
2. Use cyclic quadrilateral and straight-line facts to find the other angles.
3. Give the full name of each theorem you use.

Example. A, B, C and D lie on a circle, in that order. The straight line PAQ is the tangent to the circle at A, with P nearer to D and Q nearer to B. Angle BAQ = 70° and angle BCD = 95°.



(a) Work out the size of angle ADB. Give a reason. (2 marks)
(b) Work out the size of angle PAD. Give reasons. (3 marks)

Model answer.

(a) Angle ADB = 70° (B1) by the alternate segment theorem (C1)

(b) Angle BAD = 180 − 95 = 85° (M1) because opposite angles of a cyclic quadrilateral add up to 180°

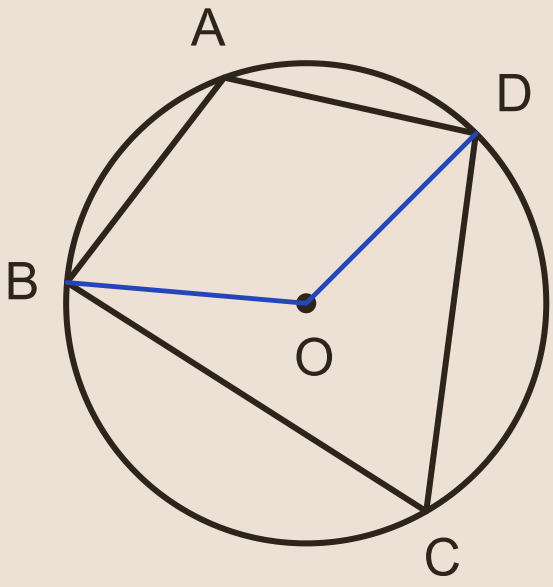
Angle PAD = 180 − 70 − 85 = 25° (A1) because angles on a straight line add up to 180° (C1)

Prove a result

4 marks | Grade 9

1. Let one angle be x.
2. Use a theorem to write another angle in terms of x, giving the reason.
3. Continue step by step until you reach the result, then write a concluding statement.

Example. ABCD is a cyclic quadrilateral. The centre of the circle is O.



Use the fact that the angle at the centre is twice the angle at the circumference to prove that angle BAD + angle BCD = 180°.

Model answer.

Let angle BAD = x. The angle BOD subtended at the centre by the same arc as angle BAD is 2x, because the angle at the centre is twice the angle at the circumference (M1)

The other angle BOD at the centre = 360° − 2x, because angles around a point add up to 360° (M1)

Angle BCD is subtended by the other arc BD, so angle BCD = ½(360° − 2x) = 180° − x (A1)

So angle BAD + angle BCD = x + 180° − x = 180° (C1)

Don’t lose marks

- Using the angle at the centre theorem when the two angles are not subtended by the same arc.
- Calling a quadrilateral cyclic when one of its vertices is the centre, not a point on the circle.
- Halving when you should double: the angle at the centre is the larger one.
- Writing shorthand reasons such as 'semicircle' or 'bow tie' instead of the full theorem.
- Assuming a line is a diameter or a tangent because it looks like one; it must be stated.

More tips

Memory tricks

- Spot the shape: a 'bow tie' of two triangles on one chord means angles in the same segment; an 'arrowhead' from the centre means the angle at the centre; a triangle on a diameter means 90°.
- Every pair of radii makes an isosceles triangle: mark the equal base angles at once.
- A tangent in the diagram almost always means a 90° angle with a radius.
- Two tangents and two radii make a kite with two 90° angles, so its other two angles add up to 180°.

Exam technique

- Learn each theorem in exact words, e.g. 'the angle at the centre is twice the angle at the circumference'. Full, correct reasons earn the C mark.
- Mark every angle you find on the diagram, then write your steps in order with a reason for each.
- In 'prove' questions, use letters for angles and finish with a concluding sentence.
- If you are stuck, draw in a radius; it often creates an isosceles triangle or a right angle.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 6 — **Use the tangent and radius facts**
A tangent meets a radius at 90°, and the two tangents from a point outside a circle are equal in length.

Grade 6 — **Use the angle in a semicircle**
An angle at the circumference subtended by a diameter is 90°.

Grade 7 — **Use the angle at the centre theorem**
The angle at the centre is twice the angle at the circumference subtended by the same arc.

Grade 7 — **Use same-segment and cyclic quadrilateral facts**
Angles in the same segment are equal, and opposite angles of a cyclic quadrilateral add up to 180°.

Grade 7 — **Use the perpendicular bisector of a chord**
The perpendicular from the centre to a chord bisects the chord.

Grade 8 — **Use the alternate segment theorem**
The angle between a tangent and a chord equals the angle in the alternate segment.

Grade 8 — **Solve multi-step problems with full reasons**
Combine several theorems and angle facts, stating the reason for each step.

Grade 9 — **Prove circle theorems and related results**
Use isosceles triangles made by radii, and algebra, to prove a result in general.

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