Equation of a circle and tangents

A16 · Algebra · revision notes

Higher only. Recognising $x^2 + y^2 = r^2$ as the equation of a circle with centre the origin and radius r, and finding the equation of the tangent to the circle at a given point. Often combined with straight-line graphs and simultaneous equations.

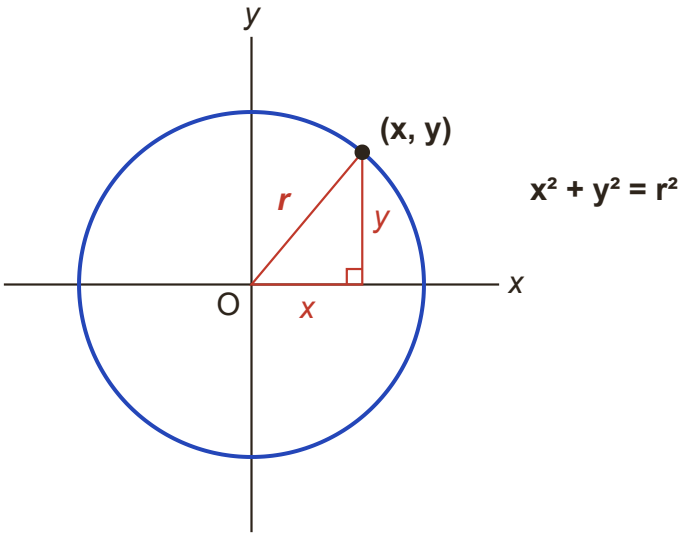
Key facts

- $x^2 + y^2 = r^2$: centre (0, 0), radius r
- A tangent is perpendicular to the radius at the point where it touches
- Gradient of the tangent at (a, b) is $-\frac{a}{b}$
- Radius = square root of the number on the right
- Gradient of the radius from (0, 0) to (a, b) is $\frac{b}{a}$
- Check: the tangent to $x^2 + y^2 = r^2$ at (a, b) is $ax + by = r^2$

Notes

Equation of a circle

- A circle with centre (0, 0) and radius r has equation $x^2 + y^2 = r^2$. This comes from Pythagoras: every point (x, y) on it is r away from the origin.

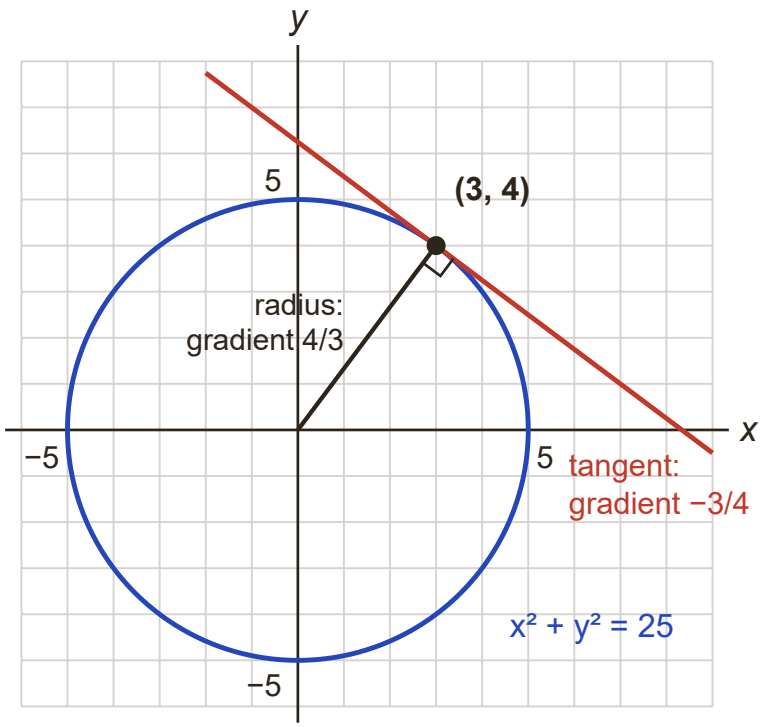


Pythagoras on the red triangle gives the equation of the circle.

- $x^2 + y^2 = 49$ has radius 7. $x^2 + y^2 = 20$ has radius $\sqrt{20} = 2\sqrt{5}$
- It crosses the axes at $(\pm r, 0)$ and $(0, \pm r)$
- A point is on the circle if its coordinates fit the equation: (6, −8) is on $x^2 + y^2 = 100$ because $36 + 64 = 100$
- At GCSE the centre is always the origin.

Tangent at a point

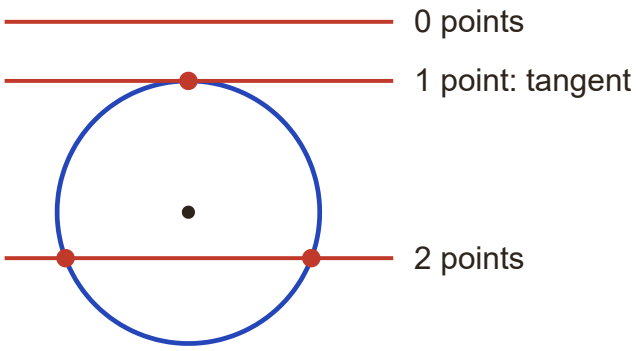
- A tangent touches the circle at one point and is perpendicular to the radius at that point.
- Step 1: the gradient of the radius from (0, 0) to (a, b) is $\frac{b}{a}$
- Step 2: the tangent's gradient is the negative reciprocal, $-\frac{a}{b}$
- Step 3: substitute (a, b) into $y = mx + c$ to find c.
- Tangent to $x^2 + y^2 = 25$ at (3, 4): radius gradient $\frac{4}{3}$, so tangent gradient $-\frac{3}{4}$. $4 = -\frac{9}{4} + c$, so $c = \frac{25}{4}$. The tangent is $y = -\frac{3}{4}x + \frac{25}{4}$, or $3x + 4y = 25$



The tangent is perpendicular to the radius, so its gradient is the negative reciprocal.

Lines meeting circles Grade 8+ only

- To find where a line meets a circle, substitute the line into the circle's equation and solve the quadratic.
- Two solutions: the line crosses the circle twice. One repeated solution: the line is a tangent. No solutions: they do not meet.



The number of solutions of the quadratic matches the number of meeting points.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, P = a process mark, C = a communication mark.

Find the equation of a circle 2 marks Grade 7

1. Use Pythagoras with the given point to find r^2
2. Write $x^2 + y^2 = r^2$

Example. A circle has centre (0, 0) and passes through the point (−5, 12). Find the equation of the circle.

Model answer.

$r^2 = (-5)^2 + 12^2 = 169$ M1

$x^2 + y^2 = 169$ A1

Decide whether a point lies on a circle 2 marks Grade 7

1. Substitute the point into $x^2 + y^2$
2. Compare with r^2 and write a conclusion.

Example. Does the point (4, −5) lie on the circle $x^2 + y^2 = 40$? Show how you decide.

Model answer.

$4^2 + (-5)^2 = 41$ M1

41 is not 40, so the point is not on the circle (it is just outside it) C1

Find the equation of a tangent

4 marks | Grade 8

1. Find the gradient of the radius to the point.
2. Take the negative reciprocal for the tangent.
3. Substitute the point into $y = mx + c$ to find c.

Example. The point P(2, −4) lies on the circle $x^2 + y^2 = 20$. Find the equation of the tangent to the circle at P. Give your answer in the form $y = mx + c$

Model answer.

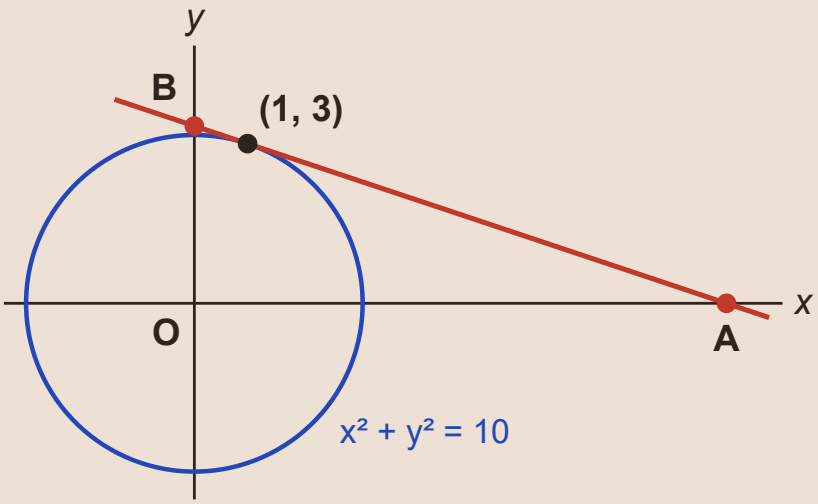
Gradient of OP $= \frac{-4}{2} = -2$	M1
Tangent gradient $= \frac{1}{2}$	M1
$-4 = \frac{1}{2} \times 2 + c$, so $c = -5$	M1
$y = \frac{1}{2}x - 5$	A1

Tangent problem with the axes

5 marks | Grade 9

1. Find the equation of the tangent.
2. Put $y = 0$ and $x = 0$ to find where it meets the axes.
3. Use these lengths to answer the question, e.g. an area.

Example. The tangent to the circle $x^2 + y^2 = 10$ at the point (1, 3) meets the x-axis at A and the y-axis at B. O is the origin. Work out the area of triangle OAB.



Model answer.

Radius gradient 3, so tangent gradient $-\frac{1}{3}$	P1
$3 = -\frac{1}{3} \times 1 + c$, so $y = -\frac{1}{3}x + \frac{10}{3}$	P1
A = (10, 0) and B = $\left(0, \frac{10}{3}\right)$	P1
Area $= \frac{1}{2} \times 10 \times \frac{10}{3}$	P1
$= \frac{50}{3}$ (16.7 to 3 s.f.)	A1

Don't lose marks

- Giving r^2 as the radius: $x^2 + y^2 = 16$ has radius 4, not 16.
- Using the radius gradient as the tangent gradient.
- Getting the negative reciprocal wrong: for a radius gradient of −2, the tangent gradient is $\frac{1}{2}$
- Slips with negative coordinates when squaring: $(-4)^2 = 16$

More tips

Memory tricks

- Check a tangent with $ax + by = r^2$: at (2, −4) on $x^2 + y^2 = 20$, $2x - 4y = 20$ gives $y = \frac{1}{2}x - 5$
- Radius at a glance: square root the number on the right-hand side.
- Sketch the circle, the point and the tangent: the tangent should look perpendicular to the radius.

Exam technique

- Always show the radius gradient and then the tangent gradient: each earns a method mark.
- Give the tangent in the form asked for; if no form is given, $ax + by = c$ is fine.
- In problem-solving questions, draw a sketch showing the centre, the point and the tangent.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 7 — **Find the radius from a circle's equation**
$x^2 + y^2 = 36$ is a circle with centre (0, 0) and radius 6.

Grade 7 — **Write the equation of a circle**
Radius 5 gives $x^2 + y^2 = 25$; through (3, −4) gives $r^2 = 9 + 16 = 25$, the same circle.

Grade 7 — **Check whether a point lies on a circle**
Substitute the coordinates: the point is on the circle only if $x^2 + y^2$ equals r^2

Grade 8 — **Find the equation of a tangent**
Find the gradient of the radius, take the negative reciprocal, then substitute the point to find c.

Grade 8 — **Find where a line meets a circle**
Substitute the line into $x^2 + y^2 = r^2$ and solve the quadratic.

Grade 9 — **Solve problems involving tangents and axes**
e.g. find where a tangent meets the axes and the area of the triangle it makes.

Free notes, Full packs, practice papers and mock exams at chhetriacademy.com. Written for this site; not taken from past papers. Stuck? 1-to-1 tutoring with a free 30-minute trial lesson.