Upper and lower bounds

N16 · Number · revision notes

Finding the upper and lower bounds of rounded measurements and using them in calculations to find the largest and smallest possible answers, and to decide on a suitable degree of accuracy. Higher tier only; usually a 2 to 5 mark calculator question about area, speed, density or a formula.

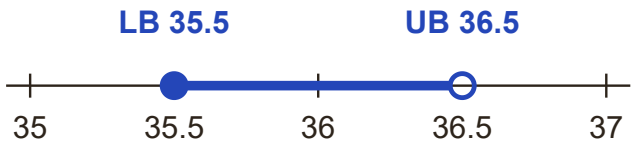
Key facts

- Bounds = value ± half the unit it was rounded to
- UB of $a \times b$ = $UB_a \times UB_b$; LB of $a \times b$ = $LB_a \times LB_b$
- UB of $a \div b$ = $UB_a \div LB_b$; LB of $a \div b$ = $LB_a \div UB_b$
- Grade 8+ only Degree of accuracy: round the UB and LB; give the most accurate value they both round to
- UB of $a + b$ = $UB_a + UB_b$; LB of $a + b$ = $LB_a + LB_b$
- UB of $a - b$ = $UB_a - LB_b$; LB of $a - b$ = $LB_a - UB_b$
- UB of speed = UB of distance ÷ LB of time

Notes

Upper and lower bounds

- A measurement rounded to the nearest unit could really be up to half a unit smaller or half a unit bigger.
- Lower bound (LB) = value − half a unit. Upper bound (UB) = value + half a unit.
- 36 cm to the nearest cm: LB 35.5 cm, UB 36.5 cm. 2.4 kg to 1 d.p.: LB 2.35 kg, UB 2.45 kg. 800 m to the nearest 50 m: LB 775 m, UB 825 m.



36 cm to the nearest cm: half a unit either side.

- Significant figures: work out the unit first. 150 to 2 s.f. is to the nearest 10, so the bounds are 145 and 155.
- Strictly the value is less than the UB (it would round up), but you still use the UB in calculations.

Bounds in calculations

- Add: UB = UB + UB; LB = LB + LB. Multiply: UB = UB × UB; LB = LB × LB.
- Subtract: UB = UB − LB; LB = LB − UB.
- Divide: UB = UB ÷ LB; LB = LB ÷ UB.
- Think: to make an answer as big as possible, make what you add or multiply by big, and what you subtract or divide by small.
- Example: a runner covers 150 m (to the nearest metre) in 12.4 s (to 1 d.p.). Distance bounds 149.5 m and 150.5 m; time bounds 12.35 s and 12.45 s.
- UB of speed = 150.5 ÷ 12.35 = 12.186... m/s. LB of speed = 149.5 ÷ 12.45 = 12.008... m/s.

A suitable degree of accuracy Grade 8+ only

- Work out the UB and LB of the answer, and write them to several figures.
- Round both to 1 s.f., then 2 s.f., then 3 s.f., ... Stop when they no longer agree.
- Give the most accurate rounding on which both bounds agree, with a reason.
- In the speed example: both bounds round to 12 to 2 s.f., but to 12.2 and 12.0 to 3 s.f. So the speed is 12 m/s (2 s.f.), because both bounds round to 12.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Work out a bound of an area or perimeter

2 to 3 marks | **Grade 6**

1. Write down the bounds of each measurement.
2. Choose the right bounds (both upper for an upper bound of a product or sum).
3. Calculate, and give units.

Example. A rectangle has length 8.6 cm and width 5.3 cm, both correct to 1 decimal place.
Work out the upper bound of the area of the rectangle.



Model answer.

Upper bounds 8.65 cm and 5.35 cm (B1 for either)

8.65×5.35 (M1)

= 46.2775 cm^2 (A1)

Work out a bound involving division (density, speed)

3 marks | **Grade 7**

1. Write down the bounds of each measurement, finding the unit first for s.f.
2. Upper bound of a quotient: UB on top ÷ LB on the bottom. Lower bound: LB ÷ UB.
3. Show the calculation, then give the answer with units.

Example. The mass of a stone is 420 g, correct to the nearest 10 g. The volume of the stone is 150 cm^3, correct to 2 significant figures.
Work out the upper bound for the density of the stone. Give your answer correct to 3 significant figures.

Model answer.

Mass UB 425 g; volume LB 145 cm^3 (B1 for either)

$\frac{425}{145}$ (M1 for UB mass ÷ LB volume)

= 2.931... = 2.93 g/cm^3 (A1)

By considering bounds, give an answer to a suitable degree of accuracy

4 to 5 marks **Grade 8**

1. Write down the bounds of every measurement.
2. Work out the upper bound and the lower bound of the answer.
3. Round both to 1 s.f., 2 s.f., ... and find the most accurate value they agree on.
4. State the answer and the reason.

Example. $c = \frac{a}{b}$

$a = 7.6$ correct to 1 decimal place. $b = 2.84$ correct to 2 decimal places.

By considering bounds, work out the value of c to a suitable degree of accuracy. You must show all your working and give a reason for your answer.

Model answer.

Bounds of a: 7.55 and 7.65; bounds of b: 2.835 and 2.845 (B1)

UB of c = 7.65 ÷ 2.835 = 2.6984... (M1)

LB of c = 7.55 ÷ 2.845 = 2.6537... (M1)

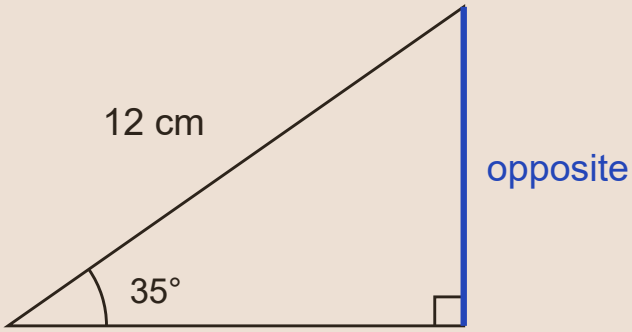
c = 2.7, because both bounds round to 2.7 to 2 s.f. but to different values (2.70 and 2.65) to 3 s.f. (A1)

Bounds in trigonometry

3 marks | Grade 8

1. Write down the bounds of each length and angle.
2. Work out which bound makes the answer largest: for angles between 0° and 90°, sin and tan increase as the angle increases, but cos decreases.
3. Substitute and calculate.

Example. In a right-angled triangle, the hypotenuse is 12 cm, correct to the nearest cm. One of the other angles is 35°, correct to the nearest degree.
Work out the upper bound of the length of the side opposite the 35° angle. Give your answer correct to 3 significant figures.



Model answer.

Bounds used: 12.5 cm and 35.5°	(B1)
Opposite = hypotenuse × sin angle, so UB = 12.5 × sin 35.5°	(M1)
= 7.2587... = 7.26 cm	(A1)

Don’t lose marks

- Using UB ÷ UB for the upper bound of a division.
- Getting the unit wrong: 150 to 2 s.f. has bounds 145 and 155, not 149.5 and 150.5.
- Writing 8.649 as the upper bound. Use 8.65 (8.6499... recurring is the same number).
- Rounding the bounds before the final step; keep full values until the end.
- In a 'suitable degree of accuracy' question, giving the answer without a reason, or to more figures than both bounds agree on.

More tips

Memory tricks

- Biggest answer: big on top, small on the bottom. Smallest answer: small on top, big on the bottom.
- Subtraction: 'big minus small' gives the UB, 'small minus big' gives the LB.
- Sense check: the UB of your answer should be a little more than the answer you get from the rounded values (8.6 × 5.3 = 45.58, UB 46.2775).
- Bounds = value ± half the unit. '150 to 2 s.f.' means the unit is 10.

Exam technique

- Write down every bound you use before calculating: there is often a B1 mark for seeing them.
- Say which bound you are using and why, e.g. 'UB density = UB mass ÷ LB volume'.
- For 'suitable degree of accuracy', write both bounds to at least 4 figures, show both rounded values and state that they agree.
- Give units, and give the bound to at least 3 significant figures unless the question says otherwise.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 5 — **Find the bounds of a rounded measurement**
Go half a unit either side: 36 cm to the nearest cm has bounds 35.5 cm and 36.5 cm.

Grade 6 — **Find the bounds of a sum or product**
Upper bound: use both upper bounds; lower bound: use both lower bounds.

Grade 7 — **Find the bounds of a difference or quotient**
For the upper bound of $a - b$ or $a \div b$, use the upper bound of a and the lower bound of b

Grade 7 — **Use bounds with compound measures**
Upper bound of speed = upper bound of distance ÷ lower bound of time.

Grade 8 — **Choose a suitable degree of accuracy**
Round both bounds; give the most accurate value that they both round to.

Grade 8 — **Use bounds in formulas and trigonometry**
Decide which bound of each value makes the answer as large (or small) as possible.

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