Arcs and sectors

G18 · Geometry and measures · revision notes

An arc is part of the circumference, and a sector is the 'slice' between two radii. You work out arc lengths, sector areas and perimeters as a fraction of the whole circle, sometimes in terms of π, and work backwards to find an angle or a radius.

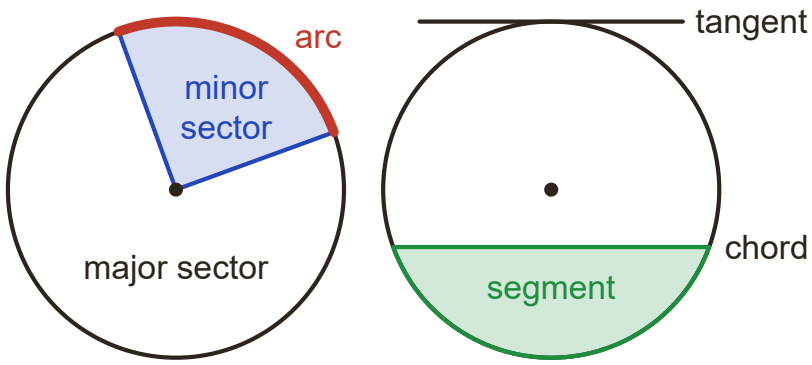
Key facts

- Circumference = 2πr = πd; area of a circle = πr²
- Arc length = $\frac{\theta}{360} \times 2\pi r$
- Sector area = $\frac{\theta}{360} \times \pi r^2$
- Perimeter of a sector = arc length + 2r
- Semicircle: θ = 180° (half); quarter circle: θ = 90° (a quarter)
- Grade 7+ only Segment area = sector area − $\frac{1}{2} r^2 \sin\theta$

Notes

Parts of a circle

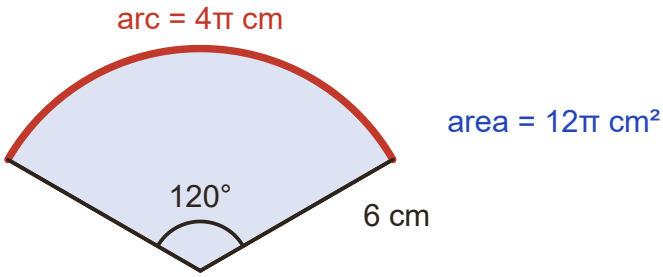
- The **radius** r goes from the centre to the circle. The **diameter** d is a chord through the centre, so d = 2r. The **circumference** is the distance around the circle.
- An **arc** is part of the circumference. A **sector** is the region between two radii and an arc.
- A **chord** is a straight line joining two points on the circle. A **segment** is the region between a chord and an arc.
- A **tangent** is a straight line that touches the circle at one point.
- The **minor** arc or sector is the smaller one (angle less than 180°); the **major** one is the larger.



A sector is cut off by two radii; a segment is cut off by a chord.

Arc length and sector area

- A sector with angle θ at the centre is $\frac{\theta}{360}$ of the whole circle.
- Arc length = $\frac{\theta}{360} \times 2\pi r$ (that fraction of the circumference).
- Sector area = $\frac{\theta}{360} \times \pi r^2$ (that fraction of the area).
- Example: r = 6 cm and θ = 120°. Arc length = $\frac{120}{360} \times 12\pi = 4\pi$ = 12.6 cm. Sector area = $\frac{120}{360} \times 36\pi = 12\pi$ = 37.7 cm².
- Perimeter of a sector = arc length + 2 radii. In the example: $4\pi + 12$ = 24.6 cm.



The sector is $\frac{120}{360} = \frac{1}{3}$ of the circle.

- For a major sector, use 360° − θ as the angle.

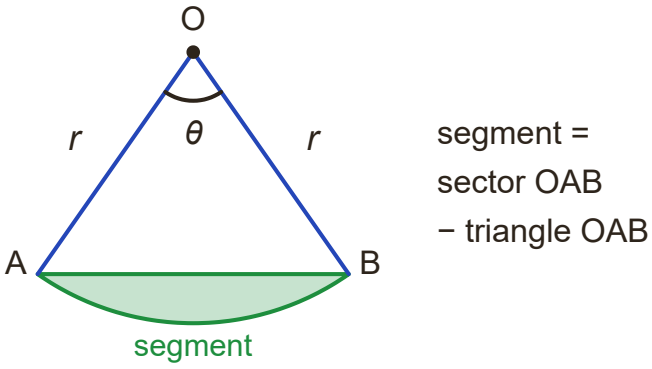
Working backwards

- If you are given the arc length or area, substitute everything you know and solve.
- A sector of radius 10 cm has area 40π cm²: $\frac{\theta}{360} \times 100\pi = 40\pi$, so $\theta = \frac{40 \times 360}{100} = 144°$

Grade 6+ only

Segments Grade 7+ only

- Area of a segment = area of the sector − area of the triangle formed by the two radii and the chord.
- Area of that triangle = $\frac{1}{2} r^2 \sin\theta$ (from ½ab sin C, with both sides equal to r).



Triangle area = ½r² sin θ; take it away from the sector.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Work out the area of a sector or the length of an arc

2 marks **Grade 5**

1. Write the fraction of the circle: θ ÷ 360.
2. Multiply by πr² for an area, or by 2πr for an arc length.
3. Round as asked and give units.

Example. A sector of a circle has radius 7.5 cm and angle 64°.
Work out the area of the sector. Give your answer correct to 3 significant figures.

Model answer.

Area = $\frac{64}{360}$ × π × 7.5² (M1)

= 31.4 cm² (A1)

Work out the perimeter of a sector

3 marks **Grade 5**

1. Work out the arc length.
2. Add both straight edges (two radii).
3. Round at the end and include units.

Example. OAB is a sector of a circle with centre O. OA = OB = 9 cm and angle AOB = 140°.
Work out the perimeter of the sector. Give your answer correct to 1 decimal place.

Model answer.

Arc AB = $\frac{140}{360}$ × 2 × π × 9 = 21.99… cm (M1)

Perimeter = 21.99… + 9 + 9 (M1)

= 40.0 cm (A1)

Work backwards to find the angle (in terms of π)

3 marks | Grade 6

1. Write the formula with the given values substituted, leaving π in.
2. Cancel π from both sides if it appears on both.
3. Rearrange to find θ (or r).

Example. A sector of a circle has radius 12 cm. The arc length of the sector is 5π cm.
Work out the angle of the sector.

Model answer.

$\frac{\theta}{360} \times 2 \times \pi \times 12 = 5\pi$ (M1)

$\frac{\theta}{360} \times 24 = 5$, so $\theta = 5 \times 360 \div 24$ (M1)

$\theta = 75°$ (A1)

Work out the area of a segment

4 marks | Grade 7

1. Work out the area of the sector.
2. Work out the area of the triangle using ½ab sin C.
3. Subtract the triangle from the sector.

Example. AB is a chord of a circle with centre O and radius 8 cm. Angle AOB = 110°.
Work out the area of the minor segment cut off by the chord AB. Give your answer correct to 3 significant figures.

Model answer.

Sector area = $\frac{110}{360} \times \pi \times 8^2 = 61.43\ldots$ cm² (M1)

Triangle area = ½ × 8 × 8 × sin 110° = 30.07… cm² (M1)

Segment = 61.43… − 30.07… (M1)

= 31.4 cm² (A1)

Don't lose marks

- Forgetting to add the two radii to find the perimeter of a sector.
- Using the diameter where the formula needs the radius.
- Using θ ÷ 180 or θ ÷ 100 instead of θ ÷ 360.
- Using the minor angle when the question asks about the major sector.
- Using 3.14 instead of the π button, which can change the last digit of your answer.

More tips

Memory tricks

- Simplify the fraction first: 120/360 = ⅓, 45/360 = ⅛, 72/360 = ⅕.
- Sense check: a 90° sector is a quarter of the circle, so its area is a quarter of πr².
- Arc: 'fraction × circumference'. Sector: 'fraction × area'.
- For answers in terms of π, cancel the numbers and write π at the end, e.g. 12π.

Exam technique

- 'Give your answer in terms of π' means leave π in: an answer like 12π is exact.
- Write the full calculation (e.g. 64/360 × π × 7.5²) before the answer to earn the method mark.
- Give units: cm for lengths and perimeters, cm² for areas.
- For a compound shape, list which edges make up the perimeter before adding them.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 3 — **Name the parts of a circle**

Radius, diameter, circumference, chord, tangent, arc, sector and segment.

Grade 5 — **Calculate the length of an arc**

Arc length = $\frac{\theta}{360} \times 2\pi r$, a fraction of the circumference.

Grade 5 — **Calculate the area of a sector**

Sector area = $\frac{\theta}{360} \times \pi r^2$, a fraction of the area of the circle.

Grade 5 — **Find the perimeter of a sector**

Add the two radii to the arc length.

Grade 5 — **Give answers in terms of π**

Leave π as a symbol and simplify, e.g. $\frac{40}{360} \times \pi \times 9^2 = 9\pi$

Grade 6 — **Find the angle or radius of a sector**

Substitute the given arc length or area into the formula and solve for θ or r.

Grade 7 — **Find the area of a segment**

Sector area minus the area of the triangle, using ½ab sin C for the triangle.

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