Angles, parallel lines and polygons

G1, G3, G4, G6 · Geometry and measures · revision notes

Angle facts on straight lines, around a point, in triangles and quadrilaterals, on parallel lines and in polygons. Questions ask you to work out angles and give reasons; harder ones chain several facts together or use algebra. This topic comes up on both tiers, on calculator and non-calculator papers.

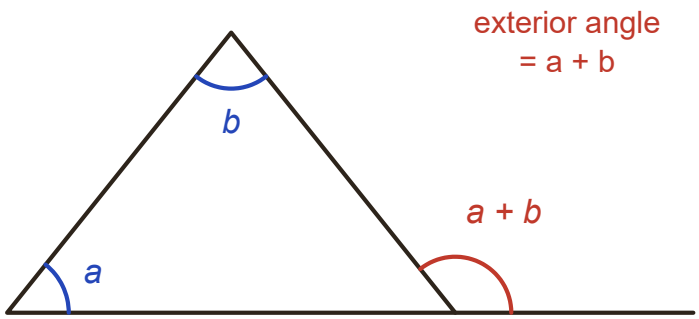
Key facts

- Angles on a straight line add up to 180°; angles around a point add up to 360°
- Triangle: 180°; quadrilateral: 360°
- Alternate angles are equal; corresponding angles are equal; co-interior angles add up to 180°
- Exterior angles of any polygon add up to 360°
- Vertically opposite angles are equal
- Base angles of an isosceles triangle are equal
- Sum of interior angles of an n-sided polygon = (n − 2) × 180°
- Regular polygon: exterior angle = 360° ÷ n; interior angle = 180° − exterior angle

Notes

Basic angle facts

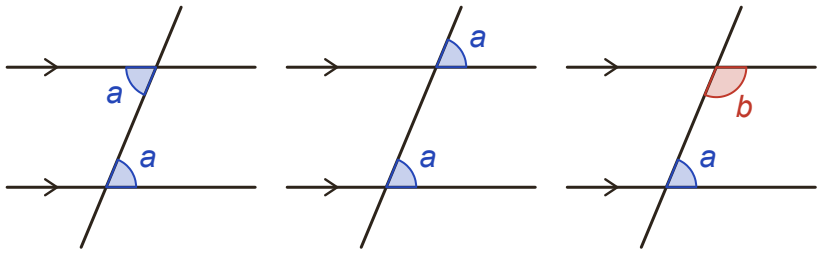
- Angles on a straight line add up to **180°**. Angles around a point add up to **360°**.
- **Vertically opposite** angles (opposite each other where two straight lines cross) are equal.
- Angles in a triangle add up to **180°**. Angles in a quadrilateral add up to **360°**.
- An **isosceles** triangle has two equal sides, and the two **base angles** (opposite the equal sides) are equal. Each angle in an equilateral triangle is 60°.
- The **exterior angle** of a triangle equals the sum of the two interior opposite angles.



The exterior angle equals the sum of the two interior opposite angles.

Parallel lines

- Parallel lines are marked with matching arrows. A line that crosses them is called a transversal.
- **Alternate** angles are equal. They are between the parallel lines, on opposite sides of the transversal (a Z shape).
- **Corresponding** angles are equal. They are in the same position at each crossing (an F shape).
- **Co-interior** angles add up to 180°. They are between the parallel lines, on the same side of the transversal (a C shape).



Alternate (Z) equal | **Corresponding (F)** equal | **Co-interior (C)** add up to 180°

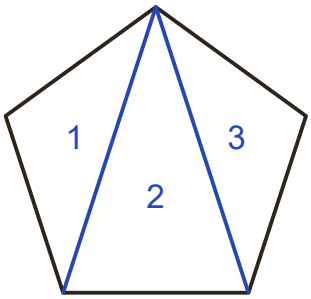
In the co-interior diagram, a + b = 180°.

Special quadrilaterals

- **Square**: 4 equal sides, 4 right angles; diagonals are equal and bisect each other at 90°.
- **Rectangle**: opposite sides equal, 4 right angles; diagonals are equal and bisect each other.
- **Parallelogram**: opposite sides equal and parallel, opposite angles equal; diagonals bisect each other.
- **Rhombus**: 4 equal sides, opposite sides parallel, opposite angles equal; diagonals bisect each other at 90°.
- **Kite**: two pairs of equal adjacent sides, one pair of equal opposite angles; diagonals cross at 90° and one diagonal bisects the other.
- **Trapezium**: one pair of parallel sides.
- Lines of symmetry: square 4, rectangle 2, rhombus 2, kite 1, isosceles trapezium 1, parallelogram 0. Order of rotational symmetry: square 4; rectangle, rhombus and parallelogram 2.

Polygons

- From one vertex, an n-sided polygon splits into n − 2 triangles, so the **sum of the interior angles** is (n − 2) × 180°.



5 sides → 3 triangles

(5 − 2) × 180°

= 540°

Diagonals from one vertex split a pentagon into 3 triangles.

- The **exterior angles** of any polygon add up to 360°. At each vertex, interior angle + exterior angle = 180°.
- In a **regular** polygon all sides and angles are equal, so each exterior angle = 360° ÷ n, and n = 360° ÷ exterior angle.
- Names: pentagon 5, hexagon 6, heptagon 7, octagon 8, nonagon 9, decagon 10 sides.

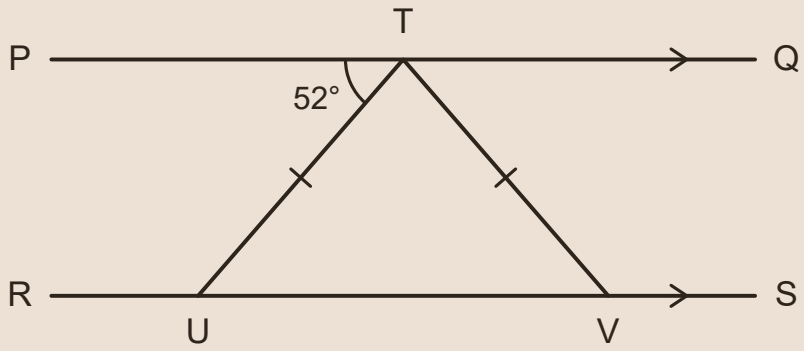
How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, C = a communication mark.

Work out the size of an angle. Give reasons for your answer. 3 to 4 marks **Grade 4**

1. Mark each angle on the diagram as soon as you find it.
2. Work towards the angle you want one fact at a time.
3. After each step, write the reason in full words, e.g. 'corresponding angles are equal'.
4. Check the answer fits the diagram (acute or obtuse).

Example. PQ and RS are parallel horizontal lines, with PQ above RS. T is a point on PQ, with P to the left of T. U and V are points on RS, with U to the left of V. TU = TV. Angle PTU = 52°.



Work out the size of angle UTV. Give a reason for each stage of your working.

Model answer.

Angle TUV = 52° because alternate angles are equal (M1)

Angle TVU = 52° because base angles of an isosceles triangle are equal

Angle UTV = 180 − 52 − 52 (M1) = 76° (A1)

Reasons given: alternate angles are equal; base angles of an isosceles triangle are equal; angles in a triangle add up to 180° (C1)

Interior and exterior angles of polygons 2 to 4 marks **Grade 4**

1. For a regular polygon, exterior angle = 360° ÷ n and interior angle = 180° − exterior angle.
2. To find the number of sides, work out 360° ÷ exterior angle. It must be a whole number.
3. For the total of the interior angles, use (n − 2) × 180°.

Example. Each interior angle of a regular polygon is 156°.
(a) Work out the number of sides of the polygon. (2 marks)
(b) Work out the sum of the interior angles of the polygon. (2 marks)

Model answer.

(a) Exterior angle = 180 − 156 = 24° (M1)

360 ÷ 24 = 15 sides (A1)

(b) (15 − 2) × 180 (M1) = 2340° (A1)

Angles in terms of x: form and solve an equation 3 to 4 marks **Grade 5**

1. Choose the angle fact that links the angles, e.g. angles in a quadrilateral add up to 360°.
2. Write the equation, collect like terms and solve for x.
3. Substitute x back in to find the angle the question asks for.

Example. The angles of a quadrilateral are x°, 2x°, (x + 30)° and (3x + 15)°.
Work out the size of the largest angle.

Model answer.

x + 2x + x + 30 + 3x + 15 = 360	(M1)
7x + 45 = 360, so 7x = 315	(M1)
x = 45	(A1)
Largest angle = 3 × 45 + 15 = 150°	(A1)

Don't lose marks

- Writing 'Z angles' or 'F angles' instead of 'alternate angles' or 'corresponding angles'; these do not score as reasons.
- Using 360 ÷ n as the interior angle of a regular polygon; it gives the exterior angle.
- Assuming lines are parallel or sides are equal because they look it; use only what is marked or stated.
- Measuring with a protractor when the diagram is labelled 'not accurately drawn'.
- Giving the value of x when the question asks for an angle.

More tips

Memory tricks

- Z for alternate, F for corresponding, C for co-interior helps you find the angles, but never write 'Z angles' as a reason.
- Polygons: exterior angles are the easy way in. Start with 360 ÷ n, then take it from 180°.
- Quick check: 360 ÷ exterior angle must be a whole number, or the polygon cannot be regular.
- Count triangles for the angle sum: a hexagon splits into 4 triangles, so its angles add up to 4 × 180° = 720°.

Exam technique

- 'Give reasons' means a reason for every step. A correct angle without full reasons loses the communication mark.
- Write the angles you find on the diagram; they can earn method marks.
- Angle ABC is the angle at B, between the lines BA and BC.
- Write the full fact, e.g. 'angles on a straight line add up to 180°', not just '180°' or 'straight line'.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 2

Use angles on a line and around a point

Angles on a straight line add up to 180° and angles around a point add up to 360°.

Grade 3

Find missing angles in triangles and quadrilaterals

Angles in a triangle add up to 180°, angles in a quadrilateral add up to 360°, and the base angles of an isosceles triangle are equal.

Grade 3

Know the properties of special quadrilaterals

Know the sides, angles, diagonals and symmetry of squares, rectangles, parallelograms, rhombuses, kites and trapezia.

Grade 4

Use alternate, corresponding and co-interior angles

On parallel lines, alternate angles and corresponding angles are equal, and co-interior angles add up to 180°.

Grade 4

Find interior and exterior angles of polygons

Sum of interior angles = (n − 2) × 180°, and the exterior angles of any polygon add up to 360°.

Grade 5

Give a geometric reason for every step

Write the angle fact in words each time you use it, e.g. 'alternate angles are equal'.

Grade 5

Form and solve equations from angle facts

When angles are given in terms of x, use an angle fact to write an equation, solve it, then find the angle.

Grade 6

Prove angle results using known facts

Use letters and a chain of reasons to show a general result, e.g. that the exterior angle of a triangle equals the sum of the two interior opposite angles.

Free notes, Full packs, practice papers and mock exams at chhetriacademy.com. Written for this site; not taken from past papers. Stuck? 1-to-1 tutoring with a free 30-minute trial lesson.