Laws of indices in algebra

A4d · Algebra · revision notes

Using the laws of indices to simplify algebraic expressions such as $3a^4 \times 5a^2$ or $(2x^3)^4$. Both tiers use the multiplying, dividing and power laws with zero and negative indices; Higher also uses fractional indices.

Key facts

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$
- $a^0 = 1$ and $a^1 = a$
- $a^{-n} = \dfrac{1}{a^n}$
- Grade 7+ only $a^{\frac{1}{n}} = \sqrt[n]{a}$
- Grade 7+ only $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$

Notes

The three main laws

- Multiplying: add the indices. $a^m \times a^n = a^{m+n}$, e.g. $x^5 \times x^3 = x^8$
- Dividing: subtract the indices. $a^m \div a^n = a^{m-n}$, e.g. $x^7 \div x^2 = x^5$

$$x^5 \times x^3 = \underbrace{(x \times x \times x \times x \times x)}_{5\text{ x's}} \times \underbrace{(x \times x \times x)}_{3\text{ x's}} = x^8$$

$$x^7 \div x^2 = \frac{\cancel{x} \times \cancel{x} \times x \times x \times x \times x \times x}{\cancel{x} \times \cancel{x}} = x^5$$

Multiplying adds the indices (5 + 3); dividing cancels, so you subtract (7 − 2).

- Power of a power: multiply the indices. $(a^m)^n = a^{mn}$, e.g. $(x^4)^3 = x^{12}$
- These laws only work for the **same base**: $x^2 \times y^3$ cannot be simplified.
- A letter on its own has index 1: $x = x^1$

Numbers and letters together

- Multiply or divide the numbers as normal, then use the laws on each letter: $6a^5b^2 \div 2a^3b = 3a^2b$
- A power outside a bracket applies to **everything** inside:

$$\begin{aligned}(2x^3y)^4 &= 2^4x^{12}y^4 \\ &= 16x^{12}y^4\end{aligned}$$

Zero and negative indices

- $a^0 = 1$ for any non-zero a, so $7x^0 = 7 \times 1 = 7$
- $a^{-n} = \dfrac{1}{a^n}$, e.g. $x^{-3} = \dfrac{1}{x^3}$
- $5x^{-2} = \dfrac{5}{x^2}$: only the x has the negative index, not the 5.

Fractional indices Grade 7+ only

- $a^{\frac{1}{n}} = \sqrt[n]{a}$, e.g. $x^{\frac{1}{2}} = \sqrt{x}$ and $x^{\frac{1}{3}} = \sqrt[3]{x}$
- $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$: take the root (denominator) first, then the power (numerator).
- $(8x^6)^{\frac{2}{3}}$: the cube root is $2x^2$, and squaring gives $4x^4$
- For a negative fractional index, take the reciprocal as well: $(9x^4)^{-\frac{1}{2}} = \dfrac{1}{3x^2}$

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Simplify using one law 1 mark each **Grade 4**

1. Decide which law applies: ×, ÷ or a power of a power.
2. Add, subtract or multiply the indices.

Example. Simplify (a) $p^6 \times p^3$ (b) $q^{10} \div q^2$ (c) $(r^4)^5$

Model answer.

(a) p^9 B1

(b) q^8 B1

(c) r^{20} B1

Simplify fully (numbers and letters)

2 marks | Grade 5

1. Divide (or multiply) the numbers.
2. Use the index laws on each letter in turn.
3. Write one number and each letter once.

Example. Simplify fully $\dfrac{12x^7y^3}{4x^2y}$

Model answer. $3x^5y^2$ B2 (B1 for two of 3, x^5, y^2 correct in a product)

Find the value of n

2 marks | Grade 7

1. Write every term as a power of the same letter, e.g. $\sqrt{x} = x^{\frac{1}{2}}$
2. Combine using the index laws.
3. Compare indices to find n.

Example. Given that $\dfrac{x^3 \times x^{-5}}{\sqrt{x}} = x^n$, find the value of n.

Model answer.

$x^3 \times x^{-5} = x^{-2}$ and $\sqrt{x} = x^{\frac{1}{2}}$ M1

$x^{-2} \div x^{\frac{1}{2}} = x^{-\frac{5}{2}}$, so $n = -\dfrac{5}{2}$ A1

Negative and fractional indices

2 marks | Grade 8

1. A negative index: take the reciprocal (flip the fraction).
2. The denominator of the index is the root: take it.
3. The numerator of the index is the power: apply it.

Example. Simplify fully $\left(\dfrac{x^6}{64}\right)^{-\frac{2}{3}}$

Model answer.

$\left(\dfrac{64}{x^6}\right)^{\frac{2}{3}}$ M1

Cube root $\dfrac{4}{x^2}$, squared: $\dfrac{16}{x^4}$ A1

Don’t lose marks

- Multiplying indices instead of adding: $x^3 \times x^4$ is x^7, not x^{12}
- Adding the numbers in front instead of multiplying: $3x^2 \times 4x^3 = 12x^5$, not $7x^5$
- Not applying the power to the number: $(2x^3)^4 = 16x^{12}$, not $2x^{12}$
- Thinking a negative index makes the answer negative: $x^{-2} = \frac{1}{x^2}$, not $-x^2$
- Writing $x^0 = 0$: it equals 1.

More tips

Memory tricks

- Multiplying terms means adding indices; dividing means subtracting; a power of a power means multiplying.
- A negative index means 'one over': $x^{-1} = \frac{1}{x}$
- Higher: in a fractional index, the denominator is the root (roots are at the bottom, like a tree's).
- Check with a number: put $x = 2$ into the question and your answer.

Exam technique

- 'Simplify fully' means one number and each letter written once with a single index.
- If the question asks for a particular form (e.g. 'without negative indices'), rewrite your answer to match it.
- In 'find the value of n' questions, write everything as a power of the same base before combining.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 4 — **Multiply powers of the same letter**
Add the indices: $a^4 \times a^3 = a^7$

Grade 4 — **Divide powers of the same letter**
Subtract the indices: $y^9 \div y^4 = y^5$

Grade 5 — **Raise a power to a power**
Multiply the indices: $(x^3)^5 = x^{15}$

Grade 5 — **Simplify terms with numbers and letters**
Deal with the numbers and each letter separately, e.g. $4a^2b \times 3ab^5 = 12a^3b^6$

Grade 5 — **Raise a whole term to a power**
The number is raised to the power too: $(3x^4)^2 = 9x^8$

Grade 5 — **Use zero and negative indices**
$x^0 = 1$ and $x^{-n} = \frac{1}{x^n}$, e.g. $x^{-2} = \frac{1}{x^2}$

Grade 7 — **Use fractional indices in algebra**
The denominator is a root and the numerator a power, e.g. $(16x^8)^{\frac{1}{2}} = 4x^4$

Grade 8 — **Simplify with negative fractional indices**

Root, then power, then take the reciprocal, e.g. $(27x^6)^{-\frac{2}{3}} = \frac{1}{9x^4}$

Free notes, Full packs, practice papers and mock exams at chhetriacademy.com. Written for this site; not taken from past papers. Stuck? 1-to-1 tutoring with a free 30-minute trial lesson.