# Algebraic fractions

A4c · Algebra · revision notes

Higher only. Simplifying, multiplying, dividing, adding and subtracting fractions that contain algebra, and solving equations with algebraic fractions. Questions are usually 3 to 5 marks and almost always need factorising first.

## Key facts

- Cancel factors, never terms
- Divide: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$
- Difference of two squares: $a^2 - b^2 = (a+b)(a-b)$, often needed before cancelling
- Multiply: $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
- Add: $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$
- A denominator can never equal 0

## Notes

### Simplifying

- You can only cancel **factors** (things that multiply the whole top and the whole bottom), never terms that are added.
- Factorise the numerator and denominator fully, then cancel common brackets.

$$\frac{x^2-9}{x^2+x-6} = \frac{\cancel{(x+3)}\,(x-3)}{\cancel{(x+3)}\,(x-2)} = \frac{x-3}{x-2}$$

- $\frac{x+6}{x}$ cannot be cancelled: the x on top is added to 6, not multiplied.

## Multiplying and dividing

- Multiplying: factorise, cancel any factor that appears on a top and a bottom, then multiply tops together and bottoms together.
- Dividing: keep the first fraction, flip the second, then multiply.

$$\begin{aligned}&\frac{2x}{x+1} \div \frac{4x^2}{x^2-1} \\ &= \frac{2x}{x+1} \times \frac{(x+1)(x-1)}{4x^2} \\ &= \frac{x-1}{2x}\end{aligned}$$

## Adding and subtracting

- Use a common denominator. For two different brackets, their product works.

$$\begin{aligned}&\frac{3}{x+2} + \frac{1}{x-1} \\ &= \frac{3(x-1)+(x+2)}{(x+2)(x-1)} \\ &= \frac{4x-1}{(x+2)(x-1)}\end{aligned}$$

- Put brackets round numerators when subtracting:

$$\begin{aligned}&\frac{5}{x} - \frac{x-2}{3} \\ &= \frac{15-x(x-2)}{3x} \\ &= \frac{15+2x-x^2}{3x}\end{aligned}$$

- You can leave the denominator factorised unless you are asked to expand it.

## Equations with algebraic fractions Grade 8+ only

- Multiply every term (on both sides) by the common denominator to clear the fractions.
- This usually gives a quadratic: rearrange it to equal 0 and solve.
- Check that no solution makes a denominator equal to 0.

## How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

**Simplify fully (factorise and cancel)**

3 marks **Grade 7**

1. Factorise the numerator.
2. Factorise the denominator.
3. Cancel the common bracket and write what is left.

**Example.** Simplify fully $\dfrac{2x^2 - 5x - 3}{x^2 - 9}$

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**Model answer.**

Numerator: $(2x + 1)(x - 3)$ M1

Denominator: $(x + 3)(x - 3)$ M1

$\dfrac{2x + 1}{x + 3}$ A1

**Write as a single fraction**

3 marks **Grade 7**

1. Choose the common denominator (the product of the two denominators if they share no factor).
2. Multiply each numerator by what its denominator was multiplied by, using brackets.
3. Expand and simplify the numerator only.

**Example.** Write $\dfrac{4}{x - 3} - \dfrac{2}{x + 1}$ as a single fraction in its simplest form.

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**Model answer.**

$\dfrac{4(x + 1) - 2(x - 3)}{(x - 3)(x + 1)}$ M1 M1

$= \dfrac{2x + 10}{(x - 3)(x + 1)}$ A1

(or $\dfrac{2(x + 5)}{(x - 3)(x + 1)}$)

### Divide (or multiply) algebraic fractions

3 marks | Grade 7

1. Flip the second fraction and change ÷ to ×.
2. Factorise every numerator and denominator.
3. Cancel common factors, then multiply.

**Example.** Simplify $\dfrac{x^2+4x}{3x-6} \div \dfrac{x+4}{x^2-4}$

**Model answer.**

$\dfrac{x^2+4x}{3x-6} \times \dfrac{x^2-4}{x+4}$ M1

$\dfrac{x(x+4)}{3(x-2)} \times \dfrac{(x+2)(x-2)}{x+4}$ M1

$\dfrac{x(x+2)}{3}$ A1

### Solve an equation with algebraic fractions

4 marks | Grade 8

1. Multiply every term by the common denominator.
2. Expand and rearrange to a quadratic equal to 0.
3. Solve, and check no solution makes a denominator 0.

**Example.** Solve $\dfrac{3}{x+1} + \dfrac{2}{x-1} = 1$

**Model answer.**

$3(x-1)+2(x+1)$
$=(x+1)(x-1)$ M1

$5x-1=x^2-1$ M1

$x^2-5x=0$, so $x(x-5)=0$ M1

$x=0$ or $x=5$ A1

## Don't lose marks

- Cancelling terms, e.g. $\dfrac{x+3}{x+5} = \dfrac{3}{5}$. This is wrong.
- Not using brackets round a numerator, so a minus sign only changes its first term.
- When solving, forgetting to multiply the right-hand side by the common denominator too.
- Not factorising fully, so a common factor is missed.

## More tips

### Memory tricks

- Before cancelling, ask: does this multiply the whole top and the whole bottom?
- Test your answer: put a value such as $x = 5$ (not one that makes a denominator 0) into the original and into your answer; both should give the same number.
- Differences of two squares hide in many of these questions: look for $x^2 - 1$, $x^2 - 4$, $x^2 - 9$, …
- Dividing: Keep, Flip, Change (keep the first fraction, flip the second, change ÷ to ×).

### Exam technique

- 'Simplify fully' nearly always means factorise and then cancel: expect a bracket to cancel.
- Show each factorised expression on its own line; each can earn a method mark.
- In 'show that' questions that end with a quadratic equal to 0, make your last line exactly the given equation.

## What each grade needs

What you need to be able to do, from the first marks up to the top grade.

**Grade 6** — **Simplify algebraic fractions by factorising**

Factorise top and bottom, then cancel common brackets, e.g. $\frac{3x+6}{x^2-4} = \frac{3}{x-2}$

**Grade 6** — **Add fractions with number denominators**

Use a common denominator, e.g. $\frac{x}{3} + \frac{x}{4} = \frac{7x}{12}$

**Grade 7** — **Multiply and divide algebraic fractions**

Factorise, flip the second fraction if dividing, then cancel and multiply.

**Grade 7** — **Add fractions with algebraic denominators**

Multiply the denominators to get a common denominator, e.g. for $\frac{2}{x+1} + \frac{3}{x-2}$ use $(x+1)(x-2)$

**Grade 7** — **Simplify fractions involving harder quadratics**

Factorise quadratics such as $2x^2 - 5x - 3$ before cancelling.

**Grade 8** — **Solve equations with algebraic fractions**

Multiply every term by the common denominator, usually giving a quadratic to solve.

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