Solving quadratic equations

A18 · Algebra · revision notes

Solve simple quadratic equations already equal to zero by factorising, or read their solutions from a graph. Each factor gives a possible value of x; check each value in the original equation.

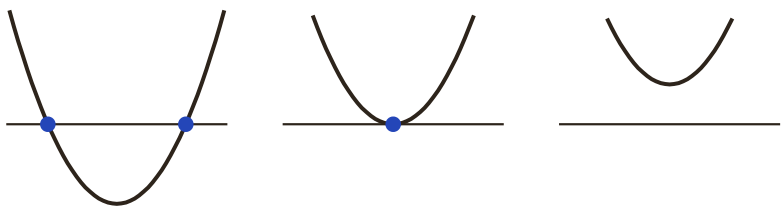
Key facts

- If $pq = 0$, then $p = 0$ or $q = 0$
- Roots are the x-coordinates where the curve crosses the x-axis
- $x^2 = k$ gives $x = \pm\sqrt{k}$
- Check roots by substitution into the original equation.

Notes

Equations equal to zero

- A quadratic equation has an x² term as its highest power. For factorising questions here, the equation is already written with zero on one side.
- In x² + 3x − 10 = 0, find two numbers that multiply to −10 and add to 3: 5 and −2.
- A quadratic usually has two solutions (roots). Some have one repeated root, and some have no solutions.



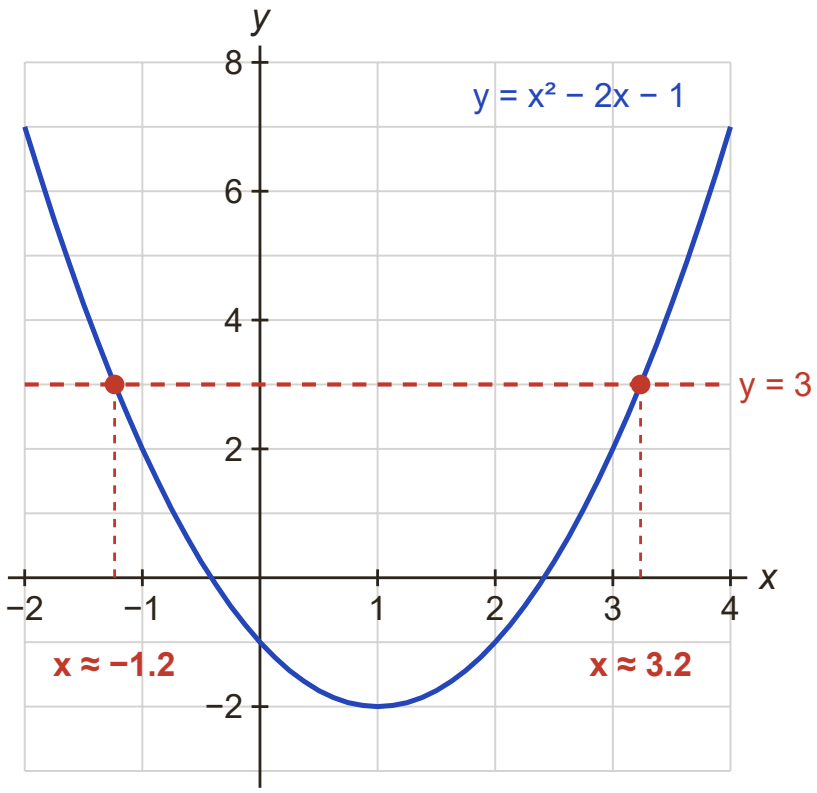
The roots are where the graph meets the x-axis.

Solving by factorising

- Factorise, then use the fact that if two things multiply to give 0, one of them must be 0.
- $x^2 + 3x - 10 = 0$ gives $(x + 5)(x - 2) = 0$, so $x = -5$ or $x = 2$
- Common factor: $x^2 - 7x = 0$ gives $x(x - 7) = 0$, so $x = 0$ or $x = 7$. Don't divide by x, or you lose $x = 0$
- $x^2 = 36$ has two solutions: $x = 6$ or $x = -6$

Using a graph

- The solutions of $x^2 + bx + c = 0$ are the x-coordinates where the graph of $y = x^2 + bx + c$ crosses the x-axis.
- To solve $x^2 - 2x - 1 = 3$ using the graph of $y = x^2 - 2x - 1$, read the x-coordinates where the curve meets the line $y = 3$



Read down from where the curve meets $y = 3$: $x \approx -1.2$ or $x \approx 3.2$

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Solve by factorising

3 marks | Grade 5

1. Check the equation equals 0.
2. Factorise.
3. Set each bracket equal to 0 and solve.

Example. Solve $x^2 + 2x - 24 = 0$

Model answer.

$(x + 6)(x - 4) = 0$ M1

$x = -6$ A1

$x = 4$ A1

Read solutions from stated graph crossings 2 marks Grade 5

1. Locate the points where the curve crosses the x-axis.
2. Read the x-coordinate of each crossing; y is zero there.

Example. The graph of y = x² − x − 6 crosses the x-axis at (−2, 0) and (3, 0). Write the two solutions of x² − x − 6 = 0.

Model answer. x = −2 (B1) or x = 3 (B1). Each crossing has y = 0.

Don't lose marks

- Wrong signs: $(x+6)(x-4)=0$ gives $x=-6$ and $x=4$, not 6 and -4
- Dividing both sides by x and losing the solution $x=0$
- Forgetting the negative square root: $x^2=25$ gives $x=5$ or $x=-5$
- Reading where the graph crosses the y-axis instead of the x-axis: the solutions are the x-coordinates where $y=0$

More tips

Memory tricks

- Check each root in the original equation: $4^2+2\times4-24=0$
- For $x^2+bx+c=0$, the two roots add up to $-b$ and multiply to give c: a quick check.
- If your calculator has an equation solver, use it to check, but always write out the method to get the marks.

Exam technique

- 'Solve' needs values of x: if you only factorise, you only get the method mark.
- Substitute each solution into the equation to check it makes both sides equal.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 4 **Read solutions from a quadratic graph**
The solutions of $x^2+bx+c=0$ are the x-coordinates where $y=x^2+bx+c$ crosses the x-axis.

Grade 5 **Solve simple quadratics by factorising**
Factorise, then set each bracket equal to 0, e.g. $(x-3)(x+7)=0$ gives $x=3$ or $x=-7$

Grade 5 **Solve quadratics with a missing term**
$x^2=49$ gives $x=\pm7$; $x^2-5x=0$ gives $x(x-5)=0$, so $x=0$ or $x=5$

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