Probability and relative frequency

P1–P5 · Probability · revision notes

Probability measures how likely something is, on a scale from 0 to 1. This subtopic covers equally likely outcomes, the rule that probabilities add up to 1, relative frequency from experiments, expected outcomes and frequency trees. It comes up on both tiers, usually in short questions worth 1 to 5 marks.

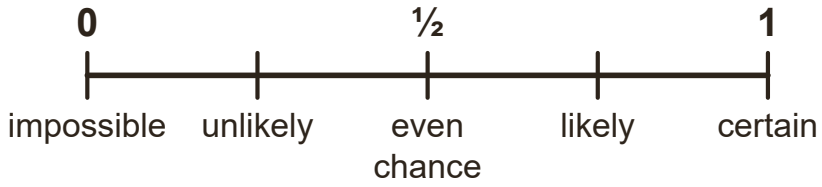
Key facts

- 0 = impossible, 0.5 = even chance, 1 = certain
- The probabilities of all the possible outcomes add up to 1
- Mutually exclusive events: P(A or B) = P(A) + P(B)
- Expected number = probability × number of trials
- Give probabilities as fractions, decimals or percentages, never ratios
- P(event) = number of ways it can happen ÷ total number of equally likely outcomes
- P(not A) = 1 − P(A)
- Relative frequency = frequency ÷ total number of trials
- More trials give a more reliable estimate

Notes

The probability scale

- Probability goes from **0** (impossible) to **1** (certain). An even chance is $\frac{1}{2}$ = 0.5.



Every probability is somewhere on this line.

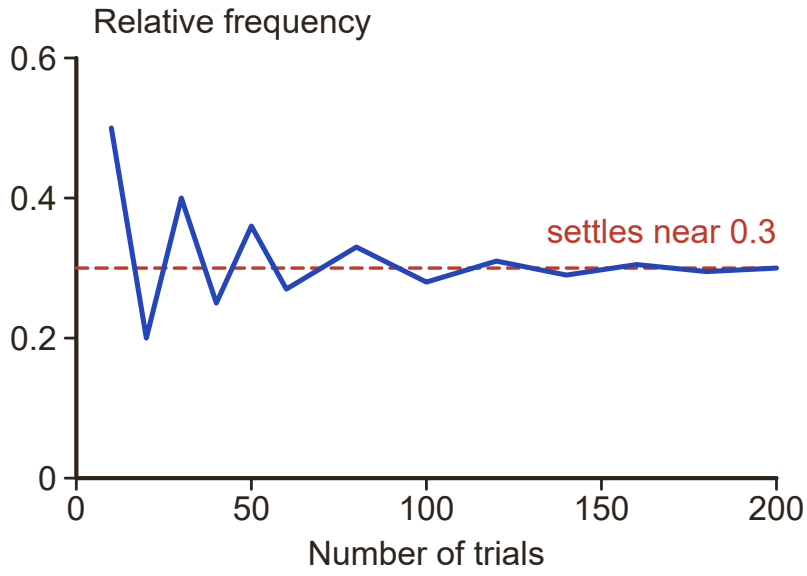
- Write a probability as a fraction, a decimal or a percentage, never as a ratio (1 : 6) or in words (1 in 6).
- When all the outcomes are **equally likely**: P(event) = number of outcomes in the event ÷ total number of outcomes.
- **At random** means every item has the same chance of being picked. A **fair** (unbiased) dice or spinner has equally likely outcomes; a **biased** one does not.

Probabilities add up to 1

- The probabilities of all the different possible outcomes add up to 1.
- So P(not A) = 1 − P(A). If P(rain) = 0.35, then P(not rain) = 1 − 0.35 = 0.65. For a missing value in a table, subtract the others from 1.
- **Mutually exclusive** events cannot happen at the same time, e.g. red and blue on one spin. For these, P(A or B) = P(A) + P(B).
- Events are **exhaustive** if together they cover every possible outcome. The probabilities of an exhaustive set of mutually exclusive events add up to 1.
- If the probabilities contain x, add them, set the total equal to 1 and solve. Then find the probability the question asks for, not just x

Relative frequency

- Relative frequency = frequency of the outcome ÷ total number of trials. It is also called experimental probability.
- Use it to estimate a probability when the outcomes are not equally likely, e.g. for a biased spinner or a drawing pin.
- The more trials, the more reliable the estimate: relative frequency tends to get closer to the true probability as the number of trials increases.



With more trials the relative frequency settles down.

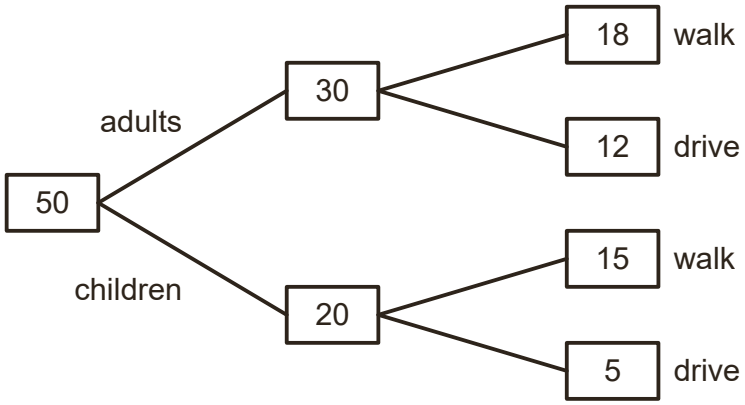
- To combine results, add all the frequencies and all the trials, then divide. Do not average the relative frequencies.
- To judge whether a dice is fair, compare each relative frequency with $\frac{1}{6}$ after a large number of throws.

Expected outcomes

- Expected number = probability × number of trials.
- If P(green) = 0.15 and a spinner is spun 300 times, expect 0.15 × 300 = 45 greens.
- This is only an estimate; the actual number will vary.

Frequency trees

- A frequency tree splits a total into groups, then splits each group again. The two numbers coming out of a box add up to the number in that box.



Each pair of branches adds up to the box it comes from: 18 + 12 = 30.

- For a probability, divide the number you want by the number in the group you are choosing from.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, C = a communication mark.

Find a missing probability from a table 2 marks **Grade 3**

1. Add up the probabilities you are given.
2. Subtract the total from 1.
3. Check your answer is between 0 and 1 and that the whole table now adds up to 1.

Example. A biased spinner can land on red, blue, green or yellow. The table shows the probability that it lands on each colour.

Colour	Red	Blue	Green	Yellow
Probability	0.35	0.2		0.1

Work out the probability that the spinner lands on green.

Model answer.

0.35 + 0.2 + 0.1 = 0.65 (M1)
1 − 0.65 = 0.35 (A1)

Complete a frequency tree and use it

3 to 5 marks | **Grade 4**

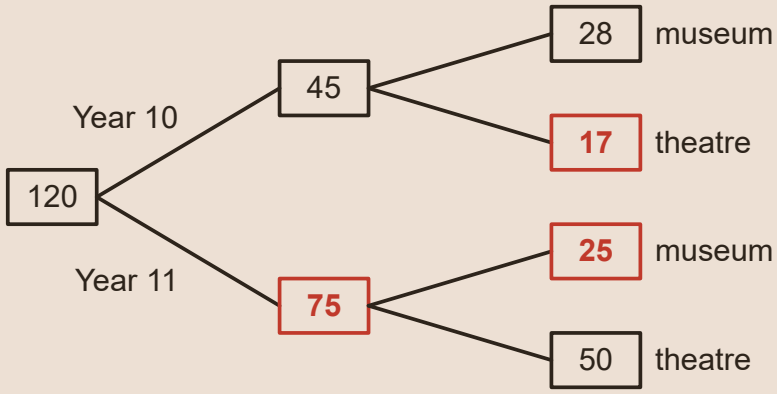
1. Put the total in the first box and fill in the numbers you are told.
2. Subtract to find each missing number: the two branches from a box add up to that box.
3. For a probability, first decide which group you are choosing from (the denominator), then count the ones you want (the numerator).

Example. 120 students each chose one trip: the museum or the theatre. 45 of the students are in Year 10 and the rest are in Year 11. 28 of the Year 10 students chose the museum. 50 of the Year 11 students chose the theatre.

(a) Draw a frequency tree to show this information. (3 marks)

(b) One of the students who chose the museum is picked at random. Work out the probability that this student is in Year 11. (2 marks)

Model answer.



(a) The numbers you work out are in red. Year 11: 120 − 45 = 75 (B1)

Year 10, theatre: 45 − 28 = 17 (B1)

Year 11, museum: 75 − 50 = 25 (B1)

(b) Students who chose the museum: 28 + 25 = 53 (M1)

P(Year 11) = $\frac{25}{53}$ (A1)

Work out a relative frequency and use it to estimate

2 to 3 marks | **Grade 4**

1. Relative frequency = number of times it happened ÷ number of trials.
2. To estimate how many times it will happen, multiply the relative frequency (or probability) by the new number of trials.
3. Check the answer is sensible: it cannot be more than the number of trials.

Example. Priya drops a drawing pin 80 times. It lands point up 28 times.
(a) Work out the relative frequency of the pin landing point up. (1 mark)
(b) Priya is going to drop the pin 500 times. Work out an estimate for the number of times it will land point up. (2 marks)

Model answer.

(a) $\frac{28}{80}$ = 0.35 (B1)

(b) 0.35 × 500 (M1) = 175 (A1)

Probabilities written in terms of x

3 to 4 marks | **Grade 5**

1. Write an equation: all the probabilities added together = 1.
2. Collect like terms and solve for x
3. Substitute x back in to find the probability you were asked for.
4. If you are told how many items there are of one kind, divide that number by its probability to find the total.

Example. A bag contains only red, blue and white counters. A counter is taken at random. The probability that it is red is 0.4. The probability that it is blue is three times the probability that it is white.
(a) Work out the probability that the counter is white. (2 marks)
(b) There are 18 blue counters in the bag. Work out the total number of counters in the bag. (2 marks)

Model answer.

(a) Let P(white) = x, so P(blue) = $3x$.

$x + 3x + 0.4 = 1$, so $4x = 0.6$ (M1)

$x = 0.15$ (A1)

(b) P(blue) = 3 × 0.15 = 0.45, so the total is 18 ÷ 0.45 (M1) = 40 counters (A1)

Is it fair? Give a reason 1 to 2 marks **Grade 5**

1. Work out what you would expect from a fair object, or work out the relative frequency.
2. Compare the two with numbers, e.g. 180 compared with 100.
3. Mention the number of trials: a big difference over many trials suggests bias, but a few trials are not enough evidence.

Example. Amir rolls an ordinary six-sided dice 600 times. It lands on 6 a total of 180 times. Amir says the dice is biased. Is Amir right? Give a reason for your answer.

Model answer.

For a fair dice you would expect $\frac{1}{6} \times 600 = 100$ sixes (M1).

Yes, the dice is probably biased: 180 is far more than 100, and 600 is a large number of rolls (C1).

(Comparing the relative frequency 0.3 with $\frac{1}{6} \approx 0.17$ also scores.)

Don’t lose marks

- Writing a probability as a ratio (1 : 5) or in words ('2 out of 7'). You lose the accuracy mark even if your working is right.
- Dividing by the wrong total, e.g. by all 120 students when the question picks only from those who chose the museum.
- Averaging relative frequencies from groups with different numbers of trials instead of adding the frequencies and the trials.
- Saying a dice is definitely biased, or definitely fair, after only a few throws.
- Solving for x and stopping: the question usually wants a probability, such as $3x$
- Rounding too early, e.g. using 0.17 for $\frac{1}{6}$ part-way through a longer calculation.

More tips

Memory tricks

- Sense check: a probability can never be less than 0 or more than 1. If yours is, look for a slip.
- 'Not' means take away from 1: P(not A) = 1 − P(A).
- Expected number is 'probability times trials', so it can never be more than the number of trials.
- 35% = 0.35 = $\frac{35}{100}$. Pick one form and stick to it within a question.
- On a calculator paper, the S⇔D key switches an answer between a fraction and a decimal, which helps when comparing with $\frac{1}{6}$ = 0.1666…

Exam technique

- 'Work out an estimate' in a probability question means use a relative frequency or probability × trials, so show that calculation.
- In 'give a reason' questions, quote numbers: compare the relative frequency (or expected number) with the theoretical value and say how many trials there were.
- A fraction does not need to be in its simplest form unless the question asks, so do not risk an arithmetic slip just to simplify.
- With tables and frequency trees, read the question twice to find exactly which group is being chosen from.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 1 — **Describe likelihood on a 0 to 1 scale**
Use impossible (0), unlikely, even chance (0.5), likely and certain (1), and mark events on a probability scale.

Grade 2 — **Find a probability for equally likely outcomes**
Divide the number of ways the event can happen by the total number of outcomes, e.g. P(5 on a fair six-sided dice) = $\frac{1}{6}$

Grade 3 — **Use the fact that probabilities sum to 1**
Find a missing probability in a table, or use P(not A) = 1 − P(A).

Grade 3 — **Complete and use a frequency tree**
Split a total into groups and then subgroups, and read off the numbers you need for a probability.

Grade 4 — **Calculate relative frequency from experiment results**
Relative frequency = number of times the outcome happened ÷ total number of trials.

Grade 4 — **Work out an expected number of outcomes**
Multiply the probability by the number of trials, e.g. 0.15 × 200 = 30.

Grade 5 — **Find probabilities written in terms of x**
Add the expressions, set the total equal to 1, solve, then substitute to find the probability asked for.

Grade 5 — **Judge fairness and reliability from experiments**
Compare relative frequency with the theoretical probability, and explain that more trials give a more reliable estimate.

Free notes, Full packs, practice papers and mock exams at chhetriacademy.com. Written for this site; not taken from past papers. Stuck? 1-to-1 tutoring with a free 30-minute trial lesson.