Graphs of functions

A12, A14 · Algebra · revision notes

Recognise, sketch and interpret quadratic, cubic and reciprocal graphs. Use graphs to read approximate solutions and to describe real situations such as distance from a starting point over time.

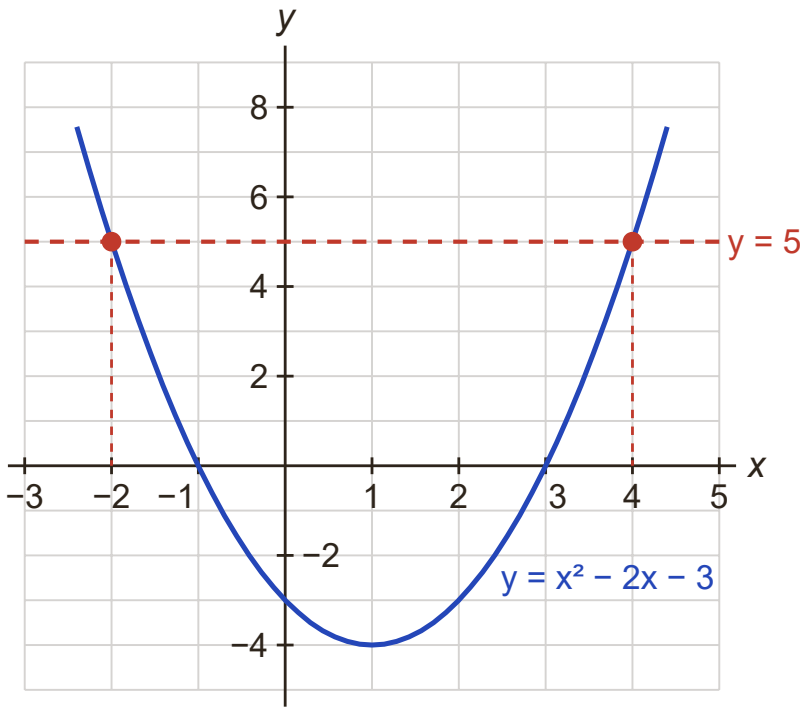
Key facts

- Quadratic: U shape (a positive) or ∩ shape (a negative), symmetrical
- Reciprocal $y = \frac{k}{x}$: two curves that never touch the axes
- Distance–time gradient = speed; velocity–time gradient = acceleration
- Cubic: S shape
- Roots: x-coordinates where the curve crosses the x-axis (y = 0)

Notes

Plotting graphs

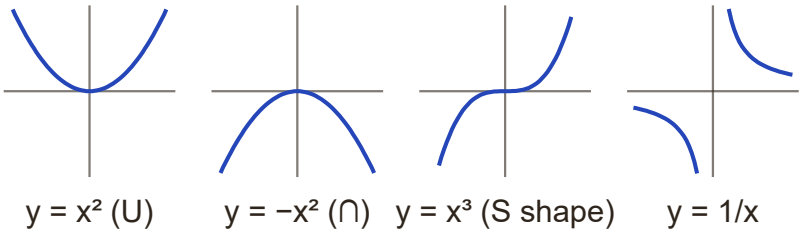
- Make a table of values, substitute each x (negatives in brackets) and plot the points accurately.
- Join the points of a curve with a smooth curve, not straight line segments. Never join across a gap (reciprocal graphs).
- To solve $x^2 - 2x - 3 = 5$ using the graph of $y = x^2 - 2x - 3$, draw $y = 5$ and read the x-coordinates where it crosses the curve.



The line $y = 5$ crosses the curve at $x = -2$ and $x = 4$: the solutions of $x^2 - 2x - 3 = 5$

Shapes to recognise

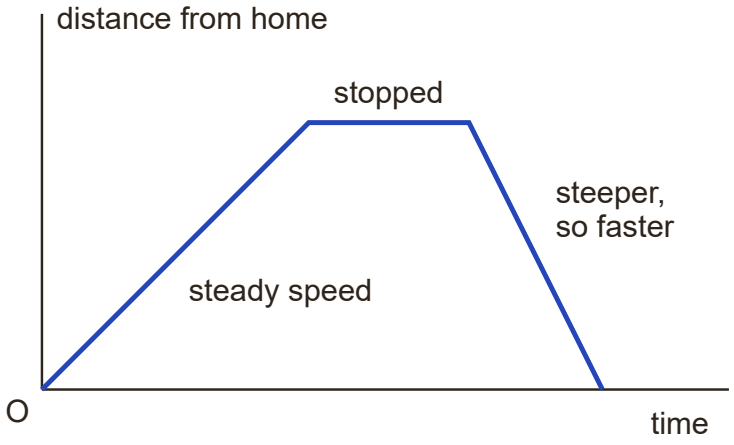
- Quadratic $y = ax^2 + bx + c$: a U shape if a is positive, a ∩ shape if a is negative. It is symmetrical about a vertical line through its turning point.
- Cubic, e.g. $y = x^3$: an S-shaped curve. $y = x^3$ passes through the origin and goes from bottom left to top right; $y = -x^3$ goes from top left to bottom right.
- Reciprocal $y = \frac{1}{x}$: two separate curves in opposite quadrants, getting closer and closer to the axes without touching them. x cannot be 0.



Learn these shapes. The reciprocal graph never touches either axis.

Real-life graphs

- Distance–time graph: the gradient is the speed; a horizontal line means stopped; steeper means faster.



Gradient = speed; flat = stopped; a line back down means returning.

- Velocity–time graph: the gradient is the acceleration.
- Read scales carefully: check what one small square represents on each axis.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Complete a table of values and draw the graph

4 marks | Grade 4

1. Substitute each x value, using brackets for negatives.
2. Plot every point accurately.
3. Join with one smooth curve.

Example. (a) Complete the table of values for $y = x^2 - 2x - 3$.

x	−2	−1	0	1	2	3	4
y			−3			0	

(b) Draw the graph of $y = x^2 - 2x - 3$ for values of x from −2 to 4.

Model answer.

(a)

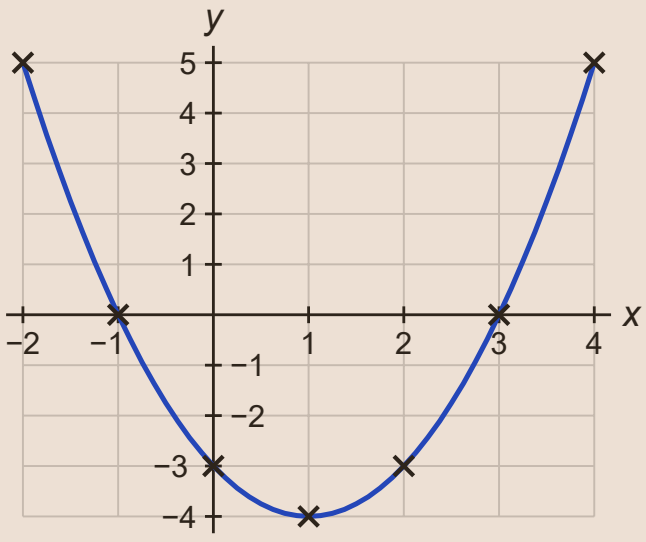
x	−2	−1	0	1	2	3	4
y	5	0	−3	−4	−3	0	5

B2

(B1 for at least 3 of the 5 missing values correct)

(b) At least 6 points plotted correctly M1

A smooth U-shaped curve through all 7 points A1

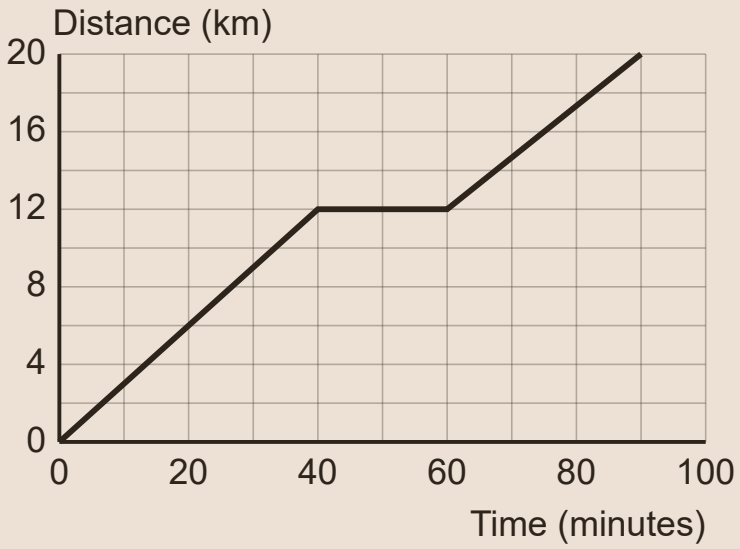


Interpret a distance–time graph

3 marks | Grade 4

1. A horizontal section means no movement.
2. Speed = distance ÷ time, using the right units.

Example. The distance–time graph shows a cyclist's journey.



(a) Describe the cyclist's journey between 40 and 60 minutes.
(b) Work out the cyclist's speed, in km/h, for the first 40 minutes.

Model answer.

(a) The graph is horizontal, so the cyclist is stationary (stopped) B1

(b) 12 km in 40 minutes: $12 \div \frac{40}{60}$ M1

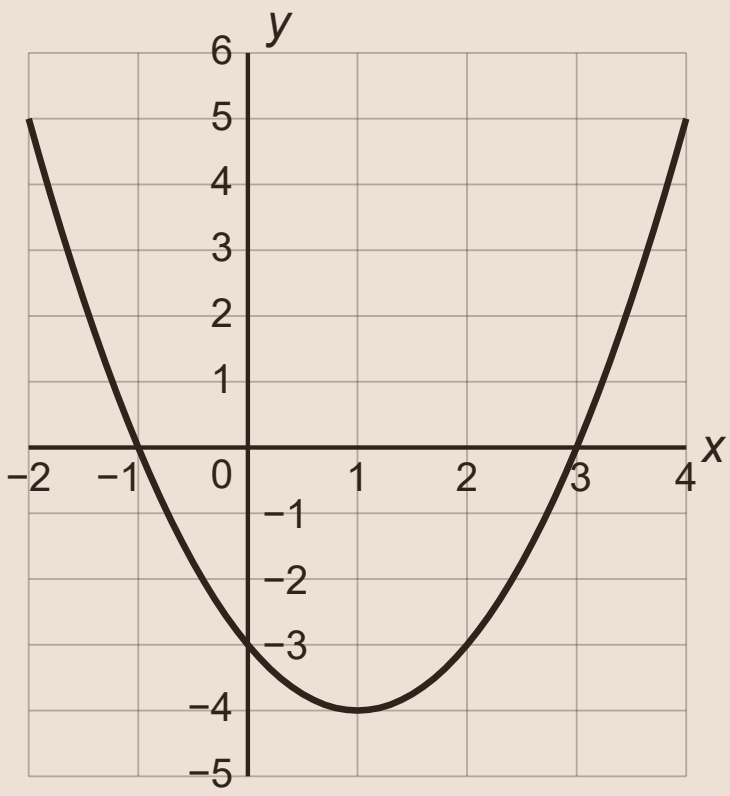
= 18 km/h A1

Use a graph to solve an equation

2 marks **Grade 5**

1. Rearrange so one side is the equation of the graph you have.
2. Draw the line for the other side.
3. Read off the x-coordinates of the crossing points.

Example. Here is the graph of $y = x^2 - 2x - 3$.



Use the graph to estimate the solutions of $x^2 - 2x - 3 = 2$

Model answer.

Draw $y = 2$ and read off where it meets the curve M1

$x \approx -1.4$ and $x \approx 3.4$ A1

(a small range is accepted)

Match equations to graphs 1 mark each **Grade 5**

1. Identify the type of each equation (quadratic, cubic, reciprocal, linear).
2. Use the sign of the leading term and the y-intercept to decide.

Example. Match each equation to its description.

(a) $y = x^3 - 2$ (b) $y = \dfrac{4}{x}$ (c) $y = 3 - x^2$

P: a ∩-shaped curve crossing the y-axis at 3. Q: two separate curves, in the first and third quadrants. R: an S-shaped curve crossing the y-axis at −2.

Model answer.

(a) R B1
(b) Q B1
(c) P B1

Don't lose marks

- Substituting negatives without brackets: for $x = -2$,

$$\begin{aligned} x^2 - 2x &= (-2)^2 - 2 \times (-2) \\ &= 4 + 4 \\ &= 8 \end{aligned}$$

- Joining points with straight lines instead of a smooth curve.
- Drawing a flat bottom on a U-shaped curve: the turning point is a smooth point.
- Joining the two parts of a reciprocal graph across $x = 0$

More tips

Memory tricks

- Quadratic tables are symmetrical, e.g. 5, 0, −3, −4, −3, 0, 5. A value that breaks the pattern is probably wrong.
- If your calculator has a table mode, use it to fill in tables of values, and check one value by hand.

Exam technique

- Draw curves in one smooth stroke with a sharp pencil, and plot points with small crosses.
- When reading solutions from a graph, give answers as accurately as the scale allows; examiners accept a small range.
- Show the line you drew (e.g. $y = 2$) on the graph: it is part of your method.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade 3 **Complete a table of values and plot**

Substitute each x (with brackets for negatives), plot the points and join them with a smooth curve.

Grade 4

Recognise the shapes of standard graphs

Linear, quadratic (U or ∩ shape), cubic (S shape) and reciprocal $y = \frac{1}{x}$ (two separate curves).

Grade 4

Interpret distance–time graphs

The gradient is the speed and a horizontal section means the object is stationary.

Grade 5

Read roots and turning points from graphs

Roots are where the curve crosses the x-axis; the turning point is the minimum or maximum point.

Grade 5

Solve equations using a graph

Draw a horizontal line such as $y = 3$ and read the x-coordinates where it meets the curve.

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