

Grade A* checklist

37 skills. Tick each one when you can do it. For grade A* you need the skills for the grades below it too.

Pure mathematics

- ☐ **Construct a full proof in an unfamiliar context** Choose a method, state every assumption, justify each step and finish with a precise conclusion. (Proof by deduction, exhaustion and counter-example)

- ☐ **Adapt the method to unfamiliar statements** e.g. show there are no integers with $x^2 - y^2 = 10$, or that $\sqrt[3]{2}$ is irrational. (Proof by contradiction)

- ☐ **Expand a fraction using partial fractions first** Split into partial fractions, expand each one, add the series, and give the narrower of the validity ranges. (Binomial expansion)

- ☐ **Solve problems combining sums, terms and convergence** Form equations from several conditions, solve for r and reject any value with $|r| \geq 1$ if a sum to infinity is involved. (Geometric sequences and series)

- ☐ **Criticise or refine a sequence model** Explain in context why the model may fail and how it could be improved. (Modelling with sequences and series)

- ☐ **Use approximations in a geometry problem** Combine them with the cosine rule or with area formulae when an angle in a diagram is small. (Small angle approximations)

- ☐ **Eliminate θ between two equations** e.g. from $x = 2 \operatorname{cosec} \theta$ and $y = 5 \cot \theta$, use $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$. (Pythagorean identities)

- ☐ **Derive new identities from compound angles** e.g. write $\sin 3x$ in terms of $\sin x$ by expanding $\sin(2x + x)$. (Compound and double angles; $R \cos(\theta \pm \alpha)$)

- ☐ **Factorise instead of dividing to keep solutions** e.g. $\sin 2x = \sin x$ gives $\sin x(2 \cos x - 1) = 0$, so the solutions of $\sin x = 0$ are not lost. (Solving trigonometric equations)

- ☐ **Prove identities using compound angles** e.g. prove that $\sin 3x \equiv 3 \sin x - 4 \sin^3 x$ by expanding $\sin(2x + x)$. (Proving trigonometric identities)

- ☐ **Use the R form to analyse a model** Combine $p \cos t + q \sin t$ into one term to find the maximum, the minimum and when they occur. (Trigonometry in context)

- ☐ **Refine a model and describe long-term behaviour** E.g. $\theta = 20 + 60e^{-0.1t}$ tends to 20 as $t \rightarrow \infty$: the temperature of the room. (Exponential growth and decay models)

- ☐ **Differentiate $\sin x$ or $\cos x$ from first principles** Use the addition formulae with $\frac{\sin h}{h} \rightarrow 1$ and $\frac{\cos h - 1}{h} \rightarrow 0$ as $h \rightarrow 0$. (First principles, second derivatives and concavity)

- ☐ **Find where tangents are parallel to an axis** Horizontal: $\frac{dy}{dx} = 0$; vertical: the denominator of $\frac{dy}{dx}$ is 0. (Implicit and parametric differentiation)

- ☐ **Simplify parametric area integrals with identities** Use trigonometric identities to turn $y \frac{dx}{dt}$ into something you can integrate, e.g. $\sin 2t \sec^2 t = 2 \tan t$. (Areas between curves and parametric areas)

- ☐ **Use trigonometric substitutions with identities** For example $x = 2 \sin \theta$ turns $\sqrt{4 - x^2}$ into $2 \cos \theta$; then use a double angle formula. (Integration by substitution)

- ☐ **Solve integrals where the original reappears** For $\int e^x \sin x \, dx$, two applications bring back the original integral I ; rearrange to find I . (Integration by parts)

- ☐ **Use partial fractions inside longer problems** Partial fractions often appear after a substitution or when separating the variables in a differential equation. (Integration using partial fractions)

- ☐ **Separate when integrals need further methods** The y -side may need partial fractions, e.g. $\int \frac{1}{y^2-1} \, dy$, and the x -side may need parts or substitution. (Separable differential equations)

- ☐ **Solve and interpret logistic-type models** Use partial fractions to solve $\frac{dP}{dt} = kP(M - P)$, then interpret the limiting value M . (Differential equations in context: interpreting solutions)

- ☐ **Derive the formula from the tangent** Write the tangent $y - f(x_n) = f'(x_n)(x - x_n)$, put $y = 0$ and solve for x . (The Newton–Raphson method)

- ☐ **Combine calculus with numerical methods** For example, set a derivative equal to zero to find a maximum, then solve the resulting equation numerically. (Numerical methods in context)

Statistics

- ☐ **Reason with knowledge of the large data set** Use facts about the stations, dates, variables and abbreviations to explain patterns or problems in a sample. (Sampling methods and the large data set)

- ☐ **Link a histogram's shape to a probability model** Recognise that a roughly symmetrical, bell-shaped histogram suggests a normal model, and treat areas as probabilities. (Histograms, box plots and cumulative frequency)

- ☐ **Find model constants from a logarithmic regression line** Match the line to $\log y = \log a + n \log x$ or $\log y = \log k + x \log b$ and undo the logs. (Correlation and regression)

- ☐ **Find unknown data values from summary statistics** Form and solve equations (sometimes quadratics) using Σx , Σx^2 , the mean or the variance. (Measures of location and spread)

- ☐ **Evaluate how cleaning affects the conclusions** Explain which statistics change a lot (mean, standard deviation, range, r) and which barely change (median, IQR). (Outliers and cleaning data)

- ☐ **Solve three-event Venn problems with conditions** Combine independence, mutual exclusivity and the total of 1 in a three-circle Venn diagram. (Mutually exclusive and independent events; Venn and tree diagrams)

- ☐ **Handle multi-step conditional problems in context** Work through several conditions in an unfamiliar context, organising them in a tree, table or Venn diagram. (Conditional probability)

- ☐ **Explain how changed assumptions affect probabilities** Say whether a more realistic model gives a higher or lower probability, and why. (Modelling with probability)

- ☐ **Combine normal and binomial models in multi-stage problems** E.g. find $p = P(X > a)$ from a normal model, then use $B(n, p)$. (The normal distribution)

- ☐ **Judge a model against evidence** E.g. show that a normal model gives impossible values, or compare an approximation with the exact probability. (The normal approximation to the binomial; choosing a distribution)

- ☐ **Find a sample size that gives significance** E.g. find the smallest n for which $X = 0$ is in the critical region. (Binomial hypothesis tests)

- ☐ **Find the sample size needed for significance** Rearrange the standardised inequality to find the smallest n . (Tests for correlation and for the mean of a normal distribution)

Mechanics

- ☐ **Find angles of projection through a point** Use the equation of the path with $\sec^2 \alpha = 1 + \tan^2 \alpha$ to get a quadratic in $\tan \alpha$. (Projectiles)

 - ☐ **Find the range of forces for equilibrium** Use both limiting cases: about to slide up (friction down the slope) and about to slide down (friction up the slope). (Friction)

 - ☐ **Solve rod-on-peg and rough-wall problems** A smooth peg's reaction is perpendicular to the rod; at every rough contact that is about to slip, $F = \mu R$. (Ladders, tilting and limiting equilibrium)
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