

Grade A checklist

112 skills. Tick each one when you can do it. For grade A you need the skills for the grades below it too.

Pure mathematics

- ☐ **Use remainders to cover every integer** Write n as $3k$, $3k + 1$ or $3k + 2$ (or similar) and prove the result for each form. (Proof by deduction, exhaustion and counter-example)

- ☐ **Prove that the square root of 2 is irrational** Assume $\sqrt{2} = \frac{a}{b}$ in lowest terms, then show a and b are both even. (Proof by contradiction)

- ☐ **Prove that there are infinitely many primes** Assume a finite list of all primes, multiply them together and add 1, and show this has a prime factor not in the list. (Proof by contradiction)

- ☐ **Simplify expressions with several different bases** e.g. $\frac{2^{x+1} \times 4^x}{8^{x-1}} = 2^{x+1+2x-3x+3} = 2^4 = 16$. (Laws of indices)

- ☐ **Solve linear equations with surd coefficients** Collect the x terms, factorise, divide and rationalise to give an exact answer. (Surd)

- ☐ **Use the discriminant for tangency conditions** Substitute a line into a curve and use $b^2 - 4ac = 0$ for the line to be a tangent. (Quadratics and the discriminant)

- ☐ **Handle curves with xy and y -squared terms** Substitute carefully, expanding brackets like $(3 - x)^2$ and $x(3 - x)$ in full. (Simultaneous equations)

- ☐ **Solve inequalities with x in the denominator** Multiply both sides by the square of the denominator, which is positive, then solve. (Inequalities)

- ☐ **Find unknown coefficients from given factors** Each known factor gives an equation, e.g. $f(-1) = 0$; solve the equations simultaneously. (Polynomials, algebraic division and the factor theorem)

- ☐ **Find values of k for a number of intersections** Use a sketch (and sometimes the discriminant) to find where a line or curve meets a graph a given number of times. (Curve sketching and the modulus function)

- ☐ **Restrict domains and find ranges of composites** Restrict a many-to-one function to one side of its turning point so it has an inverse; find the range of fg by applying f to the range of g . (Composite and inverse functions)

- ☐ **Apply combinations of transformations in order** e.g. $y = 2f(x - 1) + 3$: move right 1, stretch vertically by factor 2, then move up 3. (Transformations of graphs)

- ☐ **Split an improper algebraic fraction** If the numerator's degree is at least the denominator's, include a polynomial part, e.g. $A + \frac{B}{x-1} + \frac{C}{x+2}$. (Partial fractions)

- ☐ **Use partial fractions to integrate or expand** Each simple fraction can be integrated (giving a $\ln$ term) or expanded with the binomial series. (Partial fractions)

- ☐ **Suggest and justify refinements to a model** e.g. include air resistance, or use a function that levels off instead of growing forever. (Modelling with functions)

- ☐ **Solve multi-step problems such as triangle areas** Find vertices from intercepts and intersections, then use $\frac{1}{2} \times \text{base} \times \text{height}$. (Straight lines)

- ☐ **Use the discriminant for tangency conditions** The line is a tangent when the substituted quadratic has $b^2 - 4ac = 0$, and misses when $b^2 - 4ac < 0$. (Circles)
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- ☐ **Find the circle through three points** The centre is where the perpendicular bisectors of two chords meet. (Circles)
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- ☐ **Convert harder parametric equations with domains** e.g. using $1 + \tan^2 t = \sec^2 t$, or with a restricted parameter, and state $y = f(x)$ with its domain. (Parametric equations)
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- ☐ **Model circular motion with trigonometric parameters** e.g. $x = r \sin(kt)$, $y = h - r \cos(kt)$ for a point on a wheel turning at a steady rate. (Parametric equations in modelling)
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- ☐ **Criticise and refine a parametric model** e.g. air resistance, spin and wind are ignored, so the real path is not exactly a parabola. (Parametric equations in modelling)
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- ☐ **Use an expansion to approximate a value** Choose a value of x inside the valid range that turns the expression into the number you want, then substitute it. (Binomial expansion)
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- ☐ **Find the order and sum of periodic sequences** Find the length of the cycle, then add many terms using complete cycles plus the leftover terms. (Sequences and recurrence relations)
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- ☐ **Prove that a sequence is increasing** Show that $u_{n+1} - u_n > 0$ for every positive integer n , not just for a few terms. (Sequences and recurrence relations)
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- ☐ **Change the limits of a sum** Use $\sum_{r=k}^n f(r) = \sum_{r=1}^n f(r) - \sum_{r=1}^{k-1} f(r)$, or find the first term, last term and number of terms directly. (Sigma notation)
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- ☐ **Find an unknown limit from a given sum** Write the sum in terms of n , set it equal to the value and solve, keeping only a positive integer n . (Sigma notation)
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- ☐ **Prove the formula for the sum** Write the series forwards and backwards, add the two lines, and divide by 2. (Arithmetic sequences and series)
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- ☐ **Use logarithms to find the least n** Rearrange to get r^n on its own, take logs, and take care with the direction of the inequality. (Geometric sequences and series)
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- ☐ **Prove the formula for the sum** Write S_n , multiply by r , subtract and rearrange. (Geometric sequences and series)
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- ☐ **Model regular savings with compound interest** Each payment earns interest for a different number of years, so the total is a geometric series. (Modelling with sequences and series)
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- ☐ **Form equations from sector and triangle facts** Combine the formulae to show that θ satisfies an equation such as $3\theta = 4 \sin \theta$. (Sine and cosine rules, radians, arcs and sectors)
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- ☐ **Find the approximate value of a fraction** Approximate the top and bottom separately, then cancel the powers of θ , e.g. to get $\frac{9}{4}$. (Small angle approximations)
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- ☐ **Find constants from a graph's features** Use the maximum, the minimum and the position of a turning point to find the unknown constants. (Graphs of $\sin$, $\cos$ and $\tan$)
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- ☐ **Simplify expressions involving inverse trig functions** e.g. show that $\tan(\arcsin x) = \frac{x}{\sqrt{1-x^2}}$, using an identity or a right-angled triangle. ($\sec$, cosec , $\cot$ and inverse trig functions)
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- ☐ **Solve equations with $\sec$ and $\tan$ together** Use $\tan^2 \theta = \sec^2 \theta - 1$ to get a quadratic in $\sec \theta$, then change to $\cos \theta$. (Pythagorean identities)
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- ☐ **Solve equations using the R form** Rewrite as $R \cos(\theta - \alpha) = c$, adjust the interval for $\theta - \alpha$, solve and convert back. (Compound and double angles; $R \cos(\theta \pm \alpha)$)

- ☐ **Find maximum and minimum values using R form** The maximum of $R \cos(\theta - \alpha)$ is R and the minimum is $-R$; use this in fractions and in models. (Compound and double angles; $R \cos(\theta \pm \alpha)$)

- ☐ **Use identities before solving** Use a Pythagorean or double angle identity to get an equation in one trig function. (Solving trigonometric equations)

- ☐ **Combine fractions to prove an identity** Use a common denominator, then simplify using $\sin^2 x + \cos^2 x = 1$ and factorising. (Proving trigonometric identities)

- ☐ **Use a proved identity to solve an equation** Replace the complicated expression with the simpler side of the identity, then solve in the interval. (Proving trigonometric identities)

- ☐ **Solve a trig model for given values** Set the model equal to the value, solve the trig equation in the interval and interpret the answers as times. (Trigonometry in context)

- ☐ **Use exponential graphs to count solutions** Sketch both sides of an equation such as $e^x = 3 - x$ and explain why the graphs meet exactly once. (Exponential functions and graphs)

- ☐ **Explain why exponential models suit many situations** For $y = Ae^{kx}$, $\frac{dy}{dx} = ky$: the rate of change is proportional to the amount present. (The function e^x and its gradient)

- ☐ **Find inverses of exponential and log functions** E.g. for $f(x) = 3 + e^{2x}$, $f^{-1}(x) = \frac{1}{2} \ln(x - 3)$ with domain $x > 3$. (Logarithms)

- ☐ **Solve log equations leading to quadratics** Form and solve a quadratic, then check every root in the original equation, rejecting any that make a log's argument zero or negative. (Laws of logarithms)

- ☐ **Solve disguised quadratics in a^x** Write $9^x = (3^x)^2$, substitute $u = 3^x$, solve, then reject any $u \leq 0$. (Solving exponential equations)

- ☐ **Solve equations with e^x and e^{-x}** Multiply every term by e^x to get a quadratic in e^x . (Solving exponential equations)

- ☐ **Interpret the constants and criticise the model** E.g. k is the value when $x = 0$ and b is the multiplier per unit of x ; predictions far outside the data are unreliable. (Using log graphs to estimate parameters)

- ☐ **Find and interpret a rate of change** Differentiate the model and substitute; say whether the quantity is increasing or decreasing, with units. (Exponential growth and decay models)

- ☐ **Find and justify points of inflection** Solve $f''(x) = 0$ and show that $f''(x)$ changes sign there. (First principles, second derivatives and concavity)

- ☐ **Simplify an expression before differentiating** Split fractions, expand brackets and use log laws, e.g. $\ln x^3 = 3 \ln x$. (Differentiating standard functions)

- ☐ **Solve optimisation problems** Form an expression in one variable, differentiate, solve, then justify the maximum or minimum. (Tangents, normals and stationary points)

- ☐ **Prove the derivatives of sec, cosec and cot** Write e.g. $\sec x = (\cos x)^{-1}$ and use the chain rule or the quotient rule. (Product, quotient and chain rules)

- ☐ **Combine rules and factorise to find stationary points** Factorise answers like $2x(1 - x)e^{-2x}$ so that $\frac{dy}{dx} = 0$ is easy to solve. (Product, quotient and chain rules)

- ☐ **Use similar triangles to link variables** In a cone of water, write r in terms of h before finding $\frac{dV}{dh}$.
(Connected rates of change)
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- ☐ **Use chains of three or more rates** E.g. $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dA} \times \frac{dA}{dt}$. (Connected rates of change)
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- ☐ **Find stationary points on implicit curves** Set the numerator of $\frac{dy}{dx}$ equal to 0, then solve together with the curve's equation. (Implicit and parametric differentiation)
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- ☐ **Form equations with inflow and outflow** Net rate = rate in – rate out, e.g. $\frac{dV}{dt} = 40 - kV$. (Constructing differential equations)
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- ☐ **Use the chain rule to change the variable** Turn a rate of change of volume into a rate of change of depth with $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh}$. (Constructing differential equations)
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- ☐ **Find an unknown limit or constant** Set a definite integral equal to its given value, solve the equation and reject any value that breaks a given condition. (The fundamental theorem of calculus)
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- ☐ **Use trigonometric identities before integrating** Replace $\sin^2 x$, $\cos^2 x$ or $\tan^2 x$ using the double angle formulae or $\tan^2 x = \sec^2 x - 1$, then integrate. (Integrating standard functions)
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- ☐ **Find exact areas involving e, ln or π** Integrate standard functions, then simplify using laws of logarithms and exact trigonometric values. (Definite integrals and areas under curves)
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- ☐ **Find the area under a parametric curve** Use $\int y \frac{dx}{dt} dt$ and change the x -limits into t -limits. (Areas between curves and parametric areas)
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- ☐ **Find upper and lower bounds with rectangles** Use the greatest and least heights on each strip; the true area lies between the two totals. (Integration as the limit of a sum)
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- ☐ **Explain why narrower strips give better bounds** As $\delta x \rightarrow 0$ the upper and lower sums both approach the value of the integral. (Integration as the limit of a sum)
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- ☐ **Choose a suitable substitution yourself** Let u be the awkward part: the inside of a root, a bracket or an exponent, or the denominator. (Integration by substitution)
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- ☐ **Apply integration by parts twice** For $x^2 e^x$ or $x^2 \sin x$, the first application leaves x times a function, so apply the formula again. (Integration by parts)
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- ☐ **Divide first for improper fractions** If the degree of the numerator is at least that of the denominator, divide to get a polynomial plus a proper fraction. (Integration using partial fractions)
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- ☐ **Factorise before separating** For example $\frac{dy}{dx} = xy^2 + y^2 = y^2(x + 1)$: take out the common factor first. (Separable differential equations)
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- ☐ **Find the long-term (limiting) value** As $t \rightarrow \infty$, $e^{-kt} \rightarrow 0$ for $k > 0$; the limit is also where the rate of change is zero. (Differential equations in context: interpreting solutions)
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- ☐ **Evaluate a model and state its limitations** Compare predictions with data, say why the model may not hold, and suggest a sensible refinement. (Differential equations in context: interpreting solutions)
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- ☐ **Count roots using a sketch** Sketch $y = g(x)$ and $y = h(x)$: the number of intersections is the number of roots of $g(x) = h(x)$. (Locating roots by change of sign)
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- ☐ **Use the gradient of g to predict behaviour** Near the root, $|g'(x)| < 1$ gives convergence: a staircase if g' is positive, a cobweb if it is negative. (Iteration and staircase/cobweb diagrams)
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- ☐ **Explain why a starting value fails** If $f'(x_0) = 0$ the tangent is horizontal and never meets the axis; near a stationary point, x_1 can land far away. (The Newton–Raphson method)

- ☐ **Use the estimate for a related integral** For example $\int_a^b (k + f(x)) \, dx = k(b - a) + \int_a^b f(x) \, dx$.
(Numerical integration: the trapezium rule)

- ☐ **Compare an estimate with the exact value** Percentage error = $\frac{|\text{estimate} - \text{exact}|}{\text{exact}} \times 100\%$. (Numerical integration: the trapezium rule)

- ☐ **Judge the reliability of an estimate** Consider over- or underestimates, how often the data were recorded, and whether the model holds. (Numerical methods in context)

- ☐ **Find the angle between a vector and an axis** In 3D, $\cos \theta_x = \frac{x}{|a|}$, and similarly for the y - and z -axes.
(Magnitude and direction)

- ☐ **Find unknowns from a given magnitude** Set the sum of the squared components equal to the magnitude squared and solve, keeping both roots unless one is ruled out. (Magnitude and direction)

- ☐ **Find where lines meet by comparing routes** Write the same vector in two ways with unknown scalars, and compare the coefficients of two non-parallel vectors. (Vector arithmetic and geometry)

- ☐ **Find angles in a triangle from coordinates** Find all three side lengths, then use the cosine rule. (Position vectors and distances)

- ☐ **Find unknown coordinates from a distance** Form an equation from the distance squared and solve it, usually a quadratic. (Position vectors and distances)

- ☐ **Solve for unknown times or constants** For example, 'due north' means the $\mathbf{i}$ -components of the two position vectors are equal. (Vector problems)

Statistics

- ☐ **Critique a sampling method and suggest improvements** Identify who is left out or over-represented, and why, and propose a random method that fixes it. (Sampling methods and the large data set)

- ☐ **Compare distributions using location and spread in context** Compare a median (or mean) and an IQR (or standard deviation), with values and in the context of the data. (Histograms, box plots and cumulative frequency)

- ☐ **Linearise $y = ax^n$ or $y = kb^x$ using logarithms** Take logs of both sides: $\log y$ is linear in $\log x$ for a power model, and in x for an exponential model. (Correlation and regression)

- ☐ **Combine data sets using Σx and Σx^2** Rebuild $\Sigma x = n\bar{x}$ and $\Sigma x^2 = n(\sigma^2 + \bar{x}^2)$ for each set, add them, then recalculate. (Measures of location and spread)

- ☐ **Clean data and recalculate summary statistics** Take errors and missing values out of n , Σx and Σx^2 , then recalculate. (Outliers and cleaning data)

- ☐ **Find unknown probabilities by forming equations** Set up and solve equations from the total probability of 1 and the conditions given. (Mutually exclusive and independent events; Venn and tree diagrams)

- ☐ **Solve conditional problems with unknown probabilities** Use $P(A \cap B) = P(A)P(B | A)$ and the addition rule to form and solve equations. (Conditional probability)

- ☐ **Refine a model with more realistic assumptions** Use relative frequencies or conditional probabilities instead of equal or constant probabilities. (Modelling with probability)

- ☐ **Find n or p from a probability condition** E.g. solve $1 - (1 - p)^n > 0.9$ using logarithms. (Discrete distributions, including the discrete uniform and the binomial)

- ☐ **Find μ and σ using simultaneous equations** Form two standardised equations and solve them together. (The normal distribution)

- ☐ **Choose and justify a distribution for a context** Decide between discrete uniform, binomial and normal, and explain when none of them fits. (The normal approximation to the binomial; choosing a distribution)

- ☐ **Interpret test results and their limitations** Explain that not rejecting H_0 does not prove it, and why the actual and stated significance levels differ. (The language of hypothesis testing)

- ☐ **Carry out a complete test in context** Hypotheses, distribution, probability or critical region, decision and conclusion in context. (Binomial hypothesis tests)

- ☐ **Find a critical region for the sample mean** Use $\mu_0 \pm z \frac{\sigma}{\sqrt{n}}$ with the right z value. (Tests for correlation and for the mean of a normal distribution)

Mechanics

- ☐ **Criticise a model and suggest a refinement** Name one specific effect the model ignores, such as air resistance, and say how including it would change the answer. (SI units and modelling assumptions)

- ☐ **Solve problems with two moving objects** Write both position vectors in terms of t , then equate components or use the vector between them. (Displacement, velocity and acceleration)

- ☐ **Handle graphs with negative velocity** Area below the time axis is negative displacement; for the distance, add the sizes of all the areas. (Kinematics graphs)

- ☐ **Use suvat in vector form** Use $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ and $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with $\mathbf{i}$ and $\mathbf{j}$ components. (Constant acceleration (suvat))

- ☐ **Use calculus with exponential or trigonometric functions** Apply the Pure rules, e.g. $v = 4e^{-0.5t}$ gives $a = -2e^{-0.5t}$. (Variable acceleration using calculus)

- ☐ **Find the distance when the direction changes** Split the time interval where $v = 0$, find each displacement and add their magnitudes. (Variable acceleration using calculus)

- ☐ **Derive general projectile formulae** Find the time of flight, range and greatest height in terms of U , α and g . (Projectiles)

- ☐ **Solve equilibrium problems with strings at angles** Resolve horizontally and vertically and solve the simultaneous equations for the tensions. (Forces, resultants and equilibrium of a particle)

- ☐ **Solve problems where the forces change** When a force is removed or changes, find the new acceleration and link the stages by the velocity. (Newton's second law and vectors)

- ☐ **Handle motion starting above the ground** Use a negative displacement for a point below the start, solve the quadratic and reject the negative root. (Weight and motion under gravity)

- ☐ **Analyse motion after a string goes slack** Once a particle hits the ground the tension is zero, and the other particle continues with the speed it had. (Newton's third law, connected particles and pulleys)

- ☐ **Solve multi-stage problems on inclined planes** Find the new acceleration when a force is removed and link the stages with suvat. (Resolving forces and inclined planes)

- ☐ **Decide whether a particle moves** Compare the friction needed for equilibrium with the greatest friction available, μR . (Friction)

 - ☐ **Solve hinged rod problems with non-parallel forces** Take moments about the hinge, then resolve to find the components of the force at the hinge. (Moments, beams and rods in equilibrium)

 - ☐ **Solve ladder problems with a person climbing** Put the person at distance x up the ladder and find the value of x at which it is about to slip. (Ladders, tilting and limiting equilibrium)
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