# Conditional probability

S3.2 · Statistics · revision notes

Finding the probability of an event given that another event has happened, using two-way tables, Venn diagrams, tree diagrams and the formula $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$.

Typical questions are ‘given that’ probabilities, reversed tree diagrams and independence checks.

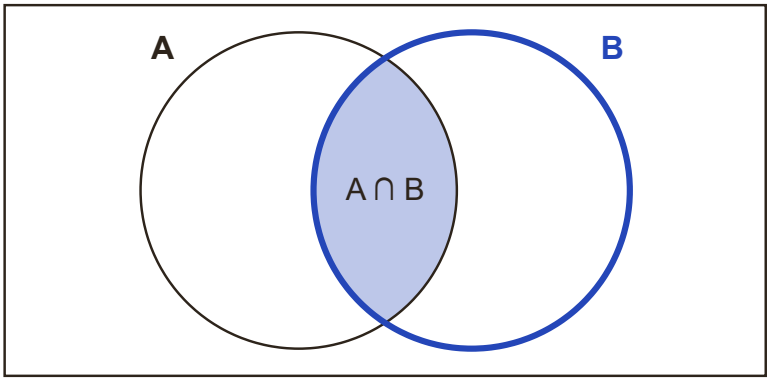
### Key facts

- $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$
- $P(A \cap B) = P(A) \times P(B \mid A)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(B) = P(A)P(B \mid A) + P(A')P(B \mid A')$
- Independent if and only if $P(A \mid B) = P(A)$
- Two-way table: divide by the total of the ‘given’ row or column

## Notes

### What conditional probability means

- $P(A \mid B)$ is the probability that A happens **given that** B has happened.
- Knowing that B has happened narrows the possibilities down to B only, so $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$




P(A | B) = P(A ∩ B) ÷ P(B): shaded part ÷ whole of B

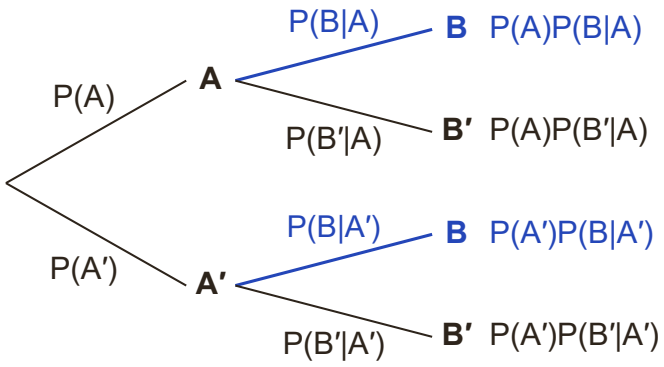
Given B, the circle B becomes the whole sample space

- With frequencies (a two-way table or Venn diagram), $P(A \mid B) = \dfrac{n(A \cap B)}{n(B)}$: divide by the total for B, not the grand total.
- In general $P(A \mid B) \neq P(B \mid A)$

## The multiplication rule and tree diagrams

$$P(A \cap B) = P(A) \times P(B \mid A)$$

- On a tree diagram the second-stage branches are conditional probabilities: the branch to B after A is $P(B \mid A)$




Second-stage branches are conditional. P(B) = sum of the blue paths

- Total probability: $P(B) = P(A)P(B \mid A) + P(A')P(B \mid A')$, adding every path that ends in B.
- Reversing a tree: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$, where the top is one path and the bottom is the total of all the paths ending in B.

## Conditional probability and independence

- A and B are independent if and only if $P(A \mid B) = P(A)$ (equivalently $P(B \mid A) = P(B)$).
- In a two-way table, compare the proportion within the given row (or column) with the overall proportion.

## Useful Venn diagram results

$$P(A \mid B') = \frac{P(A \cap B')}{P(B')}$$

- $P(A \mid A \cup B) = \dfrac{P(A)}{P(A \cup B)}$, because $A \cap (A \cup B) = A$

## How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

**‘Given that’ questions from a two-way table** 3 to 5 marks **Grade D**

1. Find the row or column for the ‘given’ event.
2. Divide the cell you want by that row or column total.
3. For independence, compare a conditional probability with the overall probability.

**Example.** The table shows how 200 students travel to school.

| | Bus | Car | Walk |
|---|---|---|---|
| **Year 12** | 40 | 25 | 35 |
| **Year 13** | 30 | 45 | 25 |

A student is chosen at random. Find

(a) $P(\text{walks} \mid \text{Year 13})$

(b) $P(\text{Year 12} \mid \text{car})$

(c) Determine whether the events ‘travels by bus’ and ‘is in Year 12’ are independent.

---

**Model answer.**

(a) $\dfrac{25}{100} = \dfrac{1}{4}$ (B1)

(b) $\dfrac{25}{25+45} = \dfrac{25}{70}$ (M1) $= \dfrac{5}{14}$ (A1)

(c) $P(\text{bus} \mid \text{Year 12}) = \dfrac{40}{100} = 0.4$ but $P(\text{bus}) = \dfrac{70}{200} = 0.35$ (M1); these are not equal, so the events are not independent (A1)

**Reverse a tree diagram**

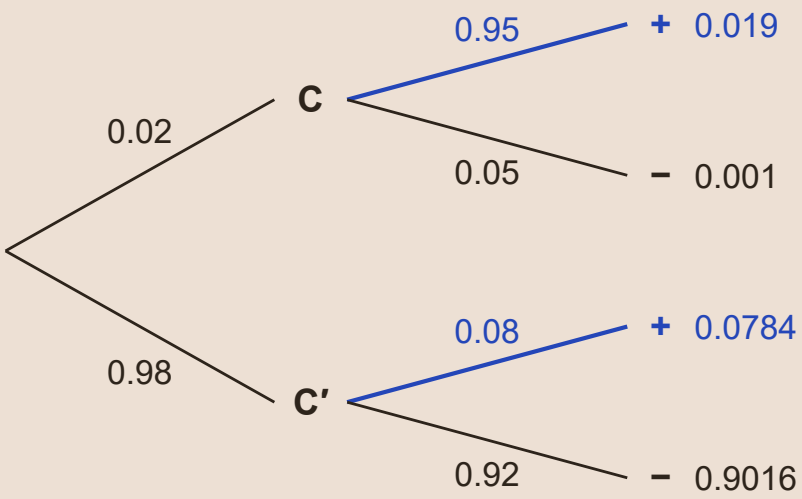
4 to 5 marks **Grade B**

1. Draw the tree in the order the information is given.
2. Find the total probability of the event you are told has happened.
3. Divide the path you want by that total.

**Example.** 2% of people in a population have a certain condition. A test gives a positive result for 95% of people with the condition and for 8% of people without it. A person is chosen at random and tested.
(a) Find the probability that the test result is positive.
(b) Given that the result is positive, find the probability that the person has the condition.

---

**Model answer.**




(a) $0.02 \times 0.95 + 0.98 \times 0.08$ (M1) $= 0.019 + 0.0784 = 0.0974$ (A1)

(b) $$P(C \mid +) = \frac{P(C \cap +)}{P(+)}$$
$$= \frac{0.019}{0.0974} \quad \text{(M1)} \quad = 0.195 \text{ (3 s.f.)} \quad \text{(A1)}$$

### Use the formulas with unknown probabilities

4 to 6 marks | **Grade A**

1. Write down what you know in set notation.
2. Use $P(A \cap B) = P(A)P(B \mid A)$ and the addition rule.
3. Draw a Venn diagram to find any other region, then use the conditional formula.

**Example.** For events A and B, $P(A) = 0.5$, $P(B \mid A) = 0.3$ and $P(A \cup B) = 0.7$. Find
(a) $P(A \cap B)$
(b) $P(B)$
(c) $P(A \mid B')$

---

**Model answer.**

(a) $0.5 \times 0.3 = 0.15$ (B1)

(b) $0.7 = 0.5 + P(B) - 0.15$ (M1), so $P(B) = 0.35$ (A1)

(c) $P(A \cap B') = 0.5 - 0.15 = 0.35$ and $P(B') = 0.65$, so $P(A \mid B') = \dfrac{0.35}{0.65}$ (M1)

$= \dfrac{7}{13}$ or 0.538 (A1)

## Don't lose marks

- Dividing by the grand total instead of the total of the given event.
- Mixing up $P(A \mid B)$ and $P(B \mid A)$
- Using $P(A \cap B) = P(A) \times P(B)$ when the events are not known to be independent.
- Missing a path when finding the total probability of the given event.

## More tips

### Memory tricks

- The 'given' event always goes on the bottom of the fraction.
- Reverse trees: find the total probability of the event you are given first, then divide.
- Sense check with test questions: when a condition is rare, a positive result can still leave the condition unlikely.

### Exam technique

- Write the formula with the events named, e.g. $P(C \mid +) = \dfrac{P(C \cap +)}{P(+)}$, before substituting: this earns the method mark.
- Draw a tree, table or Venn diagram to organise the information, even when the question doesn't ask for one.
- Give exact fractions or decimals to 3 significant figures.

## What each grade needs

What you need to be able to do, from the first marks up to the top grade.

**Grade E** — **Find conditional probabilities from a two-way table**
Divide by the total of the row or column that you are given.

**Grade D** — **Use conditional probabilities on tree diagrams**
The second-stage branches are conditional probabilities, e.g. $P(B \mid A)$

**Grade C** — **Apply the conditional probability formula**
$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$, using a Venn diagram or given values.

**Grade C** — **Show independence using conditional probability**
Check whether $P(A \mid B) = P(A)$ to decide whether A and B are independent.

**Grade B** — **Reverse a tree diagram**
Find the probability of a first-stage event given a second-stage event: one path divided by the total of all paths ending in that event.

**Grade A** — **Solve conditional problems with unknown probabilities**
Use $P(A \cap B) = P(A)P(B \mid A)$ and the addition rule to form and solve equations.

**Grade A*** — **Handle multi-step conditional problems in context**
Work through several conditions in an unfamiliar context, organising them in a tree, table or Venn diagram.

---

Free notes, Full packs, practice papers and mock exams at chhetriacademy.com. Written for this site; not taken from past papers. Stuck? 1-to-1 tutoring with a free 30-minute trial lesson.