Mutually exclusive and independent events; Venn and tree diagrams

S3.1 · Statistics · revision notes

Calculating probabilities with Venn diagrams, tree diagrams and set notation, and using the definitions of mutually exclusive and independent events. Questions often ask you to find missing probabilities in a Venn diagram or to show whether two events are independent.

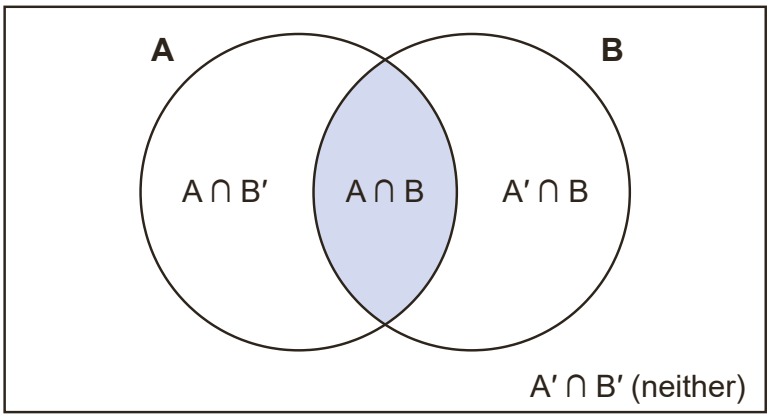
Key facts

- $P(A') = 1 - P(A)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Mutually exclusive: $P(A \cap B) = 0$
- Independent: $P(A \mid B) = P(A)$ and $P(B \mid A) = P(B)$
- $P(\text{at least one}) = 1 - P(\text{none})$
- All the regions of a Venn diagram add up to 1 (or to the total frequency)
- Independent: $P(A \cap B) = P(A) \times P(B)$

Notes

Set notation and Venn diagrams

- $A \cap B$: both A and B happen. $A \cup B$: A or B or both happen. A': A does not happen, and $P(A') = 1 - P(A)$
- In a Venn diagram the rectangle contains everything that can happen (total probability 1). Fill in the intersection first, then work outwards, then the region outside all the circles.
- $P(A \cap B') = P(A) - P(A \cap B)$ is the 'A only' region; $P(A' \cap B') = 1 - P(A \cup B)$ is 'neither'.

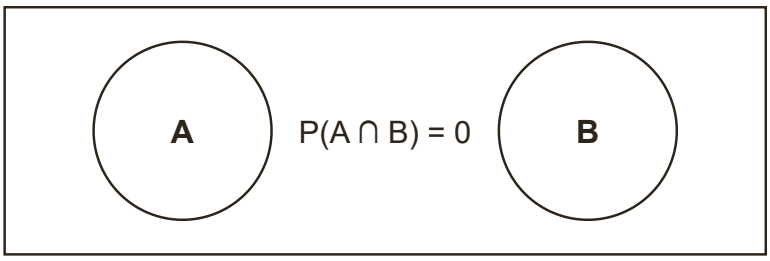


The four regions of a two-set Venn diagram; the overlap A ∩ B is shaded

- Addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Mutually exclusive events

- A and B are **mutually exclusive** if they cannot both happen: $P(A \cap B) = 0$, so their circles do not overlap.



Mutually exclusive: no overlap

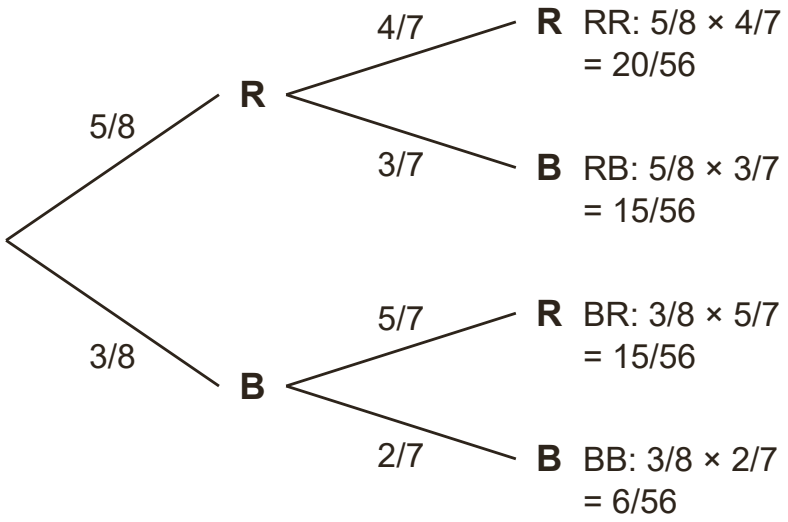
- Then $P(A \cup B) = P(A) + P(B)$

Independent events

- A and B are **independent** if one happening does not change the probability of the other:
  $P(A \cap B) = P(A) \times P(B)$
- To test for independence, work out $P(A) \times P(B)$ and compare it with $P(A \cap B)$, using exact values.
- If A and B are independent, then so are A and B', and A' and B'
- Mutually exclusive events with non-zero probabilities are never independent: if one happens, the other cannot.

Tree diagrams

- Multiply along the branches for ‘and’; add the results of different paths for ‘or’.
- The probabilities on the branches from any point add up to 1.
- Without replacement, the probabilities on the second set of branches change.



Without replacement: 5 red, 3 blue. The second-stage probabilities change

$$P(\text{at least one}) = 1 - P(\text{none})$$

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Find probabilities from a Venn diagram

3 to 4 marks | **Grade E**

1. Put the intersection in first.
2. Subtract to get the ‘only’ regions, then find the region outside the circles.
3. Add the regions you need, dividing by the total if you are using frequencies.

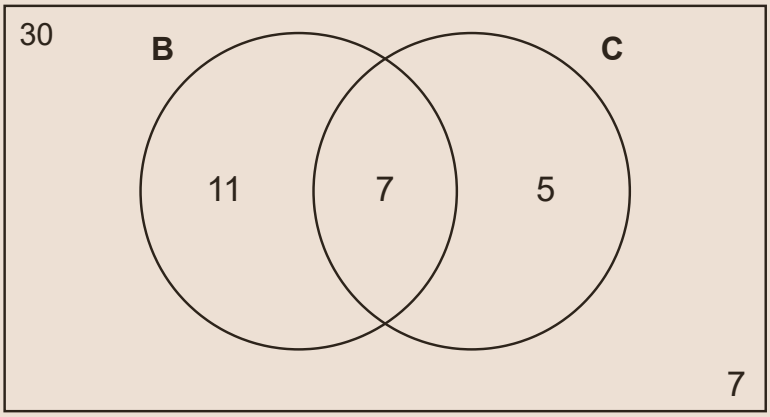
Example. In a class of 30 students, 18 study Biology (B), 12 study Chemistry (C) and 7 study both. A student is chosen at random. Find

(a) $P(B \cup C)$

(b) $P(B' \cap C)$

(c) the probability that the student studies neither subject.

Model answer.



Venn diagram: 7 in both, 11 in B only, 5 in C only, 7 outside (M1)

(a) $\dfrac{11+7+5}{30} = \dfrac{23}{30}$ (A1)

(b) $\dfrac{5}{30} = \dfrac{1}{6}$ (B1)

(c) $\dfrac{7}{30}$ (B1)

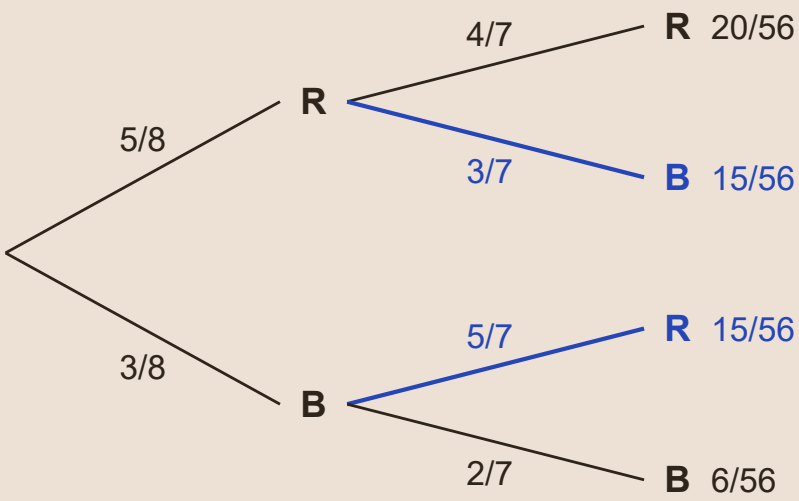
Tree diagrams, with or without replacement

4 to 5 marks | **Grade D**

1. Draw the tree with a probability on every branch.
2. Multiply along each path you need.
3. Add the paths; use $1 - P(\text{none})$ for 'at least one'.

Example. A bag contains 5 red and 3 blue counters. Two counters are taken at random, without replacement. Find the probability that
(a) the counters are different colours
(b) at least one of the counters is blue.

Model answer.



(a) $\frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{5}{7}$ (M1 M1) $= \frac{30}{56} = \frac{15}{28}$ (A1)

(b) $1 - \frac{5}{8} \times \frac{4}{7}$ (M1) $= 1 - \frac{20}{56} = \frac{9}{14}$ (A1)

Show whether two events are independent

3 to 4 marks | **Grade C**

1. Find any missing probability, e.g. $P(A \cap B)$ from the addition rule.
2. Work out $P(A) \times P(B)$
3. Compare with $P(A \cap B)$ and write a conclusion.

Example. For events A and B, $P(A) = 0.4$, $P(B) = 0.35$ and $P(A \cup B) = 0.61$.
(a) Find $P(A \cap B)$.
(b) Determine whether A and B are independent, giving a reason.

Model answer.

(a) $0.4 + 0.35 - 0.61$ (M1) $= 0.14$ (A1)

(b) $P(A) \times P(B) = 0.4 \times 0.35 = 0.14$ (M1), which equals $P(A \cap B)$, so A and B are independent (A1)

Find unknown probabilities in a Venn diagram

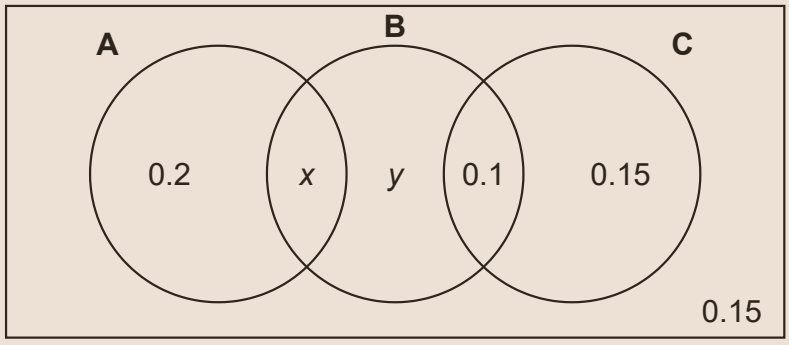
4 to 5 marks | **Grade A**

1. Label the unknown regions with letters.
2. Use the fact that all the regions add up to 1.
3. Turn each condition into an equation (mutually exclusive: no overlap; independent: the product rule).
4. Solve the equations simultaneously.

Example. Events A, B and C are such that A and C are mutually exclusive, and A and B are independent. In a Venn diagram of A, B and C, the regions have these probabilities:

Region	Probability
A only	0.2
A and B (not C)	x
B only	y
B and C (not A)	0.1
C only	0.15
Outside A, B and C	0.15

Find x and y.



Model answer.

Total: $0.2 + x + y + 0.1 + 0.15 + 0.15 = 1$ so $x + y = 0.4$ (M1)

$P(B) = x + y + 0.1 = 0.5$ and $P(A) = 0.2 + x$ (A1)

Independence: $x = 0.5(0.2 + x)$ (M1), so $0.5x = 0.1$, giving $x = 0.2$ (A1) and $y = 0.2$ (A1)

Don't lose marks

- Confusing mutually exclusive with independent.

- Writing $P(A)$ in the ‘A only’ region instead of $P(A) - P(A \cap B)$
- Not changing the second-stage probabilities when selecting without replacement.
- Deciding on independence from rounded decimals instead of exact values.
- Adding along the branches of a tree diagram instead of multiplying.

More tips

Memory tricks

- Venn diagrams: start in the middle and work outwards, then check that all the regions add up to 1.
- Independence test: multiply, then compare. Equal means independent.
- For ‘or’ questions, add the separate regions of the Venn diagram rather than whole circles, so you never count the overlap twice.
- Mutually exclusive means ‘can't both happen’; independent means ‘doesn't affect the other’.

Exam technique

- To ‘determine’ or ‘show’ independence, write the product, its value, the comparison and a conclusion.
- Draw and label a Venn or tree diagram even when you are not asked to: it organises the information and can earn method marks.
- Exact fractions are fine as final answers unless the question asks for a decimal.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade E — **Use set notation for events**
Read and write $A \cap B$ (A and B), $A \cup B$ (A or B or both) and A' (not A).

Grade E — **Complete and use a Venn diagram**
Fill it in from the intersection outwards, using probabilities or frequencies, and read off probabilities.

Grade D — **Use tree diagrams for successive events**
Multiply along the branches and add the end results, including for selections without replacement.

Grade C — **Use the addition rule for probabilities**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Grade C — **Show whether two events are independent**
Compare $P(A \cap B)$ with $P(A) \times P(B)$

Grade B — **Use mutually exclusive and independent conditions**
Use $P(A \cap B) = 0$ or $P(A \cap B) = P(A)P(B)$ to find missing probabilities.

Grade A — **Find unknown probabilities by forming equations**
Set up and solve equations from the total probability of 1 and the conditions given.

Grade A* — **Solve three-event Venn problems with conditions**
Combine independence, mutual exclusivity and the total of 1 in a three-circle Venn diagram.

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