Trigonometry in context

P5.9 · Pure mathematics · revision notes

Trigonometry is used to solve problems in context: resolving forces and velocities into components, finding the magnitude and direction of a vector, bearings with the sine and cosine rules, and modelling periodic situations such as tides and temperatures. These questions appear on the pure papers and in the mechanics section of Paper 3.

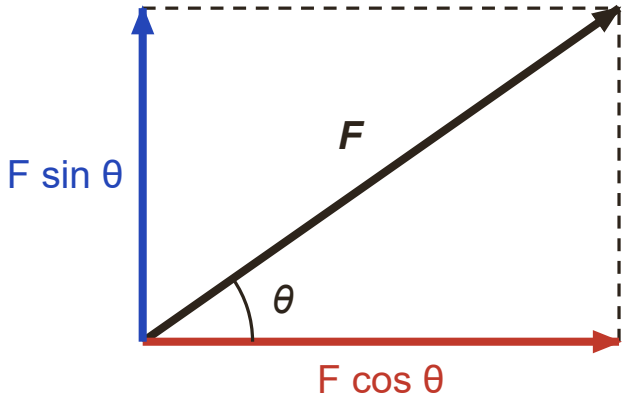
Key facts

- Components of F at angle θ to a direction: $F\cos\theta$ along that direction, $F\sin\theta$ perpendicular to it
- $\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5}, \cos\alpha = \frac{4}{5}$
- $y = a + b\sin(ct + d)$: max $a + |b|$, min $a - |b|$, period $\frac{360}{c}$ in the units of t
- $|a\mathbf{i} + b\mathbf{j}| = \sqrt{a^2 + b^2}$
- On a slope: $mg\sin\alpha$ down the slope, $mg\cos\alpha$ perpendicular to it
- Bearings: clockwise from north, three figures

Notes

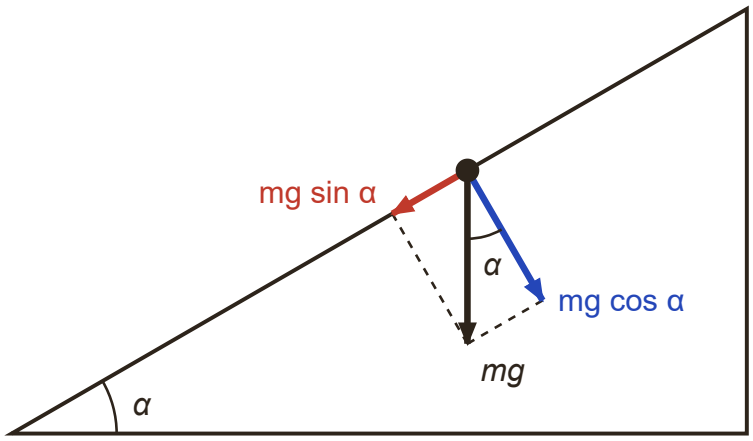
Vectors, forces and motion

- A vector of magnitude F at angle θ to the horizontal has horizontal component $F\cos\theta$ and vertical component $F\sin\theta$



The component next to the angle uses cos; the one opposite it uses sin.

- The magnitude of $a\mathbf{i}+b\mathbf{j}$ is $\sqrt{a^2+b^2}$, and the angle θ it makes with i satisfies $\tan\theta=\frac{b}{a}$. Sketch it to see which quadrant it is in.
- On a slope at angle α to the horizontal, the weight mg has a component $mg\sin\alpha$ down the slope and a component $mg\cos\alpha$ perpendicular to the slope.



The angle between mg and the perpendicular to the slope is α, so mg sin α acts down the slope.

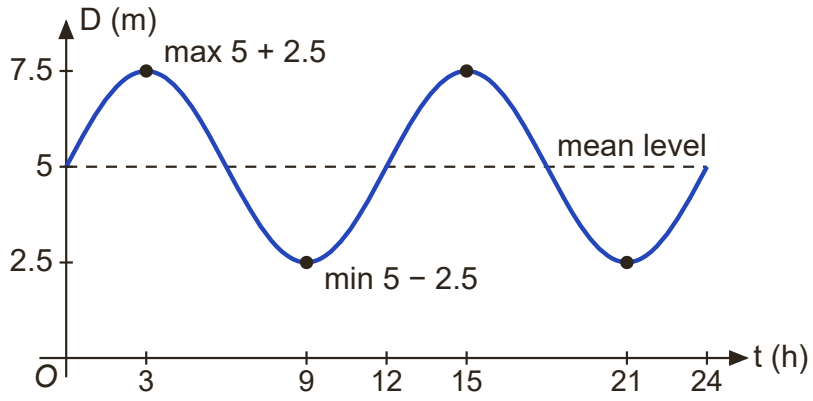
- A projectile launched at speed U at angle θ above the horizontal has initial velocity components $U\cos\theta$ horizontally and $U\sin\theta$ vertically.
- If you are given $\tan\alpha=\frac{3}{4}$, use the 3, 4, 5 triangle: $\sin\alpha=\frac{3}{5}$ and $\cos\alpha=\frac{4}{5}$

Bearings and triangles

- Bearings are measured clockwise from north and written with three figures, e.g. 060°.
- Draw a north line at every point. Use parallel lines (co-interior angles add up to 180°) or back bearings (add or subtract 180°) to find angles inside the triangle.
- Then use the sine rule, the cosine rule or $\frac{1}{2}ab\sin C$ as in any triangle.

Periodic models

- Situations that repeat, such as tides, daily temperature or the height of a seat on a big wheel, can be modelled by $y = a + b\sin(ct + d)$ or a cos version.
- a is the mean level and |b| is the amplitude, so the maximum is $a + |b|$ and the minimum is $a - |b|$. The period is $\frac{360}{c}$ (degrees) or $\frac{2\pi}{c}$ (radians), in the units of t.



D = 5 + 2.5 sin(30t)°: mean 5, amplitude 2.5, period 360 ÷ 30 = 12 hours.

- When b > 0, the maximum happens when $\sin(ct + d) = 1$: set $ct + d$ equal to 90° (or 90° + 360°, …) and solve for t, keeping the values in the interval.
- To find when y takes a value, solve the trig equation for every t in the interval, then convert to times (e.g. 0.25 hours = 15 minutes).
- If a model contains $p\cos t + q\sin t$, write it in R form first.
- Limitations: real data don't repeat exactly, the amplitude or period may change over time (e.g. with the seasons), and the model may give impossible values outside the range it was built for.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own.

Find the speed and bearing from a velocity vector

4 marks **Grade D**

1. Magnitude: the square root of the sum of the squares of the components.
2. Sketch the vector to see its direction.
3. Find an angle in the right-angled triangle, then convert it to a bearing (clockwise from north).

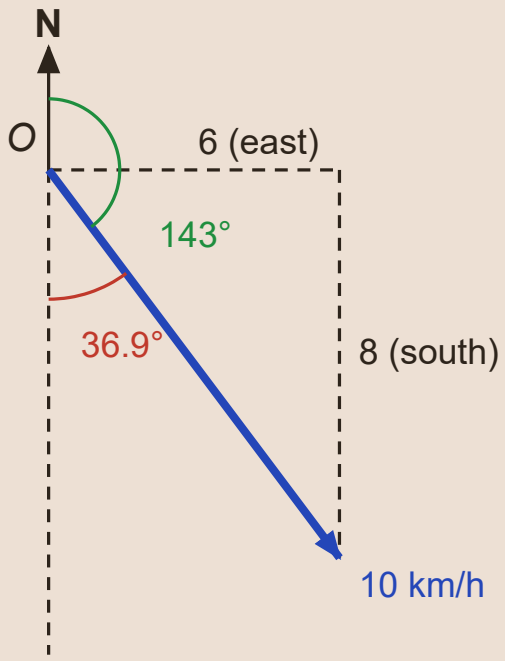
Example. A boat moves with constant velocity $(6\mathbf{i} - 8\mathbf{j})$ km h^{-1}, where i and j are unit vectors due east and due north. Find the speed of the boat and the bearing on which it moves.

Model answer.

Speed $= \sqrt{6^2 + (-8)^2} = 10$ km h^{-1} **M1 A1**

The boat moves east and south; its angle from due south is $\tan^{-1}\frac{6}{8} = 36.87°$ **M1**

Bearing $= 180° - 36.87°$
$= 143°$ (to the nearest degree) **A1**



Bearings with the cosine and sine rules

5 marks **Grade C**

1. Draw a diagram with a north line at each point.
2. Use back bearings to find the angle inside the triangle.
3. Use the cosine rule for the distance and the sine rule for an angle.
4. Add or subtract that angle from a known bearing.

Example. A ship sails 8 km from port P on a bearing of 060° to point A. It then sails 5 km on a bearing of 170° to point B. Find the distance PB and the bearing of B from P.

Model answer.

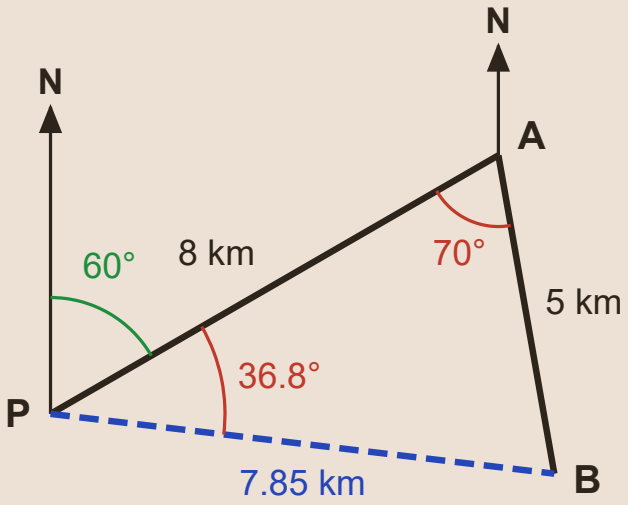
The bearing of P from A is 240°, so angle PAB = 240° − 170° = 70° **B1**

$$PB^2 = 8^2 + 5^2 - 2 \times 8 \times 5 \cos 70°$$
$$= 61.64\ldots$$

M1, so PB = 7.85 km **A1**

$$\frac{\sin \angle APB}{5} = \frac{\sin 70°}{7.851\ldots}$$, so angle APB = 36.8° **M1**

Bearing of B from P = 060° + 36.8° = 097° (to the nearest degree) **A1**



Use a periodic model: maximum and times

7 marks **Grade A**

1. Maximum = mean level + amplitude, when the sin part equals 1.
2. Set the model equal to the value and make sin the subject.
3. Find all the angles in the interval (here $0 \leq 30t < 360$ for the first 12 hours), then divide by 30.
4. Convert decimal hours into hours and minutes: multiply the decimal part by 60.

Example. The depth of water, D metres, in a harbour is modelled by $D = 5 + 2.5\sin(30t)°$, where t is the number of hours after midnight, $0 \leq t < 24$.
(a) State the maximum depth and find the first time at which it occurs.
(b) Find the times between midnight and midday when the depth is 6 m, to the nearest minute.

Model answer.

(a) Maximum = 5 + 2.5 = 7.5 m	**B1**
$30t = 90 \Rightarrow t = 3$, so at 03:00	**M1 A1**
(b) $\sin(30t)° = 0.4$	**M1**
$30t = 23.58$ or $180 - 23.58 = 156.42$	**A1**
t = 0.786 or 5.214 hours	**M1**
00:47 and 05:13	**A1**

Analyse a model using the R form

7 marks | Grade A*

1. Expand $R\cos(15t + \alpha)$ and compare coefficients to find R and α.
2. The maximum of $R\cos(\ldots)$ is R, so the maximum of the model is 12 + R.
3. Set the angle equal to 0° or 360°, whichever gives t in the interval, and solve.
4. Convert the time into hours and minutes and answer in context.

Example. The temperature, T °C, in a greenhouse t hours after midnight is modelled by $T = 12 + 3\cos(15t)° - 4\sin(15t)°$, $0 \leq t < 24$.

(a) Write $3\cos(15t)° - 4\sin(15t)°$ in the form $R\cos(15t + \alpha)°$, where R > 0 and $0 < \alpha < 90$.

(b) Find the maximum temperature predicted by the model and the time of day, to the nearest minute, at which it occurs.

(c) Give one reason why the model may not give accurate predictions.

Model answer.

(a) $R\cos\alpha = 3$ and $R\sin\alpha = 4$, so R = 5 **B1**

$\tan\alpha = \frac{4}{3}$, so α = 53.13 **M1 A1**

(b) Maximum T = 12 + 5 = 17 °C **B1**

This happens when $15t + 53.13 = 360$ **M1**, so t = 20.458, which is 20:27 **A1**

(c) The model predicts the same temperatures every day, but real temperatures change with the weather and the seasons. **B1**

Don't lose marks

- Swapping sin and cos when resolving, e.g. using $mg\cos\alpha$ for the component down a slope.
- Using radian mode for a model written in degrees, or the other way round.
- Converting 0.46 hours to 46 minutes; it is about 28 minutes.
- Giving a bearing measured from east, or without three figures.
- Giving only the first time when the question asks for all the times in the interval.

More tips

Memory tricks

- Resolving: the component along the direction the angle is measured from uses cos; the component at right angles to it uses sin.
- For a periodic model, write down the mean level, the amplitude and the period before you start solving.
- 0.1 hours = 6 minutes, so 0.786 hours is about 47 minutes.
- Sense check model answers: a depth or height must lie between the minimum and maximum of the model.

Exam technique

- Draw and label a diagram for every vector, force or bearing question.
- Answer in context with units and times of day, e.g. '7.5 m at 03:00'.
- For a limitation, refer to the real situation (weather, seasons, friction) rather than making a general comment.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade E

Resolve a vector into components

A force F at angle θ to the horizontal has components $F\cos\theta$ (horizontal) and $F\sin\theta$ (vertical).

Grade D

Find the magnitude and direction of a vector

Use $\sqrt{a^2+b^2}$ and $\tan\theta = \frac{b}{a}$, with a sketch to get the direction right.

Grade C

Solve bearings problems using sine and cosine rules

Draw north lines, find the angle inside the triangle, then use the right rule.

Grade B

Interpret the parameters of a trig model

In $y = a + b\sin(ct)$ with b > 0, a is the mean level, b is the amplitude and $\frac{360}{c}$ is the period (in the units of t).

Grade A

Solve a trig model for given values

Set the model equal to the value, solve the trig equation in the interval and interpret the answers as times.

Grade A*

Use the R form to analyse a model

Combine $p\cos t + q\sin t$ into one term to find the maximum, the minimum and when they occur.

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