sec, cosec, cot and inverse trig functions

P5.4 · Pure mathematics · revision notes

Secant, cosecant and cotangent are the reciprocals of cos, sin and tan, and arcsin, arccos and arctan are the inverse functions of sin, cos and tan on restricted domains. You need their definitions, graphs, domains and ranges, and you use them to find exact values and to solve equations.

Key facts

- $\sec x = \dfrac{1}{\cos x}$
- $\operatorname{cosec} x = \dfrac{1}{\sin x}$
- $\cot x = \dfrac{1}{\tan x} = \dfrac{\cos x}{\sin x}$
- $|\sec x| \geq 1$ and $|\operatorname{cosec} x| \geq 1$
- arcsin: domain $[-1, 1]$, range $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$
- arccos: domain $[-1, 1]$, range $[0, \pi]$
- arctan: domain all real numbers, range $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

Notes

Reciprocal functions

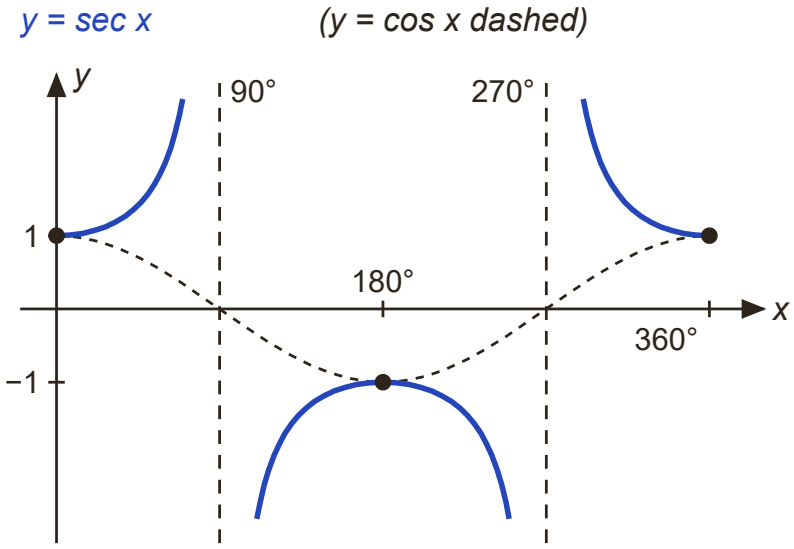
- $\sec x = \dfrac{1}{\cos x}$, $\operatorname{cosec} x = \dfrac{1}{\sin x}$ and

$$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

- Memory aid: look at the third letter. se**c** goes with **c**os, co**s**ec with **s**in, co**t** with **t**an.
- Each is undefined where its denominator is zero: sec x where $\cos x = 0$ (x = 90°, 270°, …), and cosec x and cot x where $\sin x = 0$ (x = 0°, 180°, …).
- Since $|\sin x| \leq 1$ and $|\cos x| \leq 1$, it follows that $|\sec x| \geq 1$ and $|\operatorname{cosec} x| \geq 1$

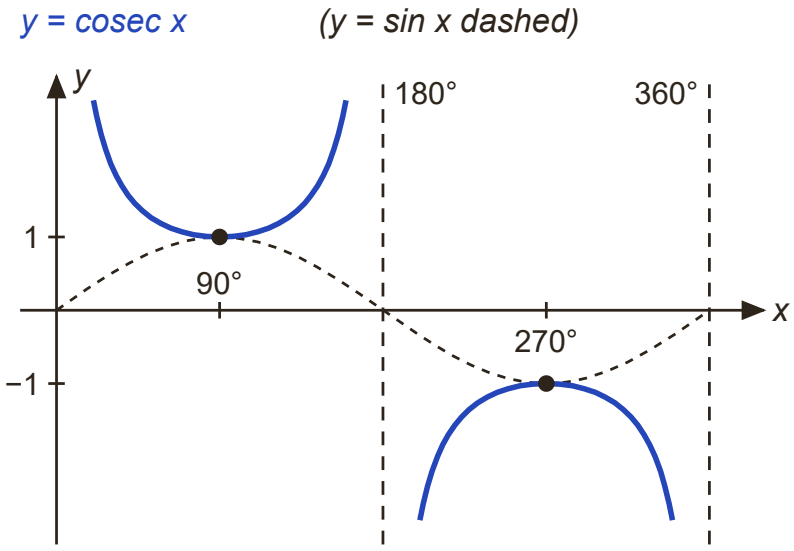
Their graphs

- $y = \sec x$: period 360°; asymptotes at x = 90°, 270°, …; minimum points at (0, 1) and (360°, 1); maximum point at (180°, −1).



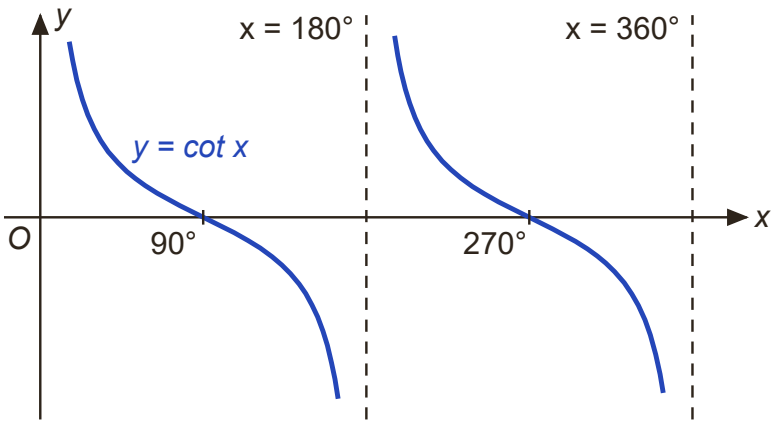
Where cos x is 1 or −1, sec x is also 1 or −1; where cos x = 0, sec x has an asymptote.

- $y = \operatorname{cosec} x$: period 360°; asymptotes at x = 0°, 180°, 360°, …; minimum at (90°, 1); maximum at (270°, −1).



cosec x has its minimum 1 where sin x = 1 and its maximum −1 where sin x = −1; asymptotes where sin x = 0.

- Both have range $y \leq -1$ or $y \geq 1$. Where cos or sin has a maximum of 1, its reciprocal has a minimum of 1.
- $y = \cot x$: period 180°; asymptotes at x = 0°, 180°, …; crosses the x-axis at 90°, 270°, …; each branch is decreasing.

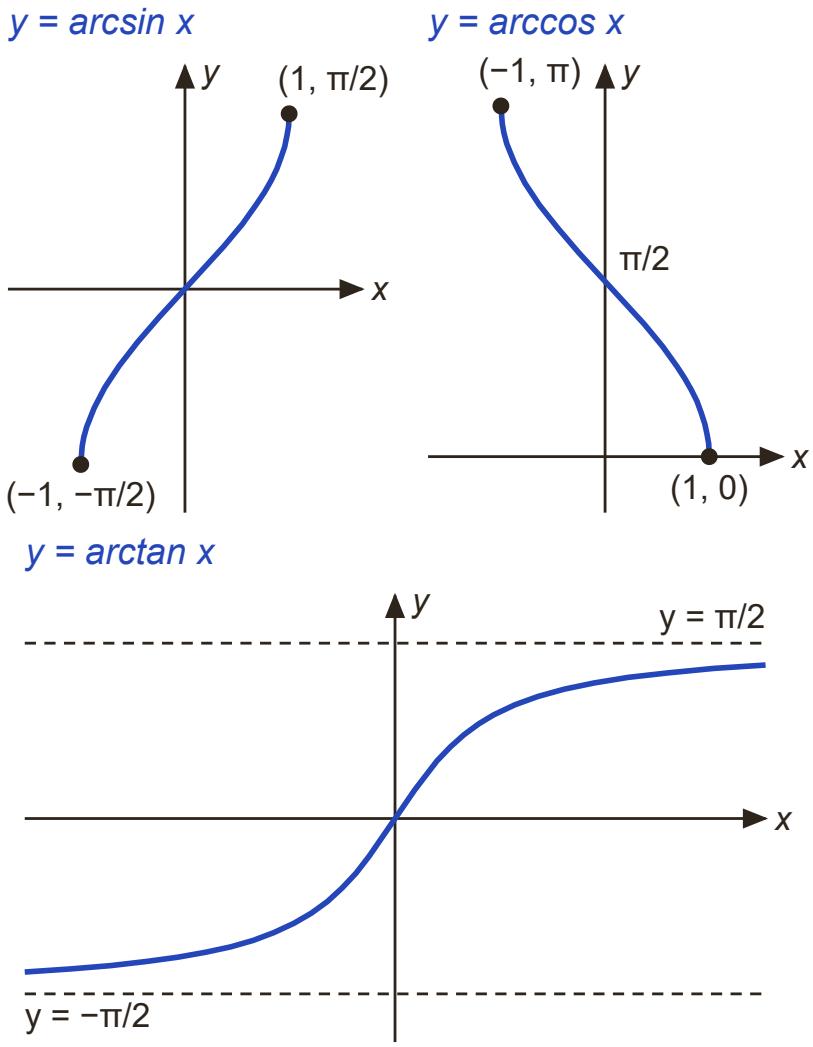


cot x repeats every 180°: asymptotes where tan x = 0, and zero where tan x has an asymptote.

- Sketch cos (or sin) lightly first, then draw the reciprocal graph from it.

Inverse trig functions

- sin, cos and tan are many-to-one, so their domains are restricted to make them one-to-one before they are inverted.
- $y = \arcsin x$: domain $-1 \leq x \leq 1$, range $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
- $y = \arccos x$: domain $-1 \leq x \leq 1$, range $0 \leq y \leq \pi$
- $y = \arctan x$: domain all real x, range $-\frac{\pi}{2} < y < \frac{\pi}{2}$, with horizontal asymptotes $y = \pm\frac{\pi}{2}$
- Each graph is the reflection of the restricted sin, cos or tan graph in the line y = x.



Each graph is the restricted sin, cos or tan graph reflected in y = x.

- $\sin^{-1} x$ means arcsin x. It does not mean $\frac{1}{\sin x}$, which is cosec x.

How to answer each type of question

M = method mark, A = answer mark, B = a mark on its own, A1* = the solution must be fully correct, with every step shown.

Find exact values 1 mark each **Grade C**

1. Rewrite each reciprocal function as 1 over sin, cos or tan.
2. Use the exact values for $\frac{\pi}{6}$, $\frac{\pi}{4}$ and $\frac{\pi}{3}$, with the sign from the quadrant.
3. For an inverse function, make sure your answer is in its range.

Example. Find the exact value of

(a) $\operatorname{cosec} \frac{\pi}{6}$

(b) $\sec \frac{3\pi}{4}$

(c) $\cot \frac{\pi}{3}$

(d) $\arccos\left(-\frac{1}{2}\right)$

Model answer.

(a) $\frac{1}{\sin \frac{\pi}{6}}$

$= \frac{1}{\frac{1}{2}}$

$= 2$ **B1**

(b) $\cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$, so $\sec \frac{3\pi}{4} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$ **B1**

(c) $\frac{1}{\tan \frac{\pi}{3}}$

$= \frac{1}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3}$ **B1**

(d) $\frac{2\pi}{3}$ (arccos always gives an angle from 0 to π) **B1**

Solve an equation involving sec, cosec or cot

4 marks **Grade B**

1. Change sec, cosec or cot into cos, sin or tan.
2. Adjust the interval if there is a multiple angle.
3. Find all the solutions in the adjusted interval, then convert back.

Example. Solve $\sec 2x = -2$ for $0 \leq x \leq \pi$, giving your answers as exact multiples of π.

Model answer.

$\cos 2x = -\frac{1}{2}$ **M1**

For $0 \leq 2x \leq 2\pi$: $2x = \frac{2\pi}{3}$ **A1** or $2x = 2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$ **M1**

$x = \frac{\pi}{3}, \frac{2\pi}{3}$ **A1**

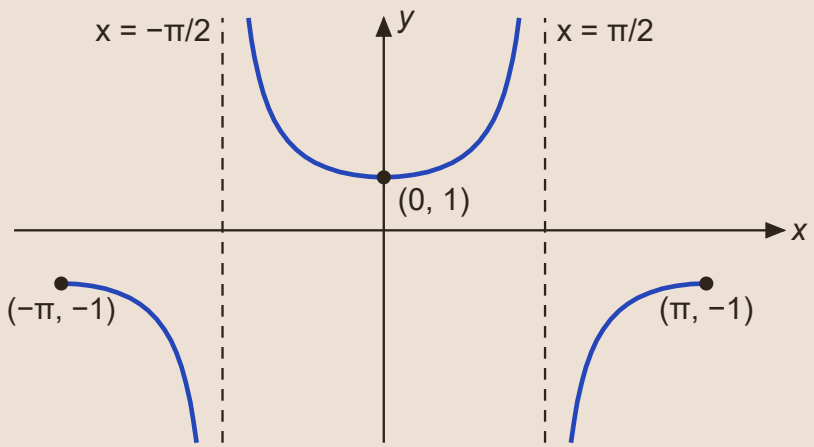
Sketch a reciprocal trig graph

3 to 4 marks **Grade B**

1. Lightly sketch $y = \cos x$ (or $y = \sin x$) first.
2. Draw a vertical asymptote wherever cos x (or sin x) is zero.
3. Where cos x is 1 or −1, sec x is 1 or −1 too: these points are the turning points.
4. As cos x gets close to 0, sec x heads towards +∞ or −∞, with the same sign as cos x.

Example. Sketch the graph of $y = \sec x$ for $-\pi \leq x \leq \pi$. Show the equations of any asymptotes and the coordinates of any turning points and end points.

Model answer.



A U-shaped branch above the x-axis between the asymptotes, and a branch below the x-axis at each end, heading down towards the asymptotes **B1**

Asymptotes $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ **B1**

Minimum point (0, 1) **B1**

End points $(-\pi, -1)$ and $(\pi, -1)$ **B1**

Work with an inverse trig function

4 marks **Grade A**

1. Rewrite $y = \arcsin x$ as $\sin y = x$, noting the range of y.
2. Use an identity (or a right-angled triangle) to find another ratio.
3. Use the range of y to choose the sign of any square root.

Example. Given that $y = \arcsin x$, where $-1 < x < 1$:
(a) show that $\cos y = \sqrt{1 - x^2}$
(b) hence write $\tan(\arcsin x)$ in terms of x.

Model answer.

(a) $\sin y = x$ **M1**

$\cos^2 y = 1 - \sin^2 y = 1 - x^2$ **M1**

Since $-\frac{\pi}{2} < y < \frac{\pi}{2}$, $\cos y > 0$, so $\cos y = \sqrt{1 - x^2}$ **A1***

(b) $\tan y = \frac{\sin y}{\cos y}$

$= \frac{x}{\sqrt{1 - x^2}}$ **B1**

Don't lose marks

- Mixing up sec and cosec, e.g. writing $\sec x = \frac{1}{\sin x}$
- Thinking that $\sin^{-1} x = \frac{1}{\sin x}$
- Giving $\arccos\left(-\frac{1}{2}\right) = -\frac{\pi}{3}$; arccos is never negative.
- Using $\leq$ instead of $<$ in the range of arctan (it never reaches $\pm\frac{\pi}{2}$).
- Trying to solve $\sec x = \frac{1}{2}$: it has no solutions, because $|\sec x| \geq 1$

More tips

Memory tricks

- Third-letter rule: sec ↔ cos, cosec ↔ sin, cot ↔ tan.
- Your calculator has no sec button: type 1 ÷ cos(…).
- arcsin and arctan give answers between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$; arccos gives answers from 0 to π. An answer outside these is wrong.
- Wherever the original graph crosses the x-axis, its reciprocal graph has an asymptote.

Exam technique

- On sketches, label asymptotes with their equations and turning points with their coordinates.
- State domains and ranges with the correct inequality signs ($\leq$ or $<$).
- Give exact values when asked. You only need to rationalise $\frac{1}{\sqrt{3}}$ to $\frac{\sqrt{3}}{3}$ if the question asks for a particular form.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade C

Define sec, cosec and cot

$\sec x = \frac{1}{\cos x}$, $\operatorname{cosec} x = \frac{1}{\sin x}$ and

$$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

Grade C

Find exact values of reciprocal and inverse functions

e.g. $\sec \frac{\pi}{3} = 2$ and $\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

Grade B

Sketch the graphs of sec, cosec and cot

The asymptotes are where the function you take the reciprocal of is zero; where cos or sin is 1 or −1, so is its reciprocal.

Grade B

State domains and ranges of inverse functions

arcsin: domain $[-1, 1]$, range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$; arccos: domain $[-1, 1]$, range $[0, \pi]$; arctan: domain all real numbers, range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Grade B

Solve equations in sec, cosec and cot

Turn them into cos, sin or tan, e.g. $\sec 2x = -2$ becomes $\cos 2x = -\frac{1}{2}$

Grade A

Simplify expressions involving inverse trig functions

e.g. show that $\tan(\arcsin x) = \frac{x}{\sqrt{1 - x^2}}$, using an identity or a right-angled triangle.

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