Graphs of sin, cos and tan

P5.3 · Pure mathematics · revision notes

You need to know the shapes of $y = \sin x$, $y = \cos x$ and $y = \tan x$ in degrees and in radians, their symmetries and periods, and how transformations change them. Sketches, related angles and the maximum or minimum of a transformed graph all appear in the exam.

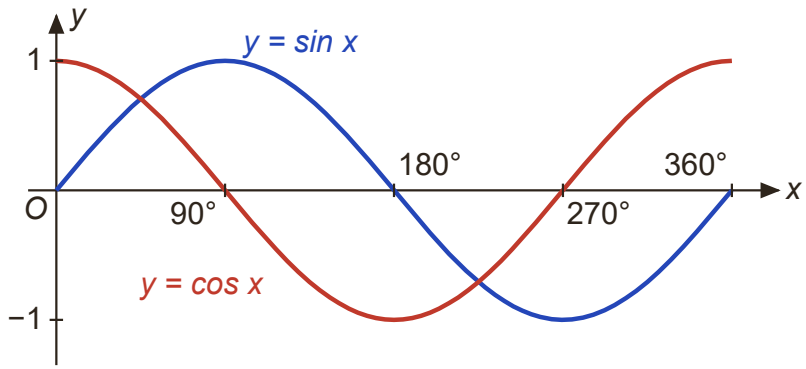
Key facts

- sin and cos: period 360° = 2π, range $-1 \le y \le 1$
- $\sin(180° - x) = \sin x$
- $\sin(90° - x) = \cos x$
- tan: period 180° = π, asymptotes at $x = 90° + 180°n$
- $\cos(-x) = \cos x$; $\sin(-x) = -\sin x$
- $y = a\sin(bx + c) + d$ (b > 0): max $d + |a|$, min $d - |a|$, period $\dfrac{360°}{b}$
- CAST: positive ratios in quadrant 1 (all), 2 (sin), 3 (tan), 4 (cos)

Notes

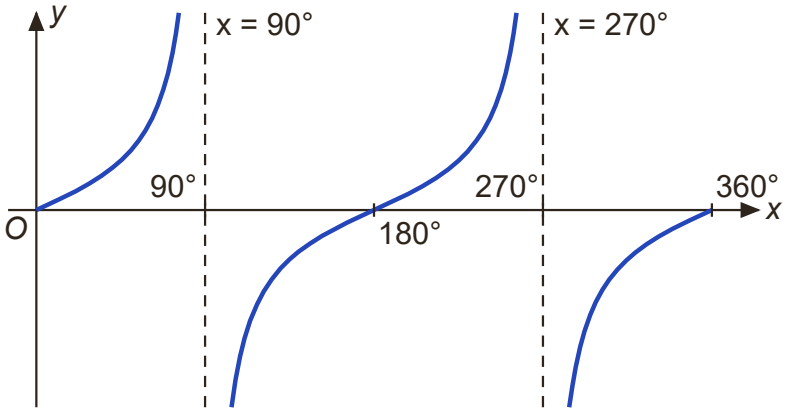
The three graphs

- $y = \sin x$: starts at (0, 0), has a maximum of 1 at 90° $\left(\frac{\pi}{2}\right)$, is back to 0 at 180° (π) and has a minimum of −1 at 270° $\left(\frac{3\pi}{2}\right)$. Period 360° (2π).
- $y = \cos x$: starts at (0, 1), is 0 at 90°, −1 at 180°, 0 at 270° and 1 at 360°. Period 360° (2π).



sin starts at 0, cos starts at 1; both repeat every 360° and stay between −1 and 1.

- Both have range $-1 \leq y \leq 1$. The cos graph is the sin graph translated 90° to the left.
- $y = \tan x$: passes through (0, 0) and (180°, 0) and has vertical asymptotes at x = 90°, 270°, … $\left(x = \frac{\pi}{2} + n\pi\right)$. Period 180° (π). It takes every real value.

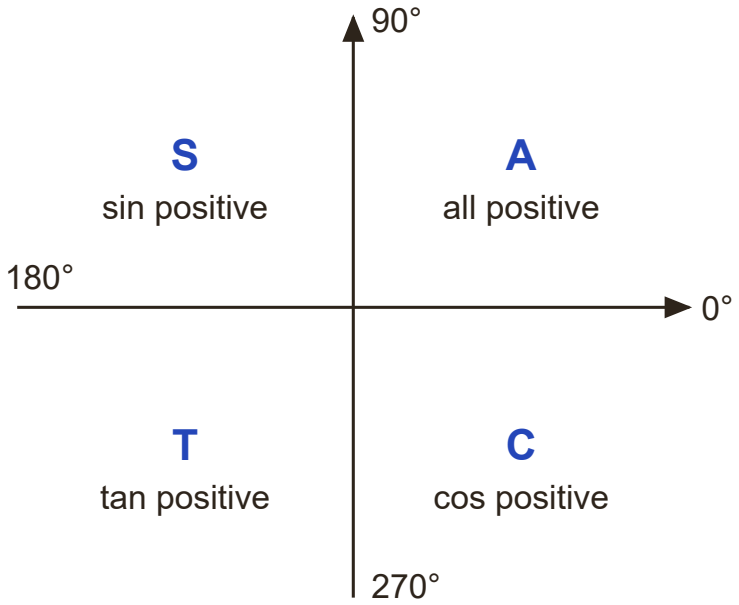


tan x repeats every 180° and has asymptotes where cos x = 0.

- Know these graphs in radians as well as in degrees, and for negative x.

Symmetries

- $\sin(180° - x) = \sin x$ and $\sin(-x) = -\sin x$
- $\cos(-x) = \cos x$ (the graph is symmetrical in the y-axis), $\cos(360° - x) = \cos x$ and $\cos(180° - x) = -\cos x$
- $\tan(x + 180°) = \tan x$ and $\tan(-x) = -\tan x$
- $\sin(90° - x) = \cos x$ and $\cos(90° - x) = \sin x$
- Signs: all three ratios are positive between 0° and 90°, only sin between 90° and 180°, only tan between 180° and 270°, and only cos between 270° and 360° (the CAST diagram).



CAST: which ratios are positive in each quadrant, counting anticlockwise from 0°.

Transformations

- $y = a\sin x$: vertical stretch with scale factor a. The amplitude is |a|.
- $y = \sin bx$: horizontal stretch with scale factor $\frac{1}{b}$. The period becomes $\frac{360°}{b}$ (or $\frac{2\pi}{b}$); for tan it becomes $\frac{180°}{b}$
- $y = \sin(x + c)$: translation c units to the **left**. $y = \sin x + d$: translation d units up.
- To find the key points of $y = \sin(bx + c)$, set $bx + c$ equal to 0°, 90°, 180°, … and solve for x.

How to answer each type of question

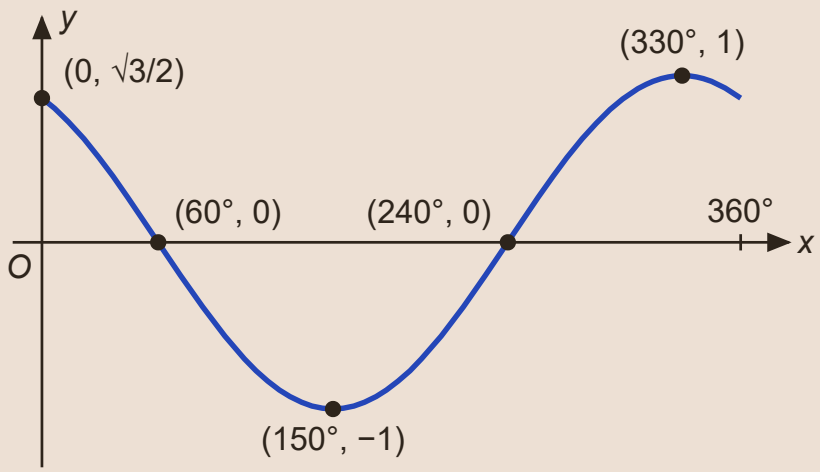
M = method mark, A = answer mark, B = a mark on its own.

Sketch a transformed graph and give where it meets the axes 3 marks **Grade C**

1. Start from the basic graph and apply the transformation (+c inside the bracket moves the graph left).
2. Find the y-intercept by putting x = 0.
3. Find the x-intercepts by setting the bracket equal to 90°, 270°, … and solving.
4. Only draw, and only list points, inside the given interval.

Example. Sketch the graph of $y = \cos(x + 30°)$ for $0 \le x \le 360°$. Give the coordinates of the points where the graph meets the coordinate axes.

Model answer.



A cosine curve translated 30° to the left: it starts at $\left(0, \frac{\sqrt{3}}{2}\right)$, has a minimum at (150°, −1) and a maximum at (330°, 1) **B1**

y-intercept: $\cos 30° = \frac{\sqrt{3}}{2}$, so $\left(0, \frac{\sqrt{3}}{2}\right)$ **B1**

x-intercepts: $x + 30° = 90°$ or $270°$, so (60°, 0) and (240°, 0) **B1**

State the period and maximum of a transformed graph

4 marks | Grade B

1. The period of $\sin bx$ or $\cos bx$ is $\dfrac{2\pi}{b}$
2. The maximum of sin is 1, so the maximum of $a\sin(\ldots)$ is |a|, plus any vertical translation.
3. Set the inside of the bracket equal to $\dfrac{\pi}{2}$ (where sin is 1) and solve for x.
4. Check that x is in the given interval.

Example. The curve C has equation $y = 3\sin\left(2x - \dfrac{\pi}{3}\right)$, $0 \le x \le \pi$.

(a) State the period of the curve.

(b) State the maximum value of y and find the exact value of x at which it occurs.

Model answer.

(a) Period $= \dfrac{2\pi}{2} = \pi$ **B1**

(b) Maximum y = 3 **B1**

$2x - \dfrac{\pi}{3} = \dfrac{\pi}{2}$ **M1**

$x = \dfrac{5\pi}{12}$ **A1**

Use symmetry to write related values

3 marks | Grade C

1. Sketch the graph or use the CAST diagram to see whether the new angle gives the same sign or the opposite sign.
2. Related angles give the same size of ratio.
3. Decide the sign from the quadrant.

Example. Given that $\cos x = p$, where $0° < x < 90°$, write each of the following in terms of p.

(a) $\cos(180° - x)$

(b) $\sin(90° + x)$

(c) $\cos(x + 180°)$

Model answer.

(a) $-p$ **B1**

(b) $\sin(90° + x) = \cos x = p$ **B1**

(c) $-p$ **B1**

Find constants from the features of a graph

3 marks **Grade A**

1. Maximum = C + A and minimum = C − A: solve these simultaneously.
2. Use where the maximum (or minimum) occurs to find the horizontal translation.
3. Check that your values give the stated point.

Example. The curve $y = A\cos(x - B) + C$, where A > 0 and $0° < B < 180°$, has a maximum point at (40°, 7) and a minimum value of −1. Find the values of A, B and C.

Model answer.

$C + A = 7$ and $C - A = -1$	**M1**
A = 4, C = 3	**A1**
cos has its maximum when $x - B = 0$, so B = 40°	**B1**

Don't lose marks

- Translating the wrong way: $y = \sin(x + 30°)$ moves left, not right.
- Thinking $y = \sin 2x$ has period 720°; it is 180°, because the graph is squashed.
- Giving tan a period of 360°, or drawing its asymptotes in the wrong places.
- Drawing curves with pointed peaks, or not reaching 1 and −1.
- Mixing degrees and radians on the same axis.

More tips

Memory tricks

- The sin and cos graphs have a key point every quarter period: mark these on the x-axis first.
- Inside the bracket, '+' moves the graph left and '−' moves it right.
- Use your calculator's table function to check points on a sketch.
- Symmetry: the sin graph is symmetrical about x = 90°, and the cos graph about x = 0° and x = 180°.

Exam technique

- Label every intercept, turning point and asymptote with coordinates or equations.
- Stick to the interval given in the question.
- Give exact values such as $\frac{\sqrt{3}}{2}$ and $\frac{5\pi}{12}$ when the question asks for exact answers.

What each grade needs

What you need to be able to do, from the first marks up to the top grade.

Grade E **Sketch the sin, cos and tan graphs**
Mark the key points: where they cross the axes, the maximum and minimum values, and the asymptotes of tan.

Grade D

State the period of each graph

sin and cos repeat every 360° (2π); tan repeats every 180° (π).

Grade C

Use symmetry to find related angles

e.g. $\sin(180° - x) = \sin x$, $\cos(-x) = \cos x$ and $\tan(x + 180°) = \tan x$

Grade C

Sketch translated and stretched trig graphs

e.g. $y = \cos(x + 30°)$ is $y = \cos x$ translated 30° to the left.

Grade B

Find maximum, minimum and period after transformations

For $y = a\sin(bx + c) + d$ with b > 0: maximum $d + |a|$, minimum $d - |a|$, period $\frac{360°}{b}$

Grade A

Find constants from a graph's features

Use the maximum, the minimum and the position of a turning point to find the unknown constants.

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